Modeling of current mismatch induced by random dopant fluctuation in nano-MOSFETs

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Abstract: Deviation of threshold voltage and effective mobility due to random dopant fluctuation is proposed. An improved 65 nm average drain current MOS model called α law is utilized after fitting HSPICE simulating data and extracting process parameters. Then, a current mismatch model of nanoscale MOSFETs induced by random dopant fluctuation is presented based on propagation of variation theory. In test conditions, the calculated standard deviation applying this model, compared to 100 times Monte–Carlo simulation data with HSPICE, indicates that the average relative error and relative standard deviation is 0.24% and 0.22%, respectively. The results show that this mismatch model is effective to illustrate the physical mechanism, as well as being simple and accurate.

Key words: mismatch model; nano-MOSFETs; process variation; random dopant fluctuation
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1. Introduction

As silicon-based MOSFETs are scaled down to nanoscale feature size, process parameter fluctuation (process variation) plays a vital and important role in matching performance and yield analysis. It is a topic of great interest and has become one of the most important issues in IC design and manufacturing[1−7]. Parameter variations result from many aspects, such as deviations in MOS gate width W, length L and gate oxide thickness T_{OX}, also poly-silicon gate particle fluctuations or the random fluctuation of dopant atoms in the substrate and depletion[2, 3, 5, 6]. Among these factors, however, random dopant fluctuation is now the dominant cause and leads to a variety of key parameters, especially threshold voltage V_T, effective carrier mobility μ_{eff}, and current factor β to shift. This changes the drain current I_d significantly and seriously affects circuit accuracy, power and timing. Therefore, various effects induced by random dopant fluctuation have been studied with a very wide range of theoretical and experimental approaches[5−16]. Studies can be traced back to the 1990s on the threshold voltage deviation of experiments and the statistical distribution of dopant atoms[9, 12]. With the development and help of EDA tools, 3D simulators have been used to easily study the effect of device characteristics due to dopant fluctuation in MOSFETs[10, 11, 13]. Moreover, Reference [16] has claimed that dopant fluctuation accounts for about 60% to 80% of total drain current mismatch and uses a microscopic multi-transistor to model the I_d mismatch, but with more complex concepts and calculations. Furthermore, the most recent studies have again used advanced 3D simulators to investigate discrete dopant-induced device variations[4, 6, 14, 15]. Thus, the presented studies of device characteristics and standard deviation analysis of I_d induced by random dopant fluctuation rely mainly on simulation tools using a complex and time-consuming Monte–Carlo approach. This leads to the physical mechanism not being clearly known and does not make full use of existing advanced physical-based compact MOS models. All of the above is not conducive to early estimates of the deviation of drain current fluctuations and circuit performance, which in turn seriously effects IC design and the development cycle. So achieving a simple and accurate nano-scale MOSFET mismatch model induced by dopant fluctuations has become an urgent task.

2. Statistical distribution and threshold voltage deviation due to random dopant fluctuation

The results show that the probability density function (PDF) of dopant number in MOS VLSI manufacturing obeys Poisson distribution in statistical theory. Supposing that the dopant atoms are independent of each other, the number of atoms N_0 under the channel, in volume of v_B, can be expressed as[5, 9]:

$$P(N = N_0) = \frac{(N_{av}v_B)^{N_0}}{N_0!}\exp(-N_{av}v_B), \tag{1}$$

where N_{av} denotes the average or effective substrate dopant concentration. According to probability theory, the mean, standard deviation and relative standard deviation of Eq. (1) are:

$$\mu[N] = N_{av}v_B, \tag{2}$$

$$\sigma[N] = \sqrt{N_{av}v_B}, \tag{3}$$

$$\frac{\sigma[N]}{\mu[N]} = \frac{1}{\sqrt{N_{av}v_B}}. \tag{4}$$

Equations (2)–(4) show the average absolute number and the standard deviation increase. However, the relative standard deviation decreases with the number of dopant atoms. This means that smaller size devices with less atoms are prone to

