

Design and optimization of CMOS LNA with ESD protection for 2.4 GHz WSN application*

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Abstract: A new optimization method of a source inductive degenerated low noise amplifier (LNA) with electrostatic discharge protection is proposed. It can achieve power-constrained simultaneous noise and input matching. An analysis of the input impedance and the noise parameters is also given. Based on the developed method, a 2.4 GHz LNA for wireless sensor network application is designed and optimized using 0.18- μm RF CMOS technology. The measured results show that the LNA achieves a noise figure of 1.59 dB, a power gain of 14.12 dB, an input 1 dB compression point of -8 dBm and an input third-order intercept point of 1 dBm. The DC current is 4 mA under a supply of 1.8 V.

Key words: LNA; ESD protection; noise and input impedance matching; CMOS

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1. Introduction

The low noise amplifier (LNA) is one of the key components in most communication systems, because it is the first active block after the antenna and determines the dynamic range of the receiver. The conventional inductively degenerated common source LNA is very popular for narrow band applications. For this topology, there are several optimization methods proposed in published studies^[1,2], such as the power-constrained simultaneous noise and input matching (PCSNIM) technique proposed in Ref. [2]. However, this ignored the effect of the input parasitic capacitance of the electrostatic discharge (ESD) protection circuits and the pad (usually very small compared to the ESDs). Considering the reliability of products, the ESD protection of a chip is very important and necessary. At radio frequency, the parasitic capacitance of the input port has a significant effect on the performance of the LNA^[3], such as noise figure, power gain, and so on. Thus, it must be taken into account in the circuit design. Sivonen^[4] has proposed an ESD and LNA co-design method. Its biggest drawback is that it needs two off-chip lumped components to realize the input power matching, which will increase the design complexity and the system cost.

In this paper, a source inductive degenerated LNA with ESD is analyzed in detail. An optimization method, which can achieve ESD protected LNA power-constrained simultaneous noise and input matching, is described. It features also the fact that only one off-chip lumped component is needed. An LNA is designed and measured using 0.18- μm RF CMOS technology. The measured results show that the LNA achieves an NF of 1.59 dB and a power gain of 14.12 dB.

2. Circuit analysis

2.1. Input power matching of LNA with ESD

The circuit of the LNA with ESD consisting of two reverse biased diodes is shown in Fig. 1(a), and Figure 1(b) shows its small signal equivalent circuit, in which C_p represents the parasitic capacitance of the ESD and the input pad, while L_g consists of the bonding wire inductor connecting chip pad and PCB and the SMT inductor on PCB.

As shown in Fig. 1(b), the input impedance looking into the gate of M1 can be expressed as

$$Z'_{in} = \frac{1}{Y'_{in}} = sL_s + \frac{1}{sC_t} + \frac{g_m L_s}{C_t} = \frac{1}{sC_1} + R_{eq}, \quad (1)$$

where

$$C_t = C_{gs} + C_{ex}, \quad \frac{1}{C_1} = \frac{1}{C_t} - \omega^2 L_s, \quad R_{eq} = \frac{g_m L_s}{C_t}, \quad s = j\omega.$$

Through a series of parallel-series and series-parallel conversions, as shown in Fig. 2, we have

$$\begin{aligned} \text{Re}[Z_{in}] &= R_3 \approx \frac{1}{R_2 (\omega C_2)^2} \approx \frac{1}{\frac{1}{R_{eq} (\omega C_1)^2} \omega^2 (C_1 + C_p)^2} \\ &= R_{eq} \left(\frac{C_1}{C_1 + C_p} \right)^2 = \frac{g_m L_s}{C_t} \left(\frac{C_1}{C_1 + C_p} \right)^2, \end{aligned} \quad (2)$$

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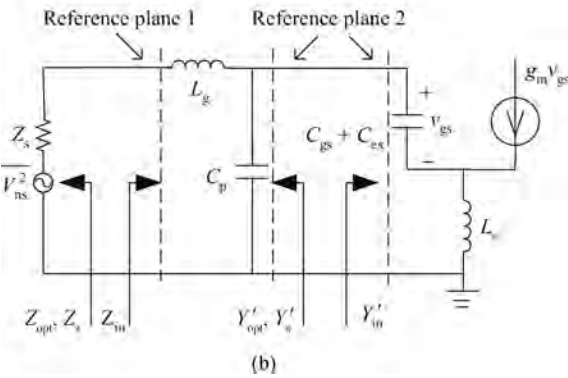
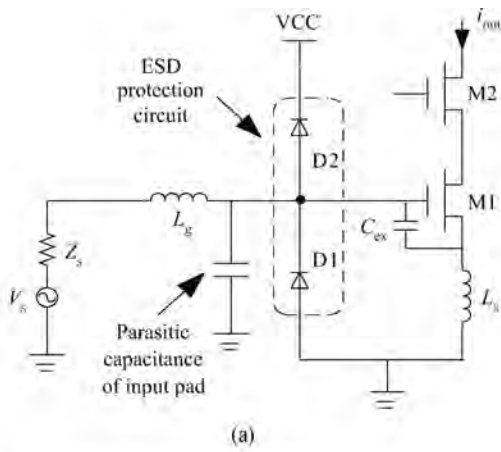


Fig. 1. (a) Circuit of LNA with ESD. (b) Its small-signal equivalent circuit.

