Guided modes in a rectangular waveguide with semiconductor metamaterial

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Abstract: The dispersion equations of bulk modes and surface modes in a rectangular waveguide of semiconductor metamaterial are derived by a modified "Marcatili's method". The cutoff frequencies of the lowest TM bulk mode are discussed, and the Brillouin diagrams of different bulk modes are drawn. They demonstrate that different heights correspond to different guidance frequency ranges which have no superposition with each other and a waveguide with a larger height possesses a wider passband of light. In addition, tendencies of degeneracy for different modes are observed. Finally, the existence of surface modes is verified by a graphical method.

Key words: semiconductor metamaterial; rectangular waveguide; bulk mode; surface mode

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1. Introduction

In 1968, Veselago theoretically proposed the concept of metamaterials exhibiting negative refraction[1], which was experimentally verified firstly by using artificial left-handed materials realized by periodic arrays of split ring resonators and wire strips in the microwave regime in 2001[2]. Because of their large number of potential applications in optics, materials science, biology, and biophysics, different metamaterials have been fabricated with chiral material[3] or photon crystals[4]. But their two-dimensional property has limited their usefulness. Recently, a Princeton-led research team has created the first three-dimensional metamaterial at IR or optical frequencies constructed entirely from semiconductors, the principal ingredient of microchips and optoelectronics[5, 6]. This new type of metamaterial is based on the strongly anisotropy of the dielectric response, and does not have any magnetic response[7]. This invention will bring us more possibilities to improve applications such as super lenses and new types of lasers.

In this paper, we investigate the guided mode properties in the proposed metamaterial by a modified "Marcatili's method". The dispersion equations of bulk modes and surface modes are derived and the transcendental equations for guided TM modes are solved numerically. According to the parameters described in Ref. [6], the guidance conditions of TM_{11} mode are discussed. Brillouin diagrams with real transverse wave numbers are drawn, including different waveguide shapes and different modes propagating in the same waveguide. Finally, we demonstrate the existence of surface modes in this waveguide at some special frequencies by a graphical method.

2. Theory

We consider a rectangular waveguide with the geometry and parameters given in Fig. 1 whose width and height are denoted by a and b, respectively. Marcatili's method for a rectangular waveguide with cross section $a \times b$ involves the solution of two problems for two dielectric slabs[8]: a horizontal slab of thickness b and a vertical slab of thickness a for the same polarization. This method allows one to derive the dispersion equations for a rectangular uniaxial anisotropic waveguide in which the optical axis coincides with the z-axis without guessing the field distribution[9].

We assume that the material inside the waveguide is nonmagnetic (μ is equal to that of vacuum), and has an anisotropic uniaxial dielectric constant ε determined by the matrix

$$\varepsilon = \begin{pmatrix} \varepsilon_{\perp} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\parallel} \end{pmatrix}, \tag{1}$$

where $\varepsilon_{\perp} = \varepsilon_{r\perp}\varepsilon_0$ denotes the permittivity in the direction normal to the optical axis and $\varepsilon_{\parallel} = \varepsilon_{r\parallel}\varepsilon_0$ denotes the permittivity parallel to the optical axis.

In Ref. [5], the metamaterial is fabricated with n$^+$-GaInAs/i-AlInAs semiconductor heterostructures with appropriately thick layers with alternating positive and negative dielectric constants. The effective permittivity tensor for the layered system is related to the AlInAs relative permittivity $\varepsilon_{\mathrm{r1}}$ and InGaAs relative permittivity $\varepsilon_{\mathrm{r2}}$ as follows[6].

$$\varepsilon_{\mathrm{r}\perp} = \frac{\varepsilon_{\mathrm{r1}} + \varepsilon_{\mathrm{r2}}}{2}, \tag{2}$$

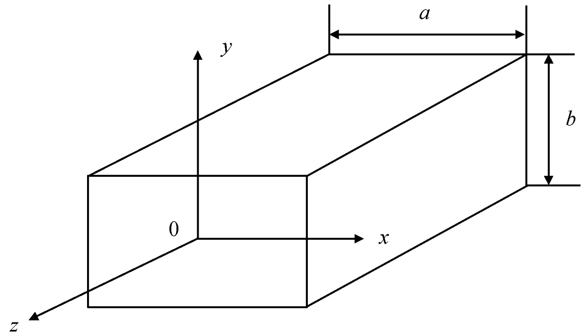

Fig. 1. Configuration of the rectangular waveguide.