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larger deviations due to a small amount of dopant atom fluctuation, which means that parameter variations caused by random dopant fluctuations in nano-MOSFET devices must be considered. The volume v_B in Eqs. (1)–(4) is defined as the multiplication of the depletion thickness and the channel area:

$$v_B = W_{eff} L_{eff} x_d, \tag{5}$$

where W_{eff} and L_{eff} are effective channel width and length, and x_d is the depletion layer thickness:

$$x_d = \sqrt{\frac{4\varepsilon_0 \varepsilon_{si} |\Phi_F|}{q N_{av}}}, \tag{6}$$

where ε_0 is the dielectric constant, and ε_{si} is the relative dielectric constant of silicon. q denotes the electronic charge and Φ_F is the Fermi potential:

$$\Phi_F = \frac{kT}{q} \ln \frac{N_{av}}{n_i}, \tag{7}$$

where n_i is the intrinsic silicon carrier concentration and $n_i = 1.256 \times 10^{16}$ m^{-3} when $T = 298$ K. An estimation of the relative deviation under various dopant atoms can be given by the above expressions. For a 65 nm MOS device process, $\sigma(N)/\mu(N) = 9.98\%$ is the basis of circuit simulation when $W_{eff} = 60$ nm, $L_{eff} = 29.5$ nm and $N_{av} = 2.54 \times 10^{18}$ cm^{-3}.

Existing approaches such as our theoretical analysis, experimental study and simulation show that the threshold voltage deviation induced by random dopant fluctuation can be simply expressed as below.

$$\sigma(V_T) = \frac{q}{2} \sqrt{\frac{N_{av} x_d}{W_{eff} L_{eff}}} \frac{T_{ox}}{\varepsilon_{ox}}. \tag{8}$$

If the substrate doping is uniform, the N_{av} is almost a constant, and if the doping is non-uniform with distribution of doping profile $N_a(x)$, then N_{av} can be expressed as an integrated equation (9), below:

$$N_{av} = 3 \int_{x=0}^{x=x_d} \frac{N_a(x)}{x_d} \left(1 - \frac{x}{x_d}\right)^2 dx. \tag{9}$$

3. Effects on effective mobility due to dopant atom fluctuation

The value of current factor β directly affects the drain current, while effective mobility μ_{eff} is one of factors in β. Therefore, effective mobility must be considered in a current mismatch model. However, theoretical analysis and experimental studies have shown that μ_{eff} changes with the horizontal and vertical electrical field. BSIM4.6.1 establishes the relationship between the lateral effective field E_{eff} (i.e. the relationship between the V_{gs} and V_T) and low field mobility μ_0:

$$\mu_{lf} = \frac{\mu_0}{1 + (E_{eff}/E_0)^{\nu}}, \tag{10}$$

where ν is a constant related to mobility degradation and

$$E_{eff} = \frac{Q_B + Q_n/2}{\varepsilon_0 \varepsilon_{si}} \approx \frac{V_{GS} + V_T}{6\mathrm{TOXE}}, \tag{11}$$

where TOXE, Q_B and Q_n are the electrical equivalent gate oxide thickness, depletion charge and channel charge respectively, which obviously indicates that both latter parameters have a direct relationship with the dopant atoms. Since calculating μ_{lf} based on Eqs. (10) and (11) is inconvenient, a universal approximate relationship in Ref. [18] is given as:

$$\mu_{lf} = 32500 E_{eff}^{-\frac{1}{3}}. \tag{12}$$

Also the vertical electric field V_{ds} or velocity saturation must be considered because the gate length is very short in nano feature sizes, then:

$$\mu_{eff} = \frac{\mu_{lf}}{1 + \dfrac{\mu_{lf} |\varepsilon_L|}{v_{sat}}} = \frac{\mu_{lf}}{1 + \dfrac{\mu_{lf} V_{ds}}{L_{eff} v_{sat}}}, \tag{13}$$

where v_{sat} is the saturation velocity of the carrier in a channel and given as 12.43×10^4 m/s in a 65 nm SPICE model. It should be noted that μ_{eff} is equivalent and changes from source to drain in an MOSFET. Difrenza *et al.* has found the relationship between the standard deviation μ_{eff} and V_T in studies of $\beta = \mu_{eff} C_{ox} W_{eff}/L_{eff}$ mismatch[17]:

$$\frac{\sigma(\mu_{eff})}{\mu_{eff}} = \mu_{eff} \alpha_d C_{ox} \sigma(V_T), \tag{14}$$

where α_d is the Coulomb scattering coefficient.

