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Improved Statistical Interconnect Timing Analysis Considering Scattering Effect*

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Abstract: We propose an improved statistical approach for modeling interconnect slew that takes into account the scattering effect of a nanoscale wire. We first propose a simple, closed-form scattering effect resistivity model, considering the effects of both width and thickness. Then we use this model to derive statistical expressions of the slew metrics using the SS2M model. We find that the delay and slew can be greatly increased when considering the scattering effect. The proposed statistical SS2M model has an average error of 4.16% with respect to SPICE Monte Carlo simulations, with an average error of standard deviation of only 3.06%.

Key words: interconnect; scattering effect; timing analysis
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1 Introduction

As the feature size of CMOS technology continues to scale down, the lateral dimension of conductors is entering the deep nanometer regime, in which wire diameter is in the range of or smaller than the mean free path of an electron. In this regime, due to the scattering effect, including surface scattering and grain boundary scattering, the effective electrical resistivity of metallic conductors is higher than that of bulk metal[1,2]. Until recently, few experimental results on copper (Cu) film or interconnect have been reported. The size effect of copper thin film was studied in Ref. [3], and the resistivity of copper wires with widths less than 50nm was studied in Ref. [4]. It was found that the resistivity of copper wires increases significantly as the wire widths decrease, but no potential solutions to this problem have been proposed[1].

Electron scattering models have been improved and can now predict the rise of Cu resistivity as a function of line width and aspect ratio[2]. However, they are so complicated that it is hard to use them for interconnect delay and slew calculation or optimization. The authors of Ref. [5] therefore proposed a simple, empirical model for wire sizing. However, they only considered the width of the interconnect wire, neglecting the effect of thickness on the resistivity. In this paper, we further modify their method to propose a new, simple and effective resistivity model that takes into account the effects of both width and thickness of the interconnect wire.

Another problem that has arisen as technology enters the nanometer regime is that the impact of process variation on a circuit's performance has become critical. Traditionally, corner-based analysis has been used to determine the impact of process variation. However, it was found in Ref. [6] that this method can cause error when applied to interconnect analysis. In previous work, a way was proposed to capture the effect of interconnect variability on timing analysis. For example, the authors of Ref. [6] proposed an approach for modeling interconnect delay under process variation for timing analysis and physical design optimization. However, the expressions of 10%～90% slew, which is as important as delay, are not provided. Furthermore, the inductance is also not considered. The authors of Ref. [7] considered the statistical slew modeling (S2M model) of the interconnect line. However, little analysis was provided for the validation of that model, and we have found that it is only accurate when the node

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is far away from the driver source. In this paper, we modify the statistical S2M model and propose a new statistical SS2M model.

2 Scattering effect modeling and impact analysis

2.1 Modeling of scattering effect

Figure 1 shows a conductor's effective resistivity in different technology nodes from ITRS 2005[1]. As the wire width decreases, the effective resistivity increases abruptly. From this figure we can also see that the effective resistivity of the global wire is smaller than that of the other types of the interconnect wires. One reason is that the thickness of the global wire is often larger than that of the other types of wires.

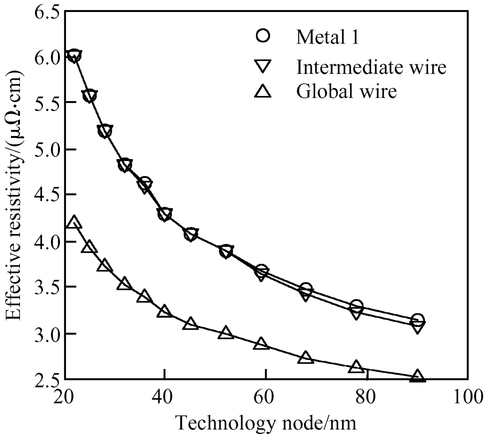

Fig. 1 Resistivity of nanoscale wire

Based on curve fitting techniques, the authors of Ref. [5] proposed an empirical equation,

$$\rho(w) = \rho_0 + \frac{K_\rho}{w} \tag{1}$$

to predict the resistivity increase in a nanoscale wire. This model is accurate compared with the measured data from Ref. [5]. However, the authors neglected the impact of thickness on the resistivity of the nanoscale wire. In this paper, using the same curve fitting techniques, we propose a new empirical equation considering both the width and thickness effects,

$$\rho(w,t) = \rho_0 + \frac{K_1}{w} + \frac{K_2}{t} \tag{2}$$

where ρ_0, K_1, and K_2 are the fitting parameters. In this paper, $\rho_0 = 1.8\mu\Omega \cdot \mathrm{cm}$, $K_1 = 1.03 \times 10^{-15}\ \Omega \cdot \mathrm{m}^2$, and $K_2 = 2.33 \times 10^{-16}\ \Omega \cdot \mathrm{m}^2$.