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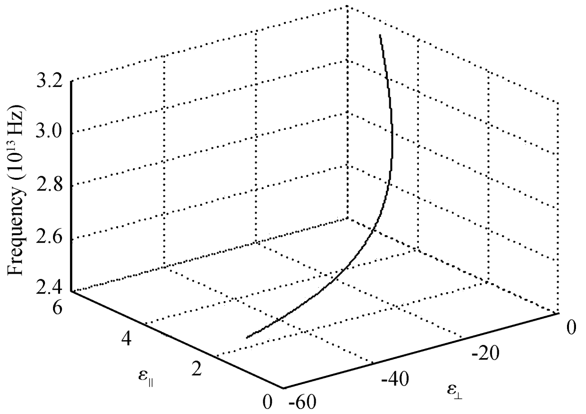

Fig. 2. Dispersion curve of semiconductor metamaterial with parameters as shown in Eqs. (2) and (3).

$$\varepsilon_{r\parallel}=\frac{2\varepsilon_{\mathrm{r1}}\varepsilon_{\mathrm{r2}}}{\varepsilon_{\mathrm{r1}}+\varepsilon_{\mathrm{r2}}},\tag{3}$$

where $\varepsilon_{\mathrm{r1}}$ is a constant and equals 10.23 and $\varepsilon_{\mathrm{r2}}$ is described by the Drude model regardless of its imaginary part:

$$\varepsilon_{\mathrm{r2}}=12.15\times\left(1-\omega_{\mathrm{p}}^2/\omega^2\right),\tag{4}$$

where ω_{p} is the plasma frequency.

As the semiconductor metamaterial is based on the anisotropy of the dielectric response, it requires $\varepsilon_{\mathrm{r}\perp}=\frac{\varepsilon_{\mathrm{r1}}+\varepsilon_{\mathrm{r2}}}{2}>0$ and $\varepsilon_{\mathrm{r}\parallel}=\frac{2\varepsilon_{\mathrm{r1}}\varepsilon_{\mathrm{r2}}}{\varepsilon_{\mathrm{r1}}+\varepsilon_{\mathrm{r2}}}<0$. Since $\varepsilon_{\mathrm{r1}}=10.23$, we must ensure $\varepsilon_{\mathrm{r2}}=12.15\times(1-\omega_{\mathrm{p}}^2/\omega^2)<0$ and $\varepsilon_{\mathrm{r1}}>|\varepsilon_{\mathrm{r2}}|$, so $0.74\omega_{\mathrm{p}}<\omega<\omega_{\mathrm{p}}$ is derived. The dispersion curve of semiconductor LHM with parameters as shown is drawn in Fig. 2.

2.1. Bulk modes

Similar to uniaxial crystals, the waveguide may support ordinary waves and extraordinary waves. In our case of the waveguide with an anisotropic core the extraordinary wave has TM polarization which is affected by both $\varepsilon_\parallel$ and $\varepsilon_\perp$[7]. So in this paper, we mainly focus on TM waves.

We can derive the dispersion equations for symmetric TM bulk modes similar to "Marcatili's equation"[10] as follows.

For the horizontal slab,

$$\frac{k_y^2}{\varepsilon_{\mathrm{r}\parallel}^2}\tan^2\left(\frac{1}{2}k_yb-n\pi\right)+\frac{\varepsilon_\perp}{\varepsilon_\parallel}k_y^2=k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right).\tag{5}$$

By simple deduction, we can get

$$k_yb=(2n-1)\pi+2\arctan\frac{k_y}{p\varepsilon_{\mathrm{r}\parallel}},\tag{6}$$

where $n=1,2,\ldots,\infty$, $p=\sqrt{\beta^2-k_0^2}$ and $\beta^2=k_0^2\varepsilon_{r\perp}-k_y^2\frac{\varepsilon_{\mathrm{r}\perp}}{\varepsilon_{\mathrm{r}\parallel}}$.