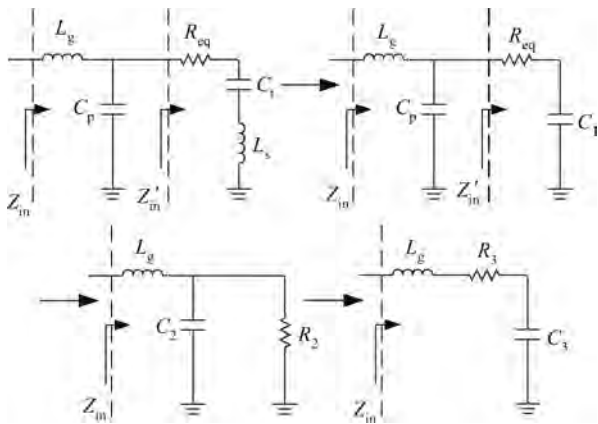


Fig. 2. Input impedance conversion.

$$\text{Im}[Z_{in}] = sL_g + \frac{1}{sC_3} \approx \frac{\frac{1}{sC_t} + sL_s}{1 + sC_p \left(\frac{1}{sC_t} + L_s \right)} + sL_g. \quad (3)$$

And the power matching requirement is that

$$\begin{cases} \text{Re}[Z_{in}] = \text{Re}[Z_s], \\ \text{Im}[Z_{in}] = -\text{Im}[Z_s]. \end{cases} \quad (4)$$

From Eq. (2), it can be seen that the parallel parasitic capacitance C_p transforms downwards the real part of the input

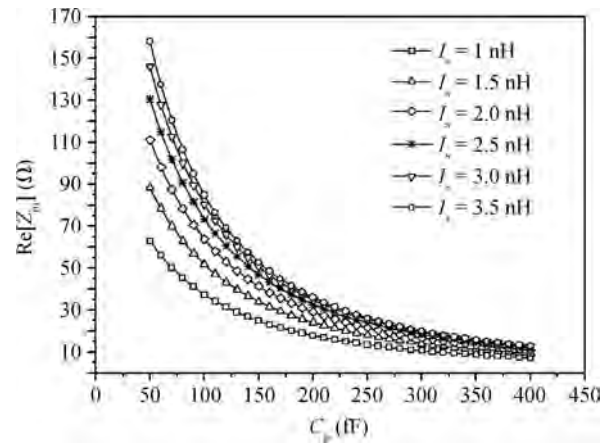


Fig. 3. Effect of C_p on the real part of the input impedance.

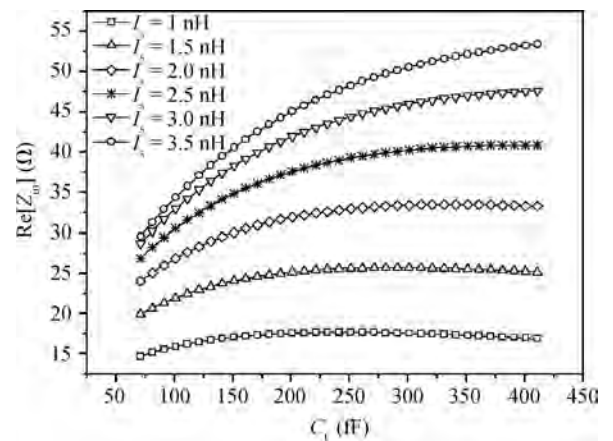


Fig. 4. Effect of C_t on the $\text{Re}[Z_{in}]$.

impedance, and the simulation results are shown in Fig. 3 with $C_{ex} = 0$. The real part of Z_{in} is below 30Ω for $C_p > 230 \text{ fF}$, which is unacceptable for a 50Ω system. Very small mismatch of $\text{Im}[Z_{in}]$ will make S_{11} be above -10 dB . C_{ex} is very important in solving this problem as will be shown in next paragraph.

We substitute C_1 in Eq. (2) with its expression, and then we get the following equation,

$$\text{Re}[Z_{in}] = \frac{g_m L_s}{\frac{a^2}{C_t} + b^2 C_t + 2ab}, \quad (5)$$

where $a = C_p$, $b = 1 - \omega^2 L_s C_p$.

From Eq. (5), it can be seen that $\text{Re}[Z_{in}]$ increases as the C_t increases when $C_t < a/b$, as shown in Fig. 4. This is quite different from that of the CS LNA without ESD, in which the effect of C_t is only reducing the real part of the input impedance and is taken advantage of in the input power match. There are two ways to increase C_t . One is by increasing C_{gs} and the other is by increasing C_{ex} . Exploiting the first method will increase the power consumption. So, C_{ex} is increased to match the real part of the input impedance to 50Ω , and the real part of the input impedance matching could be achieved by choosing appropriate g_m , L_s and C_{ex} .