2.2 Scattering effect on delay and slew

We assume that all the resistances in Fig. 4 (in the next section) are the same and that all the capacitances are also the same, and we neglect the inductance. The interconnect parameters are: $w = 138\mathrm{nm}$ is the width of the interconnect line, $t = 234\mathrm{nm}$ is the thickness of the interconnect line, $h = 206\mathrm{nm}$ is the thickness of the dielectric, and $\alpha = 138\mathrm{nm}$ is an empirical parameter, which can be found in Ref. [8]. The length of the interconnect line is assumed to be 1mm. We consider a 3-sigma tolerance of ± 10nm in w, t and h and perform 1000 Monte Carlo simulations. The results for node 3 in Fig. 4 are shown in Figs. 2 and 3.

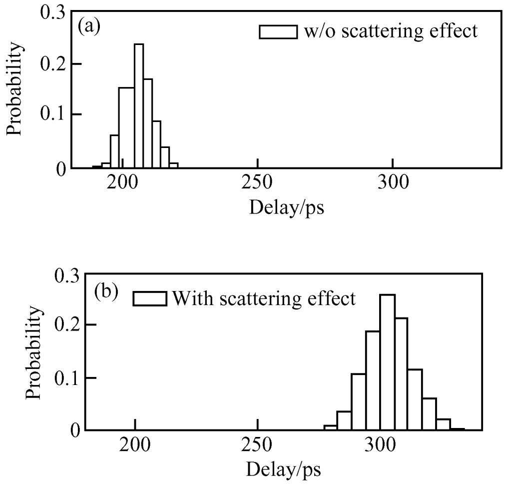

Fig. 2 Delay distribution from Monte Carlo simulations

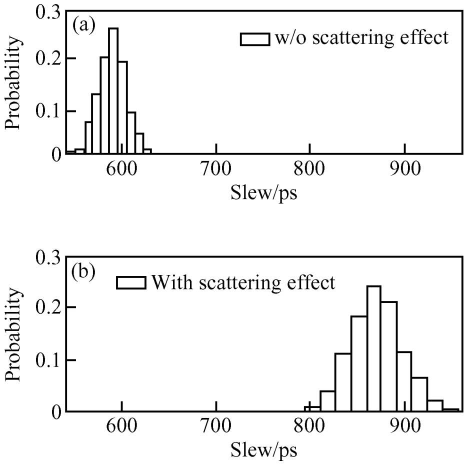

Fig. 3 Comparison of slew distributions from Monte Carlo simulations

The mean and standard deviation values of the delay when considering the scattering effect are 302.8423 and 8.9739ps, respectively. The mean and standard deviation values of the delay without considering the scattering effect are 205.0568 and 4.9663ps, respectively. This means that without considering the scattering effect, an error of 47.69% may occur when computing the delay time. Similar results are obtained for slew time analysis as we can see in Fig.3.

From these figures, we can draw the following conclusions: (1) In the nanometer regime, the scattering effect of the metal wire can not be neglected. (2) If we assume that the width, thickness, etc. have Gaussian distributions, then the delay and slew of the interconnect line also have similar Gaussian distributions. This characteristic is very important since we can then modify our modeling procedure and propose some simple analytical expressions for the delay and the slew.

3 Statistical timing analysis

3.1 Moment calculation

Instead of the RC tree presented in Ref.[6], we take an RLC tree for an example in this paper, as shown in Fig.4. The moment concept can still be applied to an RLC tree.