4. Current mismatch model

4.1. Theoretical basis

Mismatch models of current in MOSFETs are mostly from the propagation of variance relationship (POV) based on the Taylor series. The relative standard deviation is[1, 2, 19]:

$$\frac{\sigma(I_d)}{I_d} = \left[\sum_i \left(\frac{\partial I_d}{\partial p_i}\right)^2 \frac{\sigma^2(p_i)}{p_i^2}\right]^{\frac{1}{2}}, \tag{15}$$

where p_i, $\sigma^2(p_i)$ denotes the ith electrical or process parameter and its variance, respectively. According to statistical theory, all p_i should be independent of each other, otherwise, the correlation coefficient γ should be added. It can be seen from Eq. (15) that the standard deviation of I_d can be calculated as long as an analytical expression is obtained where the parameters are associated with random doping. However, the widely used BSIM4 model for nano-scale MOSFETs is difficult to use because it takes account for various physical effects and includes a lot of correlation factors, where the redundancy and correlation between parameters exist, leading it being large and complicated. Thus, applications of POV to analyze current mismatch in nano-MOSFETs induced by random doping have not yet seen and reported based on the BSIM4 model.

4.2. An improved ALPHA law model

ALPHA (α) law is a widely used submicron MOSFET model. A simple improved 65 nm α law called the average drain current model[1, 20] is in Eq. (16), where $C_{on} = \mu_{eff} C_{ox}$, index α_p is between 1–2, λ is the channel length modulation

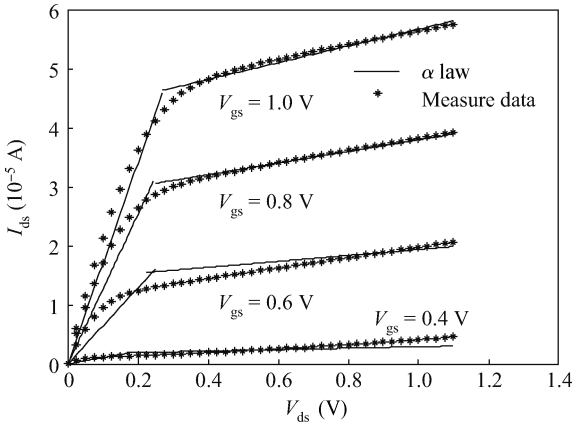

Fig. 1. α law with fitted simulation data.

factor and V_{DO} is a matching constant. When V_{ds} is the saturation voltage, $V_{DO} = V_{ds}/(1 + \lambda V_{ds})$. For determining the parameters of β, V_T, α_p, λ and V_{DO} and demonstrating its validity and reliability, it should be fitted by the current data under different V_{gs}, V_{ds} using HSPICE simulation with the BSIM4 (SPICE LEVEL 54) 65 nm process model[20].

$$I_{davg} = \begin{cases} C_{on}\dfrac{W}{L}\left(V_{gs} - V_T\right)^{\alpha_p}(1 + \lambda V_{ds}) \\ C_{on}\dfrac{W}{L}\left(V_{gs} - V_T\right)^{\alpha_p} V_{ds}/V_{DO} \end{cases}$$

$$= \begin{cases} \beta\left(V_{gs} - V_T\right)^{\alpha_p}(1 + \lambda V_{ds}), & \text{saturation,} \\ \beta\left(V_{gs} - V_T\right)^{\alpha_p} V_{ds}/V_{DO}, & \text{linear.} \end{cases} \quad (16)$$

