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A Mapping Technique to Draw Resistivity Isocontours for Slice-of-Silicon Monocrystal*

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Abstract: A resistivity distribution with a space of 3mm between test points was measured on a slice-of-silicon monocrystal (diameter 75mm) using an inclined four-point probe. This paper has determined the number of resistivity divisions and their separations by statistical methods and introduced fuzzy mathematics to place the data into different fuzzy sets, after choosing the exponent function as a membership function for fuzzy sets and suitable values of thresholds. One fuzzy set corresponds to one resistivity isocontour. Then, the resistivity isocontours can be drawn with a definite separation and finally shown in a map with MATLAB. The deviation of resistivity data on an isocontour is small and there are few residual test points without connections. So, the connection of the isocontours are high-quality and useful in application for instructing practical production.

Key words: mapping technique; silicon monocrystal; draw the resistivity isocontours
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1 Introduction

The radial resistivity profile in the cross section of silicon monocrystal is usually not even due to the dopant segregation[1]. However, the doping homogeneity directly affects the properties and production yield of the finished devices and IC. So, it is important to map the resistivity profile for cross sections of silicon monocrystals[2,3]. We have developed an automated measurement instrument using an inclined four point probe with a square configuration[4] to measure the resistivity in the area of 0.3mm[5]. The resistivity profile can be obtained, point by point, with a resolution as good as 1～3mm. However, the measurement error is about 20%.

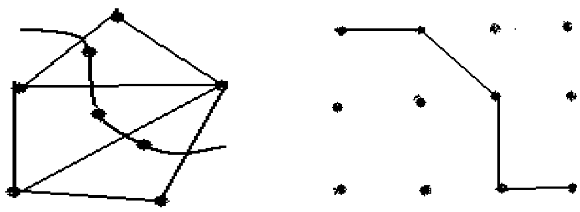
Drawing the resistivity isocontour for mapping resistivity profile is problematic despite software such as Surfer and MATLAB. However, the values of physical quantities on each node are assigned definitively and accurately. The isocontours can be easily drawn and they will intersect with the network, as shown in Fig. 1(a). However, the method to draw the resistivity isocontours for the cross-section of silicon monocrystals in this paper is different from the above:

(1) The distance among the resistivity test points can be adjusted from 1 to 3mm. The isocontours can go along the sides of the grids, or along the diagonal through the grids, as shown in Fig.1(b).

(2) It is difficult to draw isocontours to pass any two adjacent nodes because their tested resistivities are not strictly equal to each other. We must introduce fuzzy mathematics to ascribe the scattered resistivity into different sets[7]. They can be connected and smoothed by using software MATLAB and these smooth curves are of sufficient precision for mapping resistivity profiles to instruct the growth of ingots.

2 Determination of the value of division numbers m using statistics

The resistivity separation $\Delta\rho_0$ is equal to $(\rho_{max} - \rho_{min})/m$. m is the number of divisions with different radii, which will be determined in this section. However, m is not a sole one, but a better one. Provided



(a)　　(b)

Fig.1　Two methods of drawing the isocontour (solid line)

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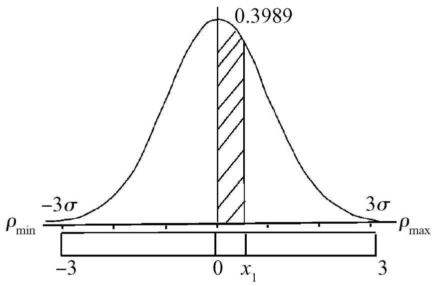