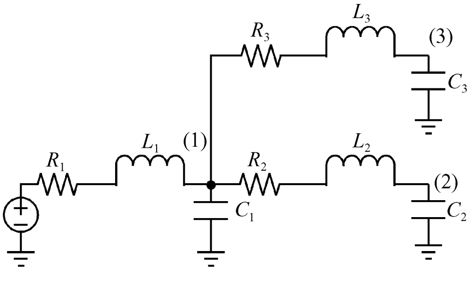

Fig.4 A simple RLC tree

We first use some empirical equations to calculate the resistance, capacitance, and inductance[8,9]:

$$\frac{R}{l} = \frac{\rho}{wt} = \frac{1}{wt}\left(\rho_0 + \frac{K_1}{w} + \frac{K_2}{t}\right)$$

$$\frac{C_g}{\varepsilon l} = \beta_0 + \beta_1\frac{w}{\alpha} + \beta_2\frac{t}{\alpha} + \beta_3\frac{\alpha}{h} + \beta_{11}\left(\frac{w}{\alpha}\right)^2 + \beta_{13}\frac{w}{h} + \beta_{23}\frac{t}{h} + \beta_{33}\left(\frac{\alpha}{h}\right)^2$$

$$L = \frac{\mu_0}{2\pi}\left[\lg\left(\frac{2l}{w+t}\right) + \frac{1}{2}\right] \tag{3}$$

Here l is the length, w the width, and t the thickness of the interconnect line; h is the thickness of the dielectric; and all the parameters of α and β can be found in Ref.[8]. In order to obtain the sensitivity of R, L, and C to the variation of w, t, and h, we can perform the following operations:

$$\begin{aligned}
R &= R_{\text{nom}} + \left(\frac{\partial R}{\partial w}\right)_{\text{nom}}\Delta w + \left(\frac{\partial R}{\partial t}\right)_{\text{nom}}\Delta t \\
C &= C_{\text{nom}} + \left(\frac{\partial C}{\partial w}\right)_{\text{nom}}\Delta w + \\
&\quad \left(\frac{\partial C}{\partial t}\right)_{\text{nom}}\Delta t + \left(\frac{\partial C}{\partial h}\right)_{\text{nom}}\Delta h \\
L &= L_{\text{nom}} + \left(\frac{\partial L}{\partial t}\right)_{\text{nom}}\Delta t + \left(\frac{\partial L}{\partial h}\right)_{\text{nom}}\Delta h
\end{aligned} \tag{4}$$

The first and second order moments of the RLC tree can be obtained as

$$\begin{aligned}
m_1^1 &= -R_1(C_1 + C_2 + C_3) \\
m_1^2 &= -R_1(C_1 + C_2 + C_3) - R_2C_2 \\
m_1^3 &= -R_1(C_1 + C_2 + C_3) - R_3C_3 \\
m_2^1 &= -R_1(C_1m_1^1 + C_2m_1^2 + C_3m_1^3) - \\
&\quad L_1(C_1 + C_2 + C_3) \\
m_2^2 &= -R_1(C_1m_1^1 + C_2m_1^2 + C_3m_1^3) - \\
&\quad R_2C_2m_1^2 - L_1(C_1 + C_2 + C_3) - L_2C_2 \\
m_2^3 &= -R_1(C_1m_1^1 + C_2m_1^2 + C_3m_1^3) - \\
&\quad R_3C_3m_1^2 - L_1(C_1 + C_2 + C_3) - L_3C_3
\end{aligned} \tag{5}$$

More generally, the p-order moment at node i of an RLC tree can be calculated as

$$\begin{aligned}
m_p^i &= \sum_k(-R_{ik}C_km_{p-1}^i - L_{ik}C_km_{p-2}^i) \\
m_0^i &= 1
\end{aligned} \tag{6}$$

where C_k is the capacitance at node k and $R_{ik}(L_{ik})$ denotes the total overlap resistance (inductance) in the unique paths from the source node to nodes i and k.