Researchers have pointed out that λ is not same under different V_{gs} and can be fitted by the quadratic model[20]. In this study, the model is depicted in Fig. 1 at $V_{gs} = 1$ V, 0.8 V, 0.6 V and 0.4 V, respectively, which shows that this model is suitable except for individuals at the junction of saturation and the linear region or $V_{ds} \approx 0$ V. For example of $V_{gs} = 1$ V, data of a total of 46 have been chosen to fit from $V_{ds} = 0$ V to 1.1 V every 0.025 V a total of 33 data in saturation (0.3–1.1 V) and 13 in the linear region (0–0.3 V), respectively. The interested parameters $V_T = 0.369$ V, $\alpha_p = 1.10$, $\lambda = 0.333$ V^{-1}, $V_{DO} = 0.25$ V, are extracted. It must be noted that the value of β changes with V_{gs} and V_{ds} are mainly due to the effective mobility μ_{eff} changes; in test conditions β is about 7×10^{-5} A/V^2. Statistical results show that the average relative error in saturation (plus and minus offset exist) is –0.011%, maximum 4.81%, minimum 0.04%, and the average standard deviation is 1.26%, which is better than Ref. [1]. It is slightly worse in the linear region for the average relative error of 17.1%, standard deviation of 33.4%, which is still close to the experimental results in Ref. [1]. Therefore, the α law is suitable for nano-scale processes and better in analog applications, although analog circuits usually work in the saturation.

4.3. Mismatch model

In Eq. (16), the parameters β and V_T change with dopant fluctuations and are regarded as a certain correlation. The main reason is T_{OX}, as some research papers have reported.

Nevertheless, theoretical analysis shows that the parameters μ_{eff}, C_{ox}, W_{eff} and L_{eff} are mutually independent of β[16, 17, 19], so the relative variance of β is:

$$\frac{\sigma^2(\beta)}{\beta^2} = \frac{\sigma^2(\mu_{eff})}{\mu_{eff}^2} + \frac{\sigma^2(C_{ox})}{C_{ox}^2} + \frac{\sigma^2(W_{eff})}{W_{eff}^2} + \frac{\sigma^2(L_{eff})}{L_{eff}^2}. \quad (17)$$

Existing papers and this study have shown that dopant fluctuation is only closely associated with μ_{eff}[17], so β mismatch due to random dopant can be expressed as:

$$\frac{\sigma^2(\beta)}{\beta^2} = \frac{\sigma^2(\mu_{eff})}{\mu_{eff}^2}. \quad (18)$$

Now the POV relationship based current mismatch, taking into account the correlation between parameters induced by random dopant fluctuation, can be expressed as:

$$\frac{\sigma^2(I_d)}{I_d^2} = \frac{\sigma^2(\beta)}{\beta^2} + \frac{\alpha_p^2\sigma^2(V_T)}{\left(V_{gs} - V_T\right)^2} + 2\gamma\frac{\sigma(\beta)}{\beta}\frac{\alpha_p\sigma(V_T)}{V_{gs} - V_T}$$

$$= \frac{\sigma^2(\mu_{eff})}{\mu_{eff}^2} + \frac{\alpha_p^2\sigma^2(V_T)}{\left(V_{gs} - V_T\right)^2} + 2\gamma\frac{\sigma(\mu_{eff})}{\mu_{eff}}\frac{\alpha_p\sigma(V_T)}{V_{gs} - V_T}, \quad (19)$$

where γ is the correlation coefficient between β and V_T. This mismatch model is much simpler than many others because it has less parameters and does not have complicated matrix equations. However, the correlation between parameters is considered, so accuracy can be improved. Research has shown that γ is negative and relatively small, generally about 0.1[21], so –0.1 is used in the model. Then Equation (19) together with Eq. (14) is:

$$\frac{\sigma^2(I_d)}{I_d^2} = \left[(\mu_{eff}\alpha_d C_{ox})^2 + \frac{\alpha_p^2}{\left(V_{gs} - V_T\right)^2} + 2\gamma\frac{\mu_{eff}\alpha_d C_{ox}\alpha_p}{V_{gs} - V_T}\right]\sigma^2(V_T). \quad (20)$$

5. Simulation data and results

For studying the deviation of I_d from random dopant effects, a MOSFET circuit realizing V_{ds}–I_d characteristics is designed with BSIM4 (SPICE LEVEL 54) 65 nm model. 100 Monte–Carlo simulations are run with random dopant variations in accordance with Eq. (4) at a concentration of impurities $N_{dep} = N_{sub} = N_{av} = 2.54 \times 10^{18}$ cm^{-3} to obtain the curves and output data. Figure 2 obtained in $V_{gs} = V_{ds} = 1$ V shows that the drain current varies significantly with random dopant fluctuations. The maximum (Max), minimum (Min), average (Mean), absolute standard deviation (σ) and relative standard deviation (σ/Mean) of I_d calculated in various another V_{ds} are shown in Table 1, where the theoretical analysis of (σ/Mean) is based on Eq. (20). It is clear that the theoretical and simulation

Table 1. Statistical data of current mismatch induced by random dopant fluctuation.