Regarding the vertical slab, "Marcatili's equation"[9] can be written as

$$k_x^2\tan^2\left(\frac{1}{2}k_xa-m\pi\right)+k_x^2=k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right).\tag{7}$$

By simple deduction, we can get

$$k_xa=(2m-1)\pi-2\arctan\frac{k_x}{q},\tag{8}$$

where $q=\sqrt{k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right)-k_x^2}$. $m=1,2,\ldots,\infty$. The dispersion equation requires $k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right)>k_x^2$.

Thus, the solution procedure can be as follows. Using Eq. (6), one can find β, then solve Eq. (8) for k_x and eventually, find that the final propagation constant is

$$k_z=\sqrt{\beta^2-k_x^2}.\tag{9}$$

For the dispersion equations (5), (7) and (8), we can derive the existence conditions for bulk modes $\frac{k_y^2}{k_0^2}\frac{\varepsilon_{\mathrm{r}\perp}}{\varepsilon_{\mathrm{r}\parallel}}<\varepsilon_{\mathrm{r}\perp}-1$, $\frac{k_x^2}{k_0^2}<\varepsilon_{\mathrm{r}\perp}-1$ and $\frac{k_x^2}{k_0^2}+\frac{k_y^2}{k_0^2}\frac{\varepsilon_{\mathrm{r}\perp}}{\varepsilon_{\mathrm{r}\parallel}}<\varepsilon_{\mathrm{r}\perp}-1$. We assume $X=\frac{k_x^2}{k_0^2}$, $Y=\frac{k_y^2}{k_0^2}\frac{\varepsilon_{\mathrm{r}\perp}}{\varepsilon_{\mathrm{r}\parallel}}$, $Z=X+Y=\frac{k_x^2}{k_0^2}+\frac{k_y^2}{k_0^2}\frac{\varepsilon_{\mathrm{r}\perp}}{\varepsilon_{\mathrm{r}\parallel}}$ and $Q=\varepsilon_{\mathrm{r}\perp}-1$. The existence conditions for bulk modes are simplified as "$\max(X,Y,Z)<Q$ and $Q>0$".

2.2. Surface modes

When k_y and k_x are imaginary, surface waves exist[11]. We present Eqs. (5) and (7) in equivalent forms using the notations $k_y=\mathrm{j}\alpha_y$ and $k_x=\mathrm{j}\alpha_x$ (α_y and α_x are positive real).

$$\frac{\alpha_y^2}{\varepsilon_{\mathrm{r}\parallel}^2}\tanh^2\left(\frac{1}{2}\alpha_yb\right)-\frac{\varepsilon_\perp}{\varepsilon_\parallel}\alpha_y^2=k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right),\tag{10}$$

$$\alpha_x^2\coth^2\left(\frac{1}{2}\alpha_xa\right)-\alpha_x^2=k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right).\tag{11}$$

As the values of the left sides of Eqs. (10) and (11) are always larger than zero, surface modes exist only when $\varepsilon_{\mathrm{r}\perp}>1$. Then the dispersion equations are obtained as

$$\alpha_y\tanh\left(\frac{1}{2}\alpha_yb\right)=-p_1\varepsilon_{\mathrm{r}\parallel},\tag{12}$$

$$\alpha_x\coth\left(\frac{1}{2}\alpha_xa\right)=q_1,\tag{13}$$

where $p_1=\sqrt{\beta_1^2-k_0^2}$, $\beta_1^2=k_0^2\varepsilon_{\mathrm{r}\perp}+\alpha_y^2\frac{\varepsilon_{\mathrm{r}\perp}}{\varepsilon_{\mathrm{r}\parallel}}$ and $q_1=\sqrt{k_0^2\left(\varepsilon_{\mathrm{r}\perp}-1\right)+\alpha_x^2}$.