3.2 Improved statistical slew analysis

The authors of Ref.[7] proposed D2M and S2M metrics to calculate the distributions of delay and slew. Here we propose some improved timing metrics to enhance the slew analysis accuracy. We also propose the statistical SS2M metrics. We should state that although the metrics of SS2M are not new, to the best of our knowledge this is the first time that they have been used in statistical timing analysis. Firstly, we approximate the moments by using the first order Gaussian distribution:

$$\begin{aligned}
m_1 &\approx m_{1,\text{nom}} + k_w\Delta w + k_t\Delta t + k_h\Delta h \\
m_2 &\approx m_{2,\text{nom}} + j_w\Delta w + j_t\Delta t + j_h\Delta h
\end{aligned} \tag{7}$$

The SS2M slew model of the interconnect line can then be expressed as

$$
\begin{aligned}
\mathrm{SS2M} &= \mathrm{SS2M}_{\mathrm{nom}}(1 + B_{\mathrm{w}}\Delta w + B_{\mathrm{t}}\Delta t + B_{\mathrm{h}}\Delta h) \\
B_{\mathrm{w}} &= \frac{k_{\mathrm{w}}}{2m_{1,\mathrm{mon}}} + \frac{j_{\mathrm{w}} - m_{1,\mathrm{nom}}k_{\mathrm{w}}}{2m_{2,\mathrm{nom}} - m_{1,\mathrm{nom}}^{2}} - \frac{j_{\mathrm{w}}}{4m_{2,\mathrm{nom}}} \\
B_{\mathrm{t}} &= \frac{k_{\mathrm{t}}}{2m_{1,\mathrm{mon}}} + \frac{j_{\mathrm{t}} - m_{1,\mathrm{nom}}k_{\mathrm{t}}}{2m_{2,\mathrm{nom}} - m_{1,\mathrm{nom}}^{2}} - \frac{j_{\mathrm{t}}}{4m_{2,\mathrm{nom}}} \\
B_{\mathrm{h}} &= \frac{k_{\mathrm{h}}}{2m_{1,\mathrm{mon}}} + \frac{j_{\mathrm{h}} - m_{1,\mathrm{nom}}k_{\mathrm{h}}}{2m_{2,\mathrm{nom}} - m_{1,\mathrm{nom}}^{2}} - \frac{j_{\mathrm{h}}}{4m_{2,\mathrm{nom}}}
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
\mathrm{SS2M}_{\mathrm{norm}} &= \ln(9)\,\frac{\sqrt{-m_{1,\mathrm{norm}}}}{\sqrt[4]{m_{2,\mathrm{nom}}}}\sqrt{2m_{2,\mathrm{norm}} - m_{1,\mathrm{norm}}^{2}} \\
\mathrm{Stdv} &= \mathrm{SS2M}_{\mathrm{norm}}\sqrt{B_{\mathrm{w}}^{2}\sigma_{\mathrm{w}}^{2} + B_{\mathrm{t}}^{2}\sigma_{\mathrm{t}}^{2} + B_{\mathrm{h}}^{2}\sigma_{\mathrm{h}}^{2}}
\end{aligned}
\tag{9}
$$

Note that for an RLC tree, Equation (9) can be used when $4m_2 - 3m_1^2 \geqslant 0$ is satisfied because then $2m_2 - m_1^2 \geqslant 0$. If $4m_2 - 3m_1^2 < 0$, there still exist cases when $2m_2 - m_1^2 \geqslant 0$. However, this is not globally true, and thus these metrics cannot be applied to the case of $4m_2 - 3m_1^2 < 0$ in general[10].

3.3　Experimental results

Figure 5 shows the testbench for model verification, obtained from Ref. [6]. The length of the interconnect line is 1mm, and the interconnect line is divided into 30 identical segments.

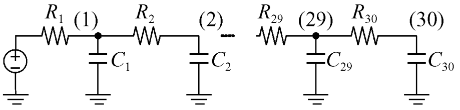

Fig. 5　Testbench for model verification[6]

Slew distribution comparisons are provided in Fig. 6 and Table 1. We performed 1000 Monte Carlo simulations for each node. From these results, we find that even with large variations in geometric dimensions, the slew distribution remains Gaussian. We also find that the S2M model is accurate only when the node is far away from the driver source. When the position of the node is near the driver source, the error can be large. For example, the mean slew time for S2M at node 5 is 63.48ns, while the SPICE Monte Carlo result is 46.49ns, an error of 35.9% occurs. However, our new model performs well in the whole range of nodes. The average mean error of the proposed model is only 4.16%, much smaller than that of the S2M model, which is about 8.42%. The average standard deviation error of the proposed model is only 3.06%, with a reduction of 63.22% compared to the S2M model.

