# Diode parameter extraction by a linear cofactor difference operation method\*

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**Abstract:** The linear cofactor difference operator (LCDO) method, a direct parameter extraction method for general diodes, is presented. With the developed LCDO method, the extreme spectral characteristic of the diode voltage–current curves is revealed, and its extreme positions are related to the diode characteristic parameters directly. The method is applied to diodes with different sizes and temperatures, and the related characteristic parameters, such as reverse saturation current, series resistance and non-ideality factor, are extracted directly. The extraction result shows good agreement with the experimental data.

**Key words:** LCDO; diode; parameter extraction; ideality factor; series resistance **DOI:** 10.1088/1674-4926/31/11/114009 **EEACC:** 2520

# 1. Introduction

The static parameters of diodes, such as series resistance, ideality factor and reverse saturation current, are of great significance in device modeling and characterization. Hence, a number of methods have been proposed to extract the device parameters of diodes<sup>[1-5]</sup>. However, these methods often rely on the complex algorithm and various approximations.

In this paper, a novel parameter extraction method, which we have named the linear cofactor difference operator (LCDO) method, is applied to extract the reverse saturation current, ideality factor and series resistance of the practice diodes. In this method, the unique extreme spectral characteristics of the current-voltage of the diode are revealed, and then the related diode parameters are determined from these extreme spectral characteristics. Its mathematical simplicity and physical concept clarity are the main advantages of this method over the traditional approaches, and thus this method will be a useful tool not only in the parameter extraction of MOSFET devices<sup>[6-8]</sup>, but also in the analysis of diode characteristics and</sup> the extraction of diode static parameters, as shown in the following discussion. Moreover, gate leakage due to the dielectric breakdown in MOSFET with ultra thin gate oxide is also an important reliability concern, and the current characteristic is modeled based on an ideal diode with a series resistance [9, 10]. Therefore, the LCDO method is powerful in extracting the breakdown model parameters.

# 2. LCDO method application

As shown in Ref. [6], If a function f(x) is strictly monotonic, non-linear, continuous and differentiable over region  $(x_0, x_1)$ , there definitely exists a point  $x_p, x_0 < x_p < x_1$ , so that

$$G'(x_{p}) = \frac{\partial G}{\partial x}\Big|_{x=x_{p}} = 0, \qquad (1)$$

where

$$G(x) = \Delta \text{LCDO}(x) = b + K_{p}x - f(x), \qquad (2)$$

is the linear cofactor difference of the measured f(x),  $\Delta LCDO(x)$  is the linear cofactor difference operator, and b and  $K_p$  are the LCDO intersection and linear factor, respectively.

The constants b and  $K_p$  can be determined via the following equations,

$$G(x_1) = b + K_p x_1 - f(x_1) = 0,$$
(3)

$$G(x_0) = b + K_p x_0 - f(x_0) = 0.$$
(4)

The detailed description of this LCDO method principle has been found in Refs. [7, 8]. Here we apply this method to the extraction of the static parameter of a general diode. According to semiconductor device physics, the current of a diode (I) is frequently modeled by the following single exponential equation<sup>[11–15]</sup>,

$$I = I_{\rm s} \left( \exp \frac{V - R_{\rm s}I}{nV_{\rm t}} - 1 \right),\tag{5}$$

where  $I_s$  is the reverse saturation current,  $R_s$  is the series resistance,  $V_t = k_B T/q$  is the thermal voltage and *n* is the diode ideality factor.

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<sup>\*</sup> Project supported by the State Key Development Program for Basic Research of China and the National Natural Science Foundation of China (Nos. 60936005, 60976066).

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Table 1. Detailed structure parameters of D1, D2 and D3.					
Parameter	Junction length	Junction width	N <sup>+</sup> Junction depth	Temperature (°C)	
D1	196	48	1	25	
D2	196	15	4	25	
D3	196	15	4	150	

Table 2. LCDO extreme points and linear cofactor difference diode voltage of D1 with different LCDO factors.

Linear cofactor difference operator factor K	Extreme point and corresponding LCDO voltage		
Elitear coractor difference operator factor Kp	Extreme point $I_{dP}$ (10 <sup>-4</sup> A)	LCDO voltage (V)	
43.6	8.36	0.684	
55.8	8.36	0.673	
76.5	4.83	0.663	

In Eq. (1), the term "-1" is negligible for the forward bias condition. Therefore, Equation (5) can be rewritten as

$$V = nV_{\rm t}\ln\frac{I}{I_{\rm s}} + IR_{\rm s}.$$
 (6)

The LCDO method is applied to Eq. (6) with b = 0, and then the following equation is obtained,

$$\Delta lcdoV(I) = K_{\rm p}I - nV_{\rm t}\ln\frac{I}{I_{\rm s}} - IR_{\rm s},\tag{7}$$

where  $K_p$  is a linear cofactor difference factor.