3. Numerical results and discussions

3.1. Bulk modes

We solve the transcendental equations (6) and (8) via exact numerical calculations to get the characters of bulk modes in semiconductor metamaterial rectangular waveguide. In calculation the component parameters of the semiconductor metamaterial are taken as described in Ref. [6]. We choose $\omega_{\mathrm{p}}=2\times10^{14}$ rad/s, $a=16$ μm and then take TM_{11} mode as an example to discuss the cutoff frequencies.

The curves of values X,Y,Z and Q versus normalized length (b/λ) are described in Fig. 3 ($b=0.5a$);, the cutoff frequencies are given by the intersections of the curves. To satisfy the condition "$\max(X,Y,Z)<Q$ and $Q>0$", TM_{11} mode

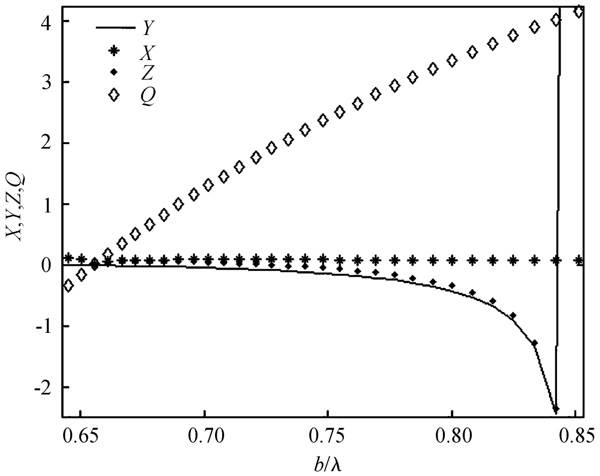

Fig. 3. Cutoff frequencies of TM_{11} mode when $\omega_{\mathrm{p}} = 2 \times 10^{14}$ rad/s, $a = 16\ \mu$m and $b = 0.5a$.

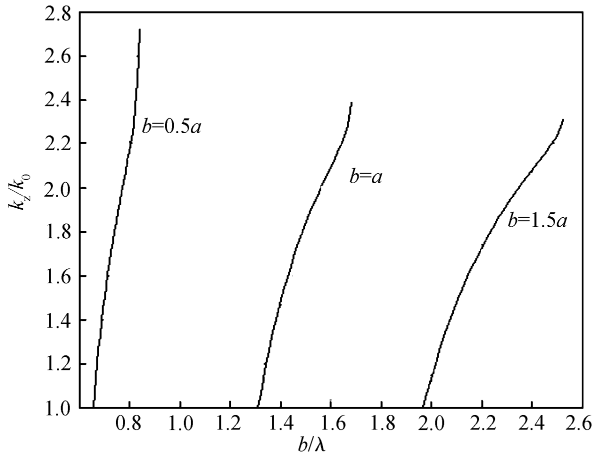

Fig. 4. Curves of k_z/k_0 versus b/λ (Brillouin diagram) for TM_{11} mode when $\omega_{\mathrm{p}} = 2 \times 10^{14}$ rad/s, $a = 16\ \mu$m, $b = 0.5a$, $b = a$, $b = 1.5a$.

exists only when $0.6557 < b/\lambda < 0.842$, so the corresponding wavelength range is $9.5\ \mu\text{m} < \lambda < 12.2\ \mu$m.

When $a = 16\ \mu$m, the curves of k_z/k_0 versus b/λ for TM_{11} mode are drawn in Fig. 4 when $b = 0.5a$, $b = a$ and $b = 1.5a$. This shows us that the height of the waveguide has a great effect on the cutoff frequencies. Different heights correspond to different guidance frequency ranges which have no superposition with each other and a larger height possesses a wider passband of light. By choosing an appropriate b it is easy to get single-mode propagation. This property is also useful when designing band-pass filters or attenuators.