Fig. 2 Probability distribution and integration of P_c (shadow area) for different m (a) $m=2$, $P_c=0.4332$; (b) $m=4$, $P_c=0.2734$; (c) $m=8$, $P_c=0.1460$

that the resistivity distribution obeys the normal distribution in the range from its minimum to maximum:

$$P = \frac{1}{\sqrt{2\pi}\sigma}\exp(-(\rho-\rho_c)^2/2\sigma^2) \tag{1}$$

where P is the resistivity distribution probability, ρ_c is the centric resistivity, and σ is the standard deviation:

$$\sigma \approx (\rho_{\max}-\rho_{\min})/6 \tag{2}$$

The probability integration P_c for half of the central division of resistivity can be calculated:

$$P_c = \frac{1}{\sqrt{2\pi}\sigma}\int_{\rho_c}^{\rho_1}[\exp(-(\rho-\rho_c)^2/2\sigma^2)]\mathrm{d}\rho$$

$$= \frac{1}{\sqrt{2\pi}}\int_0^{x_1} \mathrm{e}^{-x^2/2}\mathrm{d}x = \frac{1}{\sqrt{2\pi}}\left(\int_0^{\infty}\mathrm{e}^{-x^2/2}\mathrm{d}x - \int_{x_1}^{\infty}\mathrm{e}^{-x^2/2}\mathrm{d}x\right) \tag{3}$$

where $x=(\rho-\rho_c)/\sigma$, $\rho_1=\rho_c+(\rho_{\max}-\rho_{\min})/2m=\rho_c+6\sigma/2m$, $x_1=(\rho_1-\rho_c)/\sigma=3/m$, ρ_1 is the edge resistivity of the central half division with its corresponding x_1. The scheme of the above calculation is shown in Fig. 2 and the results are summarized in Table 1. The resistivity distribution is influenced by non-steady segregation near the facet between the ingot and the molten[1]. Generally, resistivity is lower in the centric region, but higher in the edge region of the slice. Provided that one test point corresponds to one different datum, the probability integration P_c can be used for calculating n_c of the resistivity-testing point distribution near radius r_c of the centric resistivity, as follows:

$$P_c = n_c/n = \frac{\pi[(r_c+B)^2-r_c^2]}{a^2}\bigg/\frac{\pi R^2}{a^2}$$

$$\approx\left(\sqrt{0.5}+\frac{B}{R}\right)^2-0.5 = 1.414\,\frac{B}{R},$$

$$B \approx RP_c/1.414 \tag{4}$$

where B is the half space of the adjacent isocontour in this data-concentrating region, because each division is separated by a corresponding isocontour. Although the space of the resistivity-adjacent isocontours in the data-scattering regions is lower than a, they are not effective, except for those passing through the testing points. It is appropriate for the data-scattering region to take the space of the geographical adjacent isocontours to be equal to a. There would be the best effective resolution in isocontours. However, we prefer good mapping clarity to prevent the crowding of isocontours in the data-concentrating region, at the cost of some resolution. So, we only pay attention to the data-concentrating region to determine m and take $B \geqslant a$. For our test experiments, a silicon slice with a diameter of 75mm was tested. B and m can be determined from P_c and the radii of the slices, shown in Table 1. We choose $B=3.87\text{mm} > a = 3\text{mm}$. Then $m=8$ and $P_c=0.1460$, according to Table 1. The resistivity range should be divided into 8 or 9 divisions (Here, we take 9 divisions): 21.6, 22.8, 24.0, 25.2, 26.4, 27.6, 28.8, 30, 31.2 and 32.4 Ω·cm. They are denoted respectively with 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and stand for 10 sets.

Table 1 Relation among probability integration P_c, B, B/R and m

m	2	4	8	16	32
P_c	0.4332	0.2734	0.1460	0.075	0.037
B/R	0.3063	0.1933	0.1032	0,0530	0.026
B/mm	11.49	7.25	3.87	1.99	0.098

Table 2 Standard deviation of the statistics for different isocontours

Ascribed-resist. $\rho_i/(\Omega\cdot\text{cm})$	21.6	22.8	24.0	25.2	26.4	27.6	28.8	30.0	31.2	32.4
σ_i/%	1.74	1.33	1.05	1.17	1.39	1.28	0.94	0.76	0.74	1.23