V_{ds} (V)		1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
Max (μA)		63.552	62.234	60.897	59.528	58.103	56.572	54.800	52.314	47.369
Min (μA)		51.869	50.895	49.910	48.904	47.864	46.765	45.545	44.015	41.376
Mean (μA)		57.624	56.499	55.359	54.193	52.984	51.696	50.235	48.298	44.631
σ (μA)		2.601	2.526	2.450	2.371	2.287	2.192	2.071	1.859	1.335
σ/Mean	Simulation	4.51	4.47	4.43	4.37	4.32	4.24	4.12	3.85	3.99
(%)	Theoretical	4.25	4.25	4.25	4.24	4.25	4.25	4.25	4.27	4.27

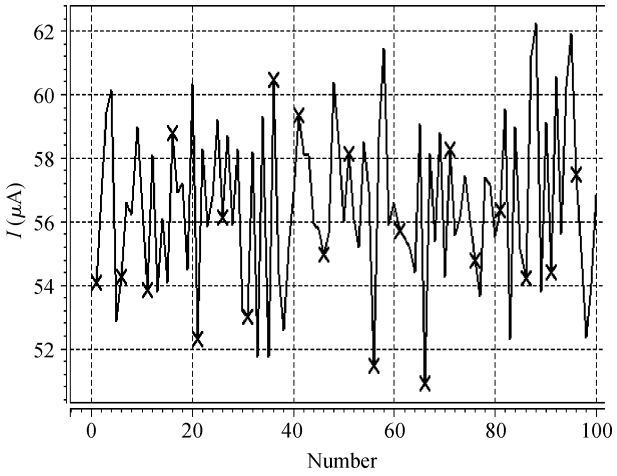

Fig. 2. Monte–Carlo simulation shows current variation.

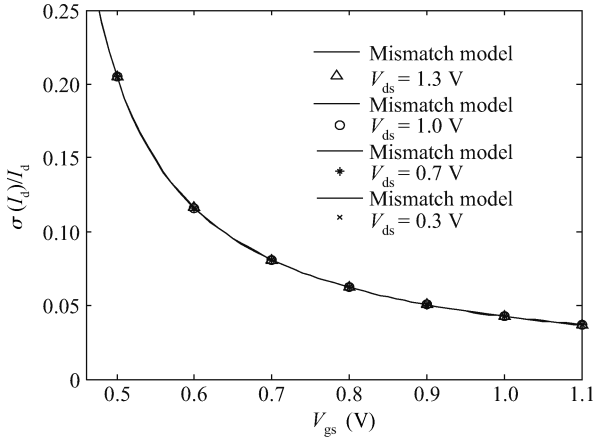

Fig. 3. Mismatch curves in different V_{ds}.

data are both close to about 4% in test conditions. In addition, the comparison of σ/Mean between the theoretical and simulated data shows that average relative error (plus or minus offset exist) and relative standard deviation is 0.24% and 0.22%, respectively. Curves of the mismatch model with V_{gs} at different V_{ds} are depicted in Fig. 3, indicating that the mismatch is sensitive to V_{gs} but not to V_{ds}. Therefore, V_{ds} is neglected as a more simple form of mismatch model is usually accepted. It should be pointed out that the above data are from an effective width and length ratio of 60 nm/29.5 nm. However, the mismatch model is accurate for different width and length ratios, such as 590 nm/29.5 nm in the simulation test.

6. Conclusions

This paper has studied the physical mechanisms and deviations of V_T and μ_{eff} associated with MOSFET current mismatch induced by random doping fluctuation. A nano-MOSFET current mismatch model is presented through applying an average drain current model called the improved α law model. The model can estimate and predict the performance of MOS devices and circuits in the early stages of circuit design. Compared to HSPICE data in 100 Monte–Carlo simulations of BSIM4 65 nm process, its average relative error and relative standard deviation is 0.24% and 0.22%, respectively, in test conditions. Consequently, the proposed mismatch model is simple, effective and also accurate.

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