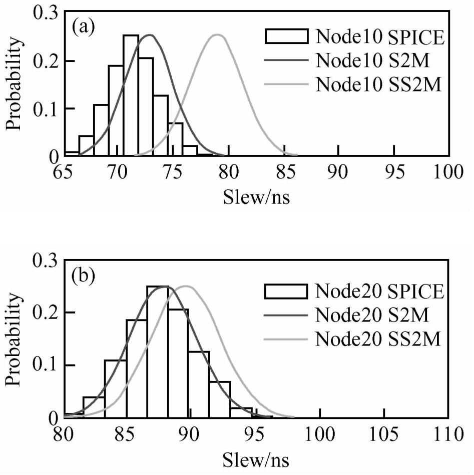

Fig. 6　Slew distribution using statistical S2M, SS2M compared to Monte Carlo simulation

Table 1　Mean and standard deviation of slew distribution along various nodes in a simple 30-segment line

Node	Norm slew	Mean/ns			Stedev/ns		
		SPICE	S2M	SS2M	SPICE	S2M	SS2M
5	46.51	46.49	63.18	50.86	1.405	1.897	1.527
10	71.40	71.36	78.89	72.75	2.157	2.367	2.183
15	82.87	82.84	85.43	84.08	2.504	2.563	2.523
20	87.70	87.66	87.81	89.58	2.650	2.634	2.688
25	88.76	88.73	88.39	91.89	2.682	2.652	2.757
30	88.82	88.79	88.43	94.43	2.684	2.654	2.777
Maximum error			35.9%	9.40%	–	35.0%	8.68%
Average error			8.42%	4.16%	–	8.32%	3.06%
Stedev of error			14.0%	3.11%	–	13.5%	2.94%

4　Conclusion

We have presented a statistical approach for modeling interconnect slew that takes into account the scattering effect of a nanoscale interconnect wire. We proposed a simple, closed-form scattering effect resistivity model. To the best of our knowledge, this is the first simple model that considers both width and thickness effects. We used this model to derive statistical expressions of the slew of interconnect lines and found that the delay and slew can be greatly increased when considering this effect's impact on the interconnect line's effective resistivity. The statistical SS2M model has an average error of 4.16% for calculating mean delay time compared to SPICE Monte

Carlo simulation, with an average error in standard deviation of only 3.06%.

References

[1] http://public.itrs.net

[2] Durkan C, Welland M E. Size effects in the electrical resistivity of polycrystalline nanowires. Phys Rev B, 2000, 61: 14215

[3] Rossnagel S M, Kuan T S. Alteration of Cu conductivity in the size effect regime. J Vac Sci Technol B, 2004, 22: 240

[4] Steinlesberger G, Engelhardt M, Schindler G, et al. Electrical assessment of copper damascene interconnects down to sub-50nm feature sizes. Microelectron Eng, 2002, 64: 409

[5] Shi S X, Pan D Z. Wire sizing with scattering effect for nanoscale interconnection. ASP-DAC, 2006: 50

[6] Agarwal K, Sylvester D, Blaauw D, et al. Variational delay metrics for interconnect timing analysis. DAC, 2004: 381

[7] Agarwal K, Agarwal M, Sylvester D, et al. Statistical interconnect metrics for physical design optimization. IEEE Trans Comput-Aided Design Circuits Syst, 2006, 25: 1273

[8] Kurokawa A, Hashimoto M, Kasebe A, et al. Second - order polynomial expressions for on-chip interconnect capacitance. IEICE Trans Fundamentals, 2005, E88-A(12): 3453

[9] Grover F W. Inductance calculations. Mineola: Dover Publications Inc, 1973

[10] Agarwal K, Sylvester D, Blaauw D. A simple metric for slew rate of RC circuits based on two circuits moments. IEEE Trans Comput-Aided Design Circuits Syst, 2004, 23(9): 1346

考虑散射效应的改进的互连线统计时序分析

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摘要： 考虑纳米尺度互连线的散射效应，提出了一种改进的时序分析方法.首先，考虑互连线宽度和厚度的影响，提出了一个简单的散射效应的解析模型.然后，利用这个模型和SS2M得到了过渡时间的统计表达式.实验结果表明，当考虑散射效应后，互连线的延时和过渡时间将进一步增加.同时，提出的统计SS2M模型与SPICE蒙特卡罗分析结果比较，均值的平均误差在4.16%以内，而方差的平均误差在3.06%以内.

关键词： 互连线；散射效应；时序分析

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