3 Ascribing the scattered resistivity into different divisions

As mentioned in introduction, we must introduce fuzzy mathematics to ascribe the scattered resistivity into different fuzzy sets. Generally, the membership function in fuzzy mathematics can be chosen as[8]:

$$S_{\tilde{A}}(x) = \exp(-x^2) \tag{5}$$

where $x=(\rho_j-\rho_i)/\Delta\rho_0$, ρ_i and ρ_j are the resistivity value at initiate point i and the concerned point j for drawing a resistivity isocontour. In addition, a suitable threshold S_0 for membership should be set according to the space and orientations for two adjacent points. Furthermore, the specified thresholds also depend upon the final results to complete this course of drawing the isocontours. They should be revised until there are scarcely points left without classification to any fuzzy sets to guarantee a good connection quality (see Table 2). The prerequisite of drawing an isocontour from point j to i is: $S_{\tilde{A}}(x) \geqslant S_0$.

4 Computer implementation of the calculation

Figure 3 shows the original tested resistivity data on the slice of a silicon mono crystal. First, we chose

NO	NO	NO	NO	NO	NO	NO	28.8	NO	28.8	28.8	NO	28.8	NO	NO	NO	NO	NO	NO	NO	NO	NO
NO	NO	NO	NO	NO	NO	28.8	27.6	27.6	27.6	27.6	27.6	27.6	28.8	30	30	NO	NO	NO	NO	NO	NO
NO	NO	NO	NO	28.8	28.8	27.6	25.2	28.8	28.8	28.8	27.6	27.6	27.6	28.8	28.8	30	30	NO	NO	NO	NO
NO	NO	NO	NO	28.8	27.6	25.2	27.6	27.6	30	30	30	27.6	27.6	28.8	30	28.8	28.8	NO	28.8	NO	NO
NO	NO	28.8	28.8	27.6	25.2	26.4	30	30	28.8	28.8	30	27.6	27.6	26.4	28.8	30	30	NO	28.8	NO	NO
NO	28.8	28.8	27.6	25.2	25.2	22.8	27.6	22.8	27.6	25.2	28.8	22.8	26.4	25.2	22.8	28.8	30	30	28.8	28.8	NO
NO	28.8	27.6	25.2	25.2	25.2	27.6	27	22.8	27.6	22.8	22.8	21.6	24	22.8	22.8	28.8	30	30	30	28.8	31.2
28.8	27.6	26.4	25.2	26.4	27.6	22.8	22.8	21.6	21.6	22.8	21.6	21.6	24	22.8	22.8	26.4	28.8	28.8	30	28.8	30
27.6	26.4	25.2		26.4	27.6	21.6	21.6	21.6	21.6	20	21.6	21.6	22.8	24	22.8	27	28.8	30	30	28.8	31.2
27.6	25.2	26.4	27.6	26.4	22.8	21.6	21.6	22.8	21.6	21.6	21.6	22.8	22.8	24	24	27	28.8	28.8	30	28.8	30
27.6	26.4	26.4	27.6	26.4	21.6	21.6	21.6	22.8	22.8	21.6	22.8	21.6	22.8	24	24	22.8	28.8	28.8	30	28.8	30
27.6	25.2	26.4	27.6	26.4	22.8	21.6	22.8	22.8	22.8	21.6	22.8	22.8	22.8	22.8	24	22.8	28.8	30	30	28.8	30
27.6	25.2	26.4	26.4	25.2	21.6	21	22.8	22.8	21.6	21.6	21.6	22.8	22.8	22.8	24	22.8	28.8	28.8	30	28.8	30
27.6	26.4	26.4	24	25.2	22.8	21.6	22.8	22.8	21.6	21.6	21.6	22.8	24	24	24	24	27.6	28.8	30	27.6	30
27.6	27.6	25.2	25.2	26.4	25.2	22.8	21.6	22.8	21.6	22.8	21.6	22.8	24	22.8	24	24	27.6	28.8	30	28.8	31.2
NO	27.6	27.6	26.4	27.6	22.8	22.8	21.6	21.6	21.6	24	25.2	22.8	22.8	24	24	24	28.8	30	30	28.8	30
NO	27.6	26.4	26.4	27.6	27.6	22.8	22.8	21.6	22.8	26.4	27.6	22.8	25.2	26.4	24	26.4	28.8	30	31.2	30	NO
NO	NO	27.6	27.6	26.4	27.6	27.6	22.8	22.8	24	32.4	27.6	32.4	28.8	27.6	26.4	27	28.8	30	32.4	NO	NO
NO	NO	NO	27.6	27.6	26.4	27.6	27.6	26.4	25.2	21	32.4	32.4	27.6	28.8	28.8	28.8	28.8	30	NO	NO	NO
NO	NO	NO	NO	27.6	27.6	26.4	27.6	27.6	26.4	25.2	32.4	27.6	27.6	28.8	28.8	28.8	30	NO	NO	NO	NO
NO	NO	NO	NO	NO	30	28.8	27.6	27.6	27.6	27.6	32.4	30	30	32.4	28.8	30	NO	NO	NO	NO	NO
NO	NO	NO	NO	NO	NO	NO	NO	26.4	27.6	27.6	32.4	NO	32.4	33	NO	NO	NO	NO	NO	NO	NO