When $b = 0.5a$ and $a = 8\ \mu$m, the curves of k_z/k_0 versus b/λ for TM_{11}, TM_{12}, TM_{21} and TM_{22} modes are drawn in Fig. 5. We can find that the values of "m" have a much smaller effect on the curves than "n". In this case, TM_{12} and TM_{22} modes (TM_{11} and TM_{21} modes) have a tendency of degeneracy at high frequency.

3.2. Surface modes

We employ a graphical method to obtain the solutions of Eqs. (12) and (13), using the parameters $\omega_{\mathrm{p}} = 2 \times 10^{14}$ rad/s

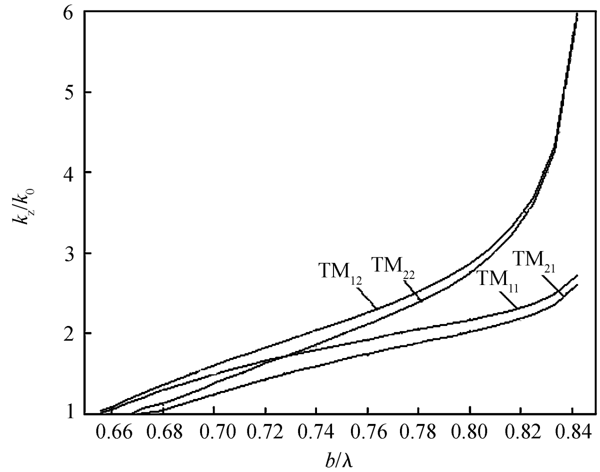

Fig. 5. Curves of k_z/k_0 versus b/λ (Brillouin diagram) for TM_{11}, TM_{12}, TM_{21} and TM_{22} modes when $\omega_{\mathrm{p}} = 2 \times 10^{14}$ rad/s, $a = 16\ \mu$m and $b = 8\ \mu$m.

and $a = b = 1.6\ \mu$m. The values of $b\alpha_y$ are determined by the intersections of two curves that indicate the values of the left-hand-side and the right-hand-side of Eq. (12) respectively in Fig. 6(a). Three cases are considered ($\lambda = 10, 9.9, 9.8\ \mu$m). By the same method we can derive the values of $a\alpha_x$ via Eq. (13) which is given by the intersections in Fig. 6(b). The intersections of the curves demonstrate solutions for surface modes at special frequencies. It is the existence of surface modes which makes its application of superlensing possible in a waveguide of semiconductor metamaterial.

In the following, we will go on studying the influence of the waveguide's shape on surface modes. In Fig. 7(a), the two curves will always meet at one point and a larger b corresponds to larger value of intersections. So along the y axis, Equation (12) always possesses nonzero solutions. On the other hand, in Fig. 7(b), a larger a corresponds to smaller value of intersections, which is different to Fig. 7(a). Furthermore, the existence of a solution in the direction of the x axis requires a more exact configuration of parameters, otherwise the two curves will easily miss each other. This result provides us with a method to control surface modes by changing the dimension of the waveguide.

4. Conclusion

In this article, the dispersion equations of bulk modes and surface modes in a rectangular waveguide of semiconductor metamaterial are derived by a modified "Marcatili's method". According to the parameters described in Ref. [6], the guidance conditions of TM_{11} mode are derived graphically, and its Brillouin diagram is drawn.