Since  $\Delta lcdoV(I)$  shows an extreme spectral at current  $I = I_p$ ,

$$\frac{\partial \Delta I c d o V(I)}{\partial I} \bigg|_{I=I_{\rm P}} = 0.$$
(8)

The extreme spectral of the linear cofactor difference diode voltage versus the current can be obtained. Substituting Eq. (8) into Eq. (7) at the extreme position point  $I_p$ , the series resistance is obtained as

$$R_{\rm s} = K_{\rm p} - \frac{nV_{\rm t}}{I_{\rm p}}.\tag{9}$$

For a given diode current versus voltage curve, the diode ideality factor can be obtained from Eq. (9) with two different linear cofactor difference operator factors,  $K_{p1}$  and  $K_{p2}$ ,

$$n = \frac{I_{\rm p1}I_{\rm p2}\left(K_{\rm p1} - K_{\rm p2}\right)}{V_{\rm t}\left(I_{\rm p2} - I_{\rm p1}\right)},\tag{10}$$

where  $I_{p1}$  and  $I_{p2}$  are current values in the extreme point positions corresponding to the two different factors  $K_{p1}$  and  $K_{p2}$ , respectively. Consequently,  $R_s$  can be determined from Eq. (9) and then  $I_s$  from Eq. (5).

### 3. Results and discussion

In order to demonstrate the validity of the LCDO method, the extraction is carried out on different N<sup>+</sup>P diodes (D1, D2 and D3) with various scales and temperatures. The detailed structure parameters are shown in Table 1. The experimental data are recorded using a semiconductor parameter analyzer HP-4156 with a step voltage of 0.02 V.



Fig. 1. Diode voltage linear cofactor difference versus current for different values of  $K_{\rm p}$ .

Figure 1 shows the linear cofactor difference voltage characteristics under different conditions with various linear cofactor difference factors  $K_p$ . The black square denotes the V-Icharacteristics of D1. The other three lines of triangles represent the linear cofactor difference with various  $K_p$  in Eq. (7). These extreme points and corresponding linear cofactor difference diode current are shown in Table 2.

Based on the obtained extreme spectral peak magnitude and positions, the related diode static parameters are easily extracted from the present formula. The ideality factor n = 1.06for D1 is calculated from Eq. (10) under different combinations of  $K_p$ . Although the LCDO peak values and peak positions are different for different  $K_p$  values, the series resistance  $R_s = 6.25 \Omega$  is extracted from Eq. (9) with any known  $K_p$ , nand I. Adjusting Eq. (5) to the experimental values, the reverse saturation current is obtained as  $I_s = 3 \times 10^{-15}$ A.

Figure 2 shows the measured and extracted I-V characteristics of D1. The extraction result, represents a good agreement with the measured data.

To demonstrate the efficiency of the LCDO method, another practical diode (D2) is used for parameter extraction with this method. Following the similar process of D1, the V-Imeasured data and  $\Delta lcdoV(I)$  with different  $K_p$  are shown in Fig. 3 and the static parameters are extracted as n = 1.06,  $R_s = 6.25 \Omega$  and  $I_s = 4.5 \times 10^{-15}$  A. The final extraction result is shown in Fig. 4, which matches well with the measured data.



Fig. 2. Comparison between measured and extracted I-V characteristics of D1 at 25 °C.



Fig. 3. Diode voltage linear cofactor difference versus current for different values of  $K_{\rm p}$ .



Fig. 4. Comparison between the measured and extracted I-V characteristics of D2 at 25 °C.

The LCDO method was also adapted to diodes at different temperatures. Taking D3 as an example, the experimental data are measured at 150 °C. The parameters are calculated as n = 1.18,  $R_s = 7.5 \Omega$  and  $I_s = 9 \times 10^{-9}$  A. The extraction result still matches well with the measurement data, as shown in Fig. 5.



Fig. 5. Comparison between the measured and extracted I-V characteristics of D3 at 150 °C.

#### 4. Conclusions

The extraction of the static parameters of three general diodes is carried out in this paper by the LCDO method. The principle of this method is to apply the special extreme spectral characteristics of the diode voltage versus the current to obtain the related physical parameters, such as the series resistance, ideality factor and reverse saturation current. The extraction results for three different diodes with different structure scales and temperatures match the experimental data very well, which demonstrates the effectiveness of the LCDO method.

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