Fig. 3 Original tested resistivity data (Ω · cm) on the slice of a silicon monocrystal (NO is denoted where are no tested resistivity data out of the slice)

an arbitrarily test point to start calculations of the membership for the nearest fuzzy sets. After comparing the specified threshold for different orientations, it can be confirmed to what fuzzy set this point should belong. The thresholds are specified as follows:

0.8 in the orientations of 0° or 90°;

0.6 in the orientation of 45°.

After the use of fuzzy mathematics, all tested resistitvity data were ascribed into 10 fuzzy sets and each was respectively denoted with a number. All adjacent points with the same number can be connected by one isocontour. These are joggling lines to represent different resistivity divisions. Afterwards, using the interpolation process in MATLAB, the joggling isocontours are made smooth, as shown in Fig. 4.

The flow of computer implementation is shown in Fig. 5.

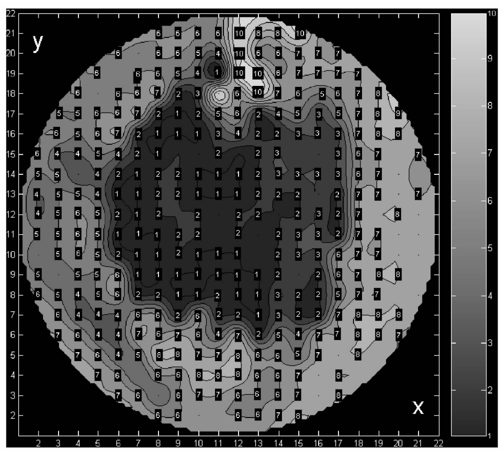

Fig. 4 Smooth isocontours, representing different resistivity divisions (Using interpolation function "griddata", interpolation method "v4" in MATLAB)