According to Eq. (4), the inductor L_g must be selected as

$$L_g = \frac{1}{C_3\omega^2} = \left[\omega^2 \left(\frac{C_t}{1 - \omega^2 C_t L_s} + C_p \right) \right]^{-1}. \quad (6)$$

In this method, a single off-chip component L_g can make the input port matching to 50Ω .

2.2. ESD protected LNA's power-constrained simultaneous noise and input power matching

After a complex derivation (the detailed derivation is shown in Appendix A), the optimum source admittance (Y'_{opt}) of the circuit shown in Fig. 1(b) can be given by

$$Y'_{opt} = G_{opt} + jB_{opt}, \quad (7)$$

where

$$\begin{aligned} B_{opt} &= -B_c \\ &= - \left[\omega C_t \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) - \omega^3 C_t^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right. \right. \\ &\quad \left. \left. + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \right] \left[1 + \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} + \frac{\alpha^2 \delta}{5\gamma} \right) \right. \\ &\quad \left. - 2\omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right]^{-1}, \end{aligned}$$

and

$$\begin{aligned} G_{opt} &= \left[B_c^2 \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right. \\ &\quad \left. + 2B_c \omega^3 C_t^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \right. \\ &\quad \left. - 2B_c^2 \omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) + \omega^2 C_t^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right. \right. \\ &\quad \left. \left. + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) - 2B_c \omega C_t \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) + B_c^2 \right]^{1/2} \\ &\quad \times \left[1 + \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} + \frac{\alpha^2 \delta}{5\gamma} + 1 \right) \right. \\ &\quad \left. - 2\omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right]^{-1/2}. \end{aligned}$$

In order to achieve the ESD protected LNA's power-constrained simultaneous noise and input matching, the following equation must be satisfied,

$$Y'_{opt} = Y'_s, \quad (8)$$

where

$$Y'_s = \frac{1}{Q^2 R_s} + j \left(\omega C_p - \frac{1}{\omega L_g} \right),$$

and

$$Q = \frac{\omega L_g}{R_s}.$$

The two conditions that satisfy Eq. (8) are as follows,

(1) $\text{Re}[Y'_{opt}] = \text{Re}[Y'_s]$:

$$\begin{aligned} &\left[B_c^2 \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right. \\ &\quad \left. + 2B_c \omega^3 C_t^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \right. \\ &\quad \left. - 2B_c^2 \omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right. \\ &\quad \left. + \omega^2 C_t^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \right. \\ &\quad \left. - 2B_c \omega C_t \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) + B_c^2 \right]^{1/2} \\ &\quad \times \left[1 + \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} + \frac{\alpha^2 \delta}{5\gamma} + 1 \right) \right. \\ &\quad \left. - 2\omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right]^{-1/2} \\ &= \frac{1}{Q^2 R_s}. \quad (9) \end{aligned}$$

(2) $\text{Im}[Y'_{opt}] = \text{Im}[Y'_s]$:

$$\begin{aligned} &- \left\{ \omega C_t \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) - \omega^3 C_t^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right. \right. \\ &\quad \left. \left. + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \right\} \left\{ 1 + \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} + \frac{\alpha^2 \delta}{5\gamma} + 1 \right) \right. \\ &\quad \left. - 2\omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma} + 1} \right) \right\}^{-1} \\ &= \omega C_p - \frac{1}{\omega L_g}. \quad (10) \end{aligned}$$

To achieve ESD protected LNA's power-constrained simultaneous noise and input matching, Equations (4) and (8) must be satisfied simultaneously. Since there are four equations and five unknowns (g_m , C_{gs} or W , C_{ex} , L_s , L_g), Equations (4) and (8) can be solved for an arbitrary value of Z_s by fixing the power consumption of the LNA.

3. Circuit design and measured results

The design is based on a $0.18 \mu\text{m}$ RF CMOS technology. In order to acquire 2 kV HBM ESD protection, the value of the ESD parasitic capacitance is about $200 \text{ fF}^{[5]}$. The input pad is only made of the top metal (M6) to reduce its parasitic capacitance. In this design, it is about 35 fF , so, the value of C_p is about 235 fF . The value of Z_s is 50Ω . The supply voltage is

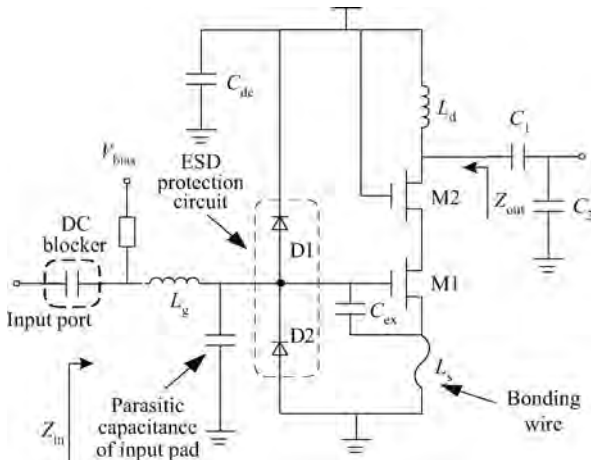


Fig. 5. Schematic of LNA with ESD protection circuit.

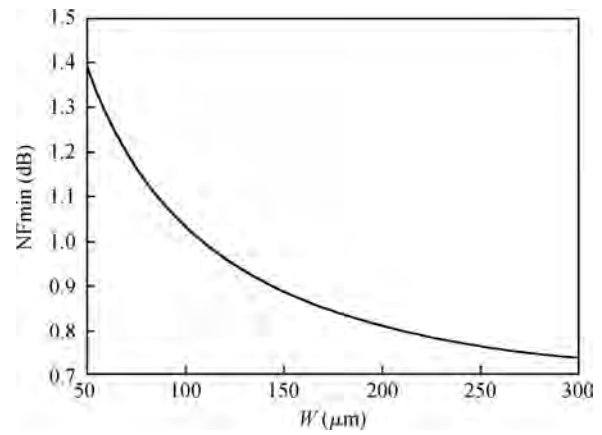


Fig. 7. NFmin versus W ($V_{bias} = 0.6 V$).

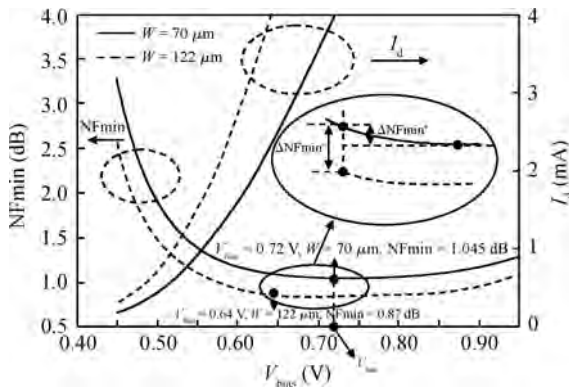


Fig. 6. DC current and NFmin versus V_{bias} .

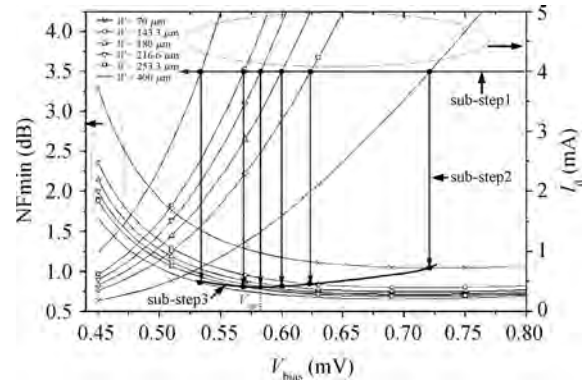


Fig. 8. DC current and NFmin versus V_{bias} relationship with different W .

fixed at 1.8 V and the power is constrained at 7.2 mW, so the DC current is constrained at 4 mA. The complete schematic of the LNA with an ESD protection circuit is shown in Fig. 5. Before the design of the LNA with ESD is described, the disadvantage of the PCSNIM^[2] is analyzed.

3.1. Analysis of the disadvantage of PCSNIM technology

The choice of bias voltage (V_{bias}) and gate width (W) of the PCSNIM is divided into two steps. First, choose the DC-bias, for example, the bias point that provides minimum NFmin. Second, choose the transistor size based on the power constraint. With this method, an NFmin is obtained, but it is not the minimum one. Figure 6 shows the simulations of DC current and NFmin at 2.4 GHz versus V_{bias} . As Figure 6 shows, according to PCSNIM, NFmin gets its minimum value, when the V_{bias} is 0.72 V. Based on the 4 mA DC current (I_d) limit, then the W is determined to be 70 μm , and the NFmin is 1.045 dB at 2.4 GHz. But when the V_{bias} is 0.6437 V, W is 122 μm , current consumption is 4 mA and the NFmin is 0.87 dB, which is smaller than 1.045 dB. This problem exists in the design process of LNA both with ESD and without. The reason for this problem is described as follows. Because of the current limit, V_{bias} and W restrict each other. If V_{bias} decreases, W will increase. Figure 7 demonstrates that with an increase in W , NFmin decreases, and more and more slowly (at a constant V_{bias}). Furthermore, NFmin rises very slightly in the vicinity

of the point V_{min} that provides the minimum NFmin (0.72 V in Fig. 6), and increases more and more quickly when V_{bias} decreases from point V_{min} . So, let the variation in NFmin be analyzed when V_{bias} is reduced from V_{min} and W is increased (maintain a constant current consumption). Divide V_{bias} into a large number of steps; for example, 100, from V_{min} to V_{TH} . These steps are called $V_1, V_2, \dots, V_n, \dots, V_{100}$, with their corresponding W are $W_1, W_2, \dots, W_n, \dots, W_{100}$. When n is small, the increase in NFmin (ΔNFmin^+) due to the decrease in V_{bias} from V_n to V_{n+1} with the W of W_n is smaller than the decrease in NFmin (ΔNFmin^-) due to the increase in W from W_n to W_{n+1} at the bias voltage of V_{n+1} (shown in Fig. 6), then at one combination of W and V_{bias} , ΔNFmin^+ will equal ΔNFmin^- , and after that, ΔNFmin^+ will be greater than ΔNFmin^- . So NFmin falls at first, reaches its minimum value and then rises. As a result, this combination of W and V_{bias} will provide the real minimum NFmin with the power constraint. This method will be described in part B.

3.2. Circuit optimization technology of LNA with ESD

The designing of an LNA is processing at reference plane 1 in Fig. 1. The solutions of Eqs. (4) and (8) also satisfy the following equations,

$$\begin{cases} \text{Re}[Z_{opt}] = \text{Re}[Z_{in}] = 50, \\ \text{Im}[Z_{opt}] = \text{Im}[Z_{in}] = 0. \end{cases} \quad (11)$$

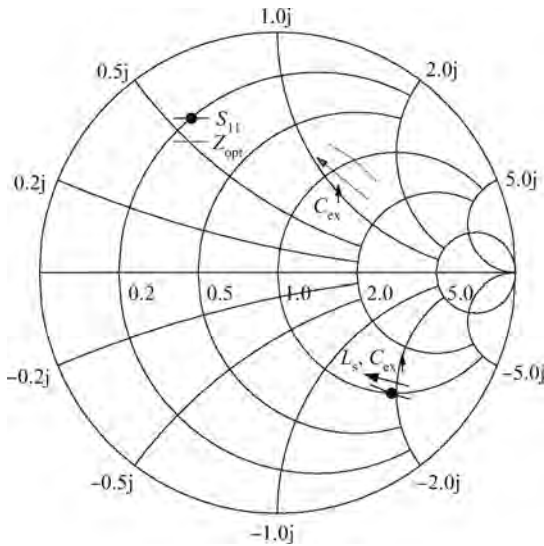


Fig. 9. Tune C_{ex} and L_s to satisfy $\text{Re}[Z'_{opt}] = \text{Re}[Z'_{in}] = R_s$.

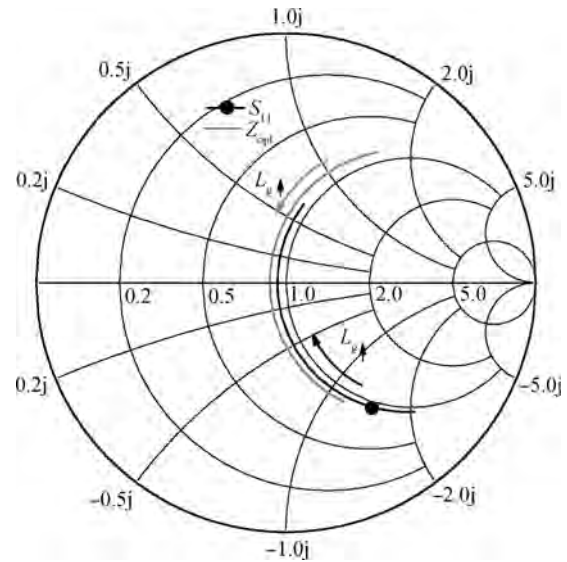


Fig. 10. Change process after setting L_g .

The design process of LNA with ESD is described as follows:

Step 1: Tune the size of the ESD circuit, and let the parasitic capacitance be equal to 200 fF. Add the ESD circuit into the LNA.

Step 2: Determine the DC-bias V_{bias} and the width W of M1 to provide the minimum NFmin. This step is divided into three sub-steps.

(1) Simulate the DC current and NFmin versus V_{bias} relationship with different W , respectively, and plot them in the same figure, as shown in Fig. 8. Then plot the constant I_d line of the maximum I_d (4 mA in this paper). This crosses with the DC current responses with different W . Mark these intersections. They represent the combinations of V_{bias} and W that meet the power requirement (4 mA). In Fig. 8, sub-step1 shows this step.

(2) Mark the intersections' corresponding points in the NFmin curves. In Fig. 8, sub-step 2 shows this step.

(3) String together these points to obtain a curve. This curve stands for NFmin of the combination of V_{bias} and W that meet the power requirement. From it, the optimal combination of W and V_{bias} can be obtained. In Fig. 8, sub-step 3 shows this step. The optimal combination is $220 \mu\text{m}$ W and 0.575 V V_{bias} . Its NFmin is only 0.78 dB, 0.265 dB smaller than that obtained though PCSNIM.

Step 3: Set L_g 's parasitic resistance (R_{Lg}) and L_s . $\text{Im}[Z_{opt}]$ of the circuit can be seen as an approximate value of L_g . According to the bond inductor's quality factor (about 75 at 2.4 GHz), a rough vaule of R_{Lg} can be obtained, and it will be adjusted in Step 6. Set the initial value of L_s to a value not too large (e.g. 1 nH) and it will be tuned in Step 4. Large L_s will reduce the power gain and worsen the noise performance.

Step 4: Set the value of L_g to 0. Tune C_{ex} and L_s to satisfy the equation $\text{Re}[Z_{opt}] = \text{Re}[Z_{in}] = 50 \Omega$. Due to the effect of C_p , usually $\text{Re}[Z_{in}]$ is much less than 50Ω , while $\text{Re}[Z_{opt}]$ is still more than it. According to Eq. (5) and the description above, it will make $\text{Re}[Z_{in}]$ increase and $\text{Re}[Z_{opt}]$ decrease to increase C_{ex} . So $\text{Re}[Z_{opt}] = \text{Re}[Z_{in}] = 50 \Omega$ will be achieved by choosing a proper value for C_{ex} . If even a very large value

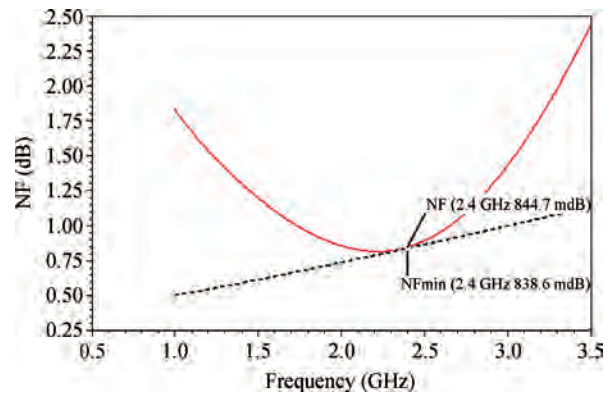


Fig. 11. Simulated NF and NFmin.

of C_{ex} can't make $\text{Re}[Z_{in}]$ match 50Ω , increase the value of L_s appropriately and rechoose the value of C_{ex} . The process is shown in Fig. 9.

Step 5: Calculate the value of L_g according to simulated $\text{Im}[Z_{opt}]$ in Fig. 9. After setting L_g to that value, $\text{Im}[Z_{opt}]$ will move down along the 50Ω circle, while $\text{Im}[Z_{in}]$ will change in the opposite direction. They will meet at the origin because the imaginary parts of Z_{opt} and Z_{in} are almost equal. The change process of S_{11} and Z_{opt} is shown in Fig. 10.

Step 6: Calculate L_g 's parasitic resistance according to its quality factor and set it to that value.

Step 7: Repeat Step 4, until the input power matching and noise matching can be achieved simultaneously through a fine tuning of L_g .

Step 8: If Step 7 can't be achieved, make a tuning of V_{bias} and W , and then repeat Steps 4 to 7. As Figure 8 shows, the increase in NFmin is very slight because of a small tuning of V_{bias} .

The values of C_{ex} , L_g and L_s , obtained after these steps, are 64 fF, 6.8 nH and 1 nH, respectively. The simulated results are shown in Figs. 11 and 12. As can be seen in these figures, in addition to good input matching, the NF of the designed LNA coincides with the NFmin at a frequency of 2.4 GHz. So the

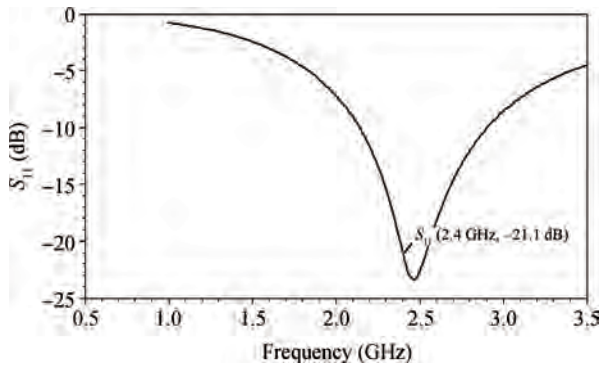


Fig. 12. Simulated S_{11} .

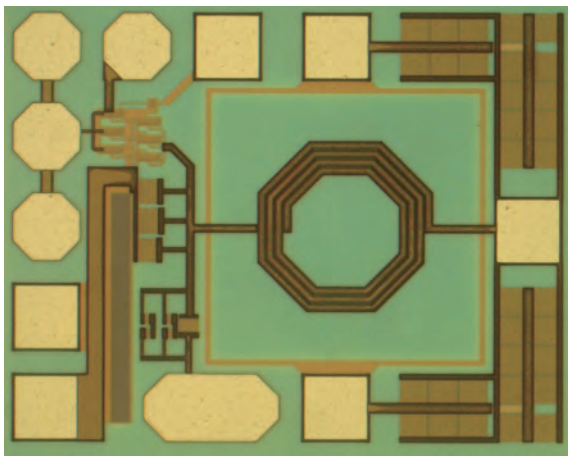


Fig. 13. Micrograph of the LNA with ESD.

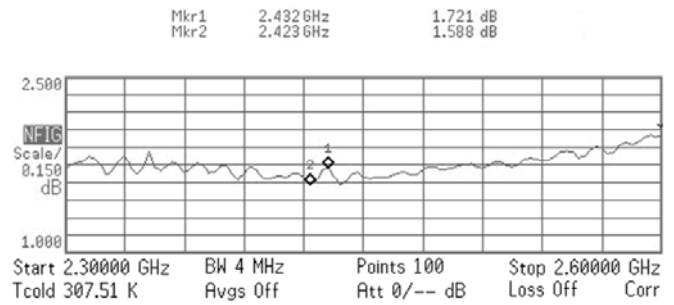


Fig. 15. Measured noise figure.

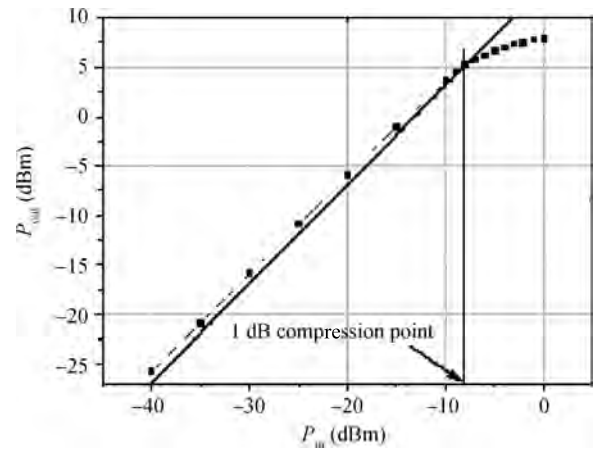


Fig. 16. Measured input 1-dB compression point.

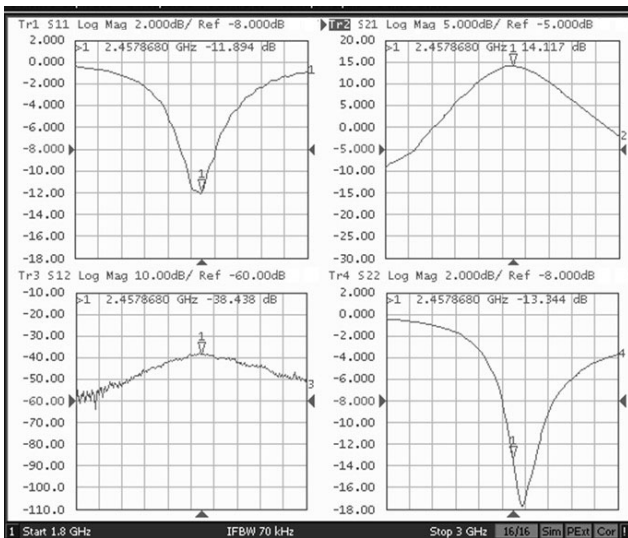


Fig. 14. Measured S -parameter.

ESD protected LNA's power-constrained simultaneous noise and input matching are realized.

3.3. Measured results

The micrograph of the LNA with ESD is shown in Fig. 13. The chip area including all bonding pads is $470 \times 600 \mu\text{m}^2$.

The chip was measured in the test laboratory of the Institute of RF- & OE-ICs, Southeast University. The major test equipment includes an Agilent network analyzer E5071B, a spectrum analyzer E4440A, an RF signal generator E4438C and a noise figure analyzer N8975A. The measured DC current is about 4 mA under a supply of 1.8 V.

Figure 14 shows the measured results of the S -parameter. It can be observed that both the input and output impedance matching value, S_{11} and S_{22} , are below -11 dB. The maximum power gain of the LNA is 14.12 dB at a frequency of 2.45 GHz.

Figure 15 shows the measured result of the noise figure. The minimum value of NF is 1.59 dB at 2.42 GHz, while the maximum value is 1.72 dB at 2.43 GHz. In most of the frequency range, the noise figure is below 1.7 dB.

The input 1 dB compression point is -8 dBm, as can be seen in Fig. 16. It is obtained with the dot method after recording the input powers and corresponding output powers.

IIP3 is obtained with the double tone multi-frequency (DTMF) test method with -23 dBm input power. The result is shown in Fig. 17 and the IIP3 is about 1 dBm.

Table 1 gives the measured results of the LNA in detail and a comparison with recently published LNAs operated at the same frequency of 2.4 GHz. From this table we can conclude that the key figures of the LNA designed in this paper are superior to those of others.

Table 1. Performance comparisons with recently published LNAs.

Ref.	Power gain (dB)	NF (dB)	S_{11} (dB)	P_{dc} (mW)	IIP3 (dBm)	Process (nm)
[6]	21.9	2.56	-12.6	12.9	-11	90 CMOS
[7]	13.0	3.60	-13.0	6.5	NA	130 SOI CMOS
[8]	12.1	2.77	-20.0	4.5	2.4	150 CMOS
[9]	10.5	4.80	-14.0	17.5	NA	250 CMOS
This work	14.12	1.59	-11	7.2	1	180 CMOS

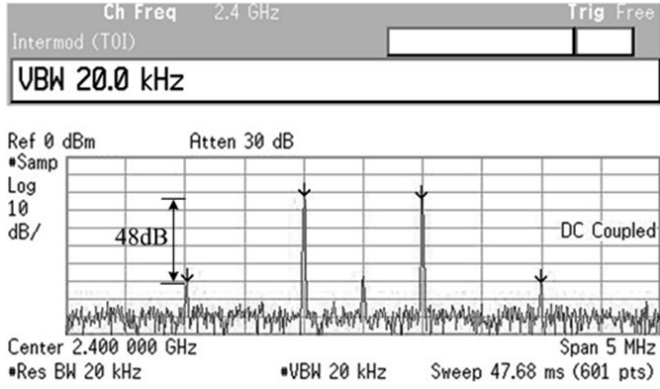


Fig. 17. Result of DTMF detection test.

4. Conclusions

In this paper, the method of PCSNIM with ESD is proposed. In addition, the input impedance and noise parameters of the LNA with ESD protection circuit are analyzed in detail. Based on this method, an LNA for 2.4 GHz WSN applications is presented. The measured results show that the proposed optimization technology is available.

Appendix A

The derivation of the source inductance degenerated CS LNA's optimum source admittance (Y_{opt}) is as follows.

In Fig. A1, i_{nd}^2 is the channel thermal noise, i_{ng}^2 is the drain induced gate noise, and v_n^2 and i_n^2 are the equivalent input noise voltage and current, respectively.

From Fig. A1, v_n^2 and i_n^2 can be worked out,

$$\begin{cases} i_n = -\frac{sC_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) i_{nd} - i_{ngu}, \\ v_n = -\frac{1}{g_m} \left[s^2 C_{gs} (L_g + L_s) \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + 1 \right] i_{nd} - s(L_g + L_s) i_{ngu}, \end{cases}$$

where

$$\alpha \triangleq \frac{g_m}{g_{d0}},$$

where g_m is the device transconductance, and g_{d0} is the zero-bias drain conductance. γ is the coefficient of channel thermal noise. δ is the coefficient of gate noise, and

$$c = \frac{i_{g'd}^*}{\sqrt{i_g^2 i_d^2}} \approx 0.395j,$$

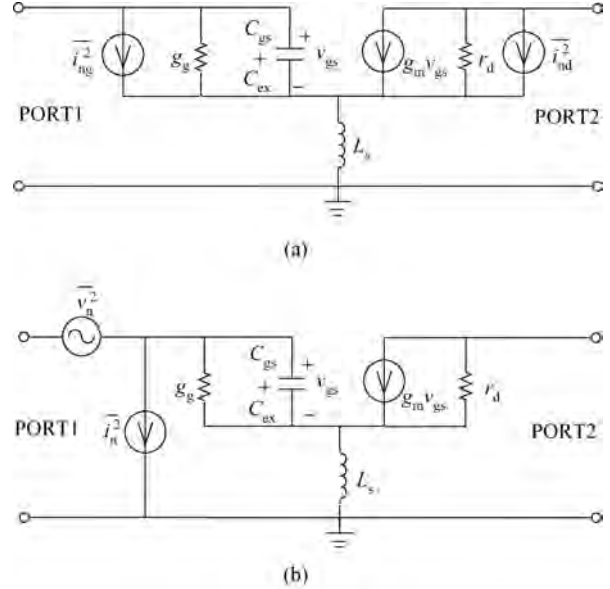


Fig. A1. (a) Noise model of the source inductance degenerated CS LNA. (b) Equivalent input noise voltage and current model with noise-less network.

$$i_{ngu} = i_{ng} - i_{ngc},$$

where i_{ngu} , i_{ngc} are the unrelated and related parts between i_{nd} and i_{ng} .

Let

$$\begin{aligned} R_n &= \frac{v_n^2}{4kT\Delta f} \\ &= \frac{\gamma}{\alpha g_m} \left[1 + \omega^4 C_{gs}^2 (L_g + L_s)^2 \right. \\ &\quad \times \left. \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + \frac{\alpha^2 \delta}{5\gamma} + 1 \right) \right] \\ &\quad - \frac{\gamma}{\alpha g_m} \left[2\omega^2 C_{gs} (L_g + L_s) \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right]. \end{aligned}$$

Y_c stands for the correlation coefficient between v_n and i_n , and is defined as

$$Y_c = c'^* \sqrt{\frac{i_n^2}{v_n^2}} = \frac{v_n i_n^*}{\sqrt{v_n^2 i_n^2}} \sqrt{\frac{i_n^2}{v_n^2}} = \frac{v_n i_n^*}{v_n^2},$$

$$\begin{aligned}
 Y_c &= \frac{sC_{gs} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) - \omega^2 C_{gs}^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right)}{1 + \omega^4 C_{gs}^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) - 2\omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right)} \\
 &= \frac{j\omega C_{gs} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) - j\omega^3 C_{gs}^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right)}{1 + \omega^4 C_{gs}^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) - 2\omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right)} \\
 &= j \frac{\omega C_{gs} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) - \omega^3 C_{gs}^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right)}{1 + \omega^4 C_{gs}^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) - 2\omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right)} \\
 &= G_c + jB_c = jB_c.
 \end{aligned}$$

so

$$B_c = \frac{\omega C_{gs} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) - \omega^3 C_{gs}^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right)}{1 + \omega^4 C_{gs}^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) - 2\omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right)},$$

$$G_c = 0.$$

According to Y_c , the unrelated part between v_n and i_n can be calculated. It is

$$\begin{aligned