It shows us that, in the defined waveguide, different heights correspond to different guidance frequency ranges which have no superposition with each other and a waveguide with a larger height possesses a wider passband of light. Then the Brillouin diagrams with real transverse wave numbers for different modes are drawn; it is seen that TM_{12} and TM_{22} modes (TM_{11} and TM_{21} modes) have a tendency of degeneracy at high frequency. Finally, we demonstrate the existence of surface modes in waveguide with semiconductor metamaterial at

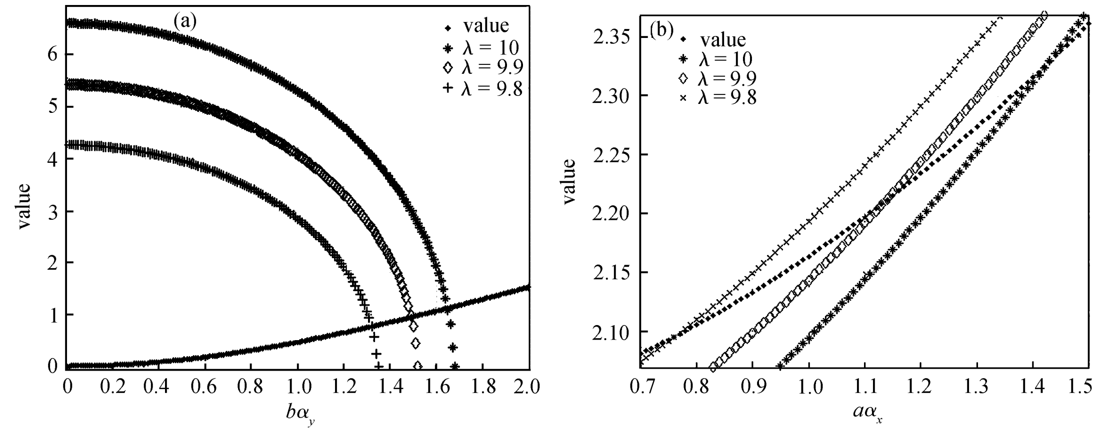

Fig. 6. Solutions of surface modes determined by Eqs. (12) and (13) when $\omega_p = 2 \times 10^{14}$ rad/s, $\lambda = 10$ μm, $\lambda = 9.9$ μm and $\lambda = 9.8$ μm. (a) $b = 1.6$ μm. (b) $a = 1.6$ μm.

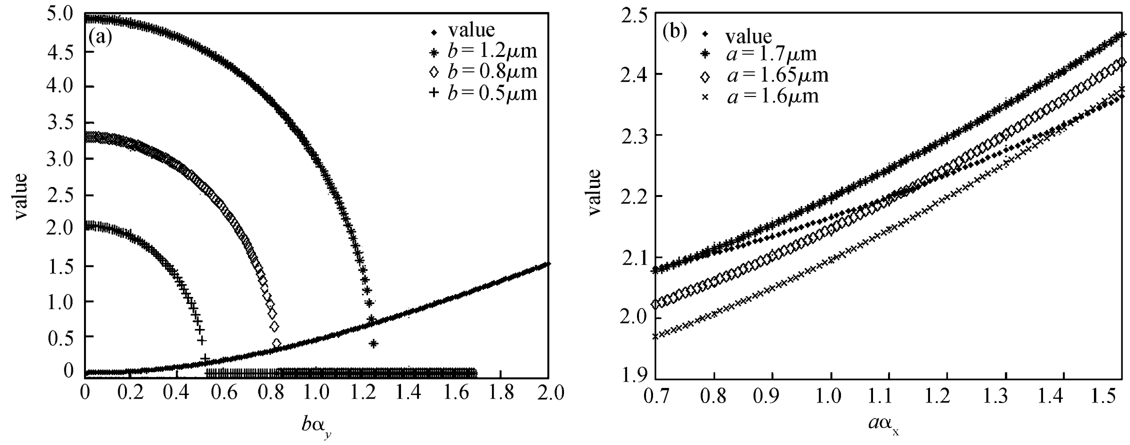

Fig. 7. Solutions of surface modes determined by Eqs. (12) and (13) when $\omega_p = 2 \times 10^{14}$ rad/s, $\lambda = 10$ μm. (a) $a = 2$ μm. (b) $b = 2$ μm.

some special frequencies by a graphical method. These properties are of particular relevance to devices requiring band-pass or single-mode propagation. As it is based solely on epitaxial III–V semiconductor growth[5], the semiconductor metamaterial will bring us more opportunities to design new devices and to have special applications.

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