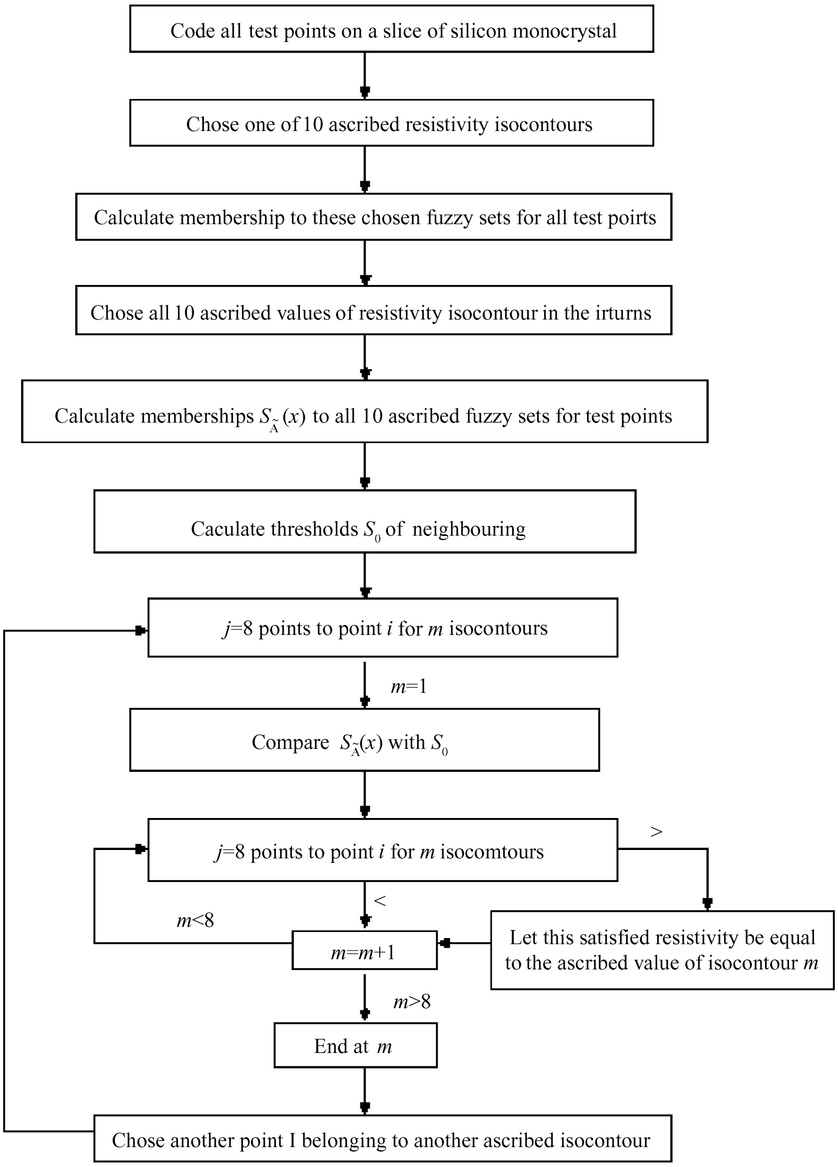

Fig. 5 Flow chart of computer implementation

5 Assessment of the connection quality

(1) Determines how many points remain without a connection to their neighboring points. Figure 4 shows that less then 5% of total test points have not been connected to their neighbors;

(2) The standard deviation of the statistics for different resistivity on an isocontour is:

$$\sigma_i = \pm\left[\frac{1}{n_i-1}\sum_{j}^{n_i}(\rho_j-\rho_i)^2\right]^{\frac{1}{2}}/\rho_i \qquad (6)$$

where n_i are the total numbers of points on ith isocontour, ρ_i is the ascribed resistivity indicated with i, and ρ_j is the resistivity at point j on this isocontour i. Table 2 shows the calculation results.

The connection quality for resistivity isocontours is good.

6 Conclusion

The spaces among testing points have been minimized to 3mm. There is no need to find the coordinates of equi-resistivity points among the test points to draw isocontours. Moreover, the relative error is about 20% in the resistivity measurement using an inclined four-point probe. It is impossible to directly draw the resistivity isocontour in a map, because there is a requirement to have a strict value on each grid. In this case, fuzzy mathematics may be introduced to ascribe the scattered test resistivity into several sets. Then, we can use MATLAB to draw their isocontours with a good connection quality.

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绘制硅单晶电阻率等值线的 Mapping 技术*

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摘要: 用斜置式四探针测定了硅单晶片(Φ75mm)上 3mm 间距测试点的电阻率分布. 本文从电阻率的统计分布出发, 确定了电阻率的分隔数和差值, 采用指数函数作为模糊集的隶属度, 并且选择合适的门槛值, 利用模糊数学将电阻率数据归类于不同的模糊集. 同一模糊集对应相同的电阻率, 这样使电阻率能以一定的间隔分布, 然后结合 MATLAB 软件画出电阻率等值线, 以构成 Mapping 图. 在同一等电阻率线条上各点具有较小的阻值偏差, 且剩余未连接点少. 连接质量好, 可以应用于指导实际生产.

关键词: mapping 技术; 硅单晶; 绘制电阻率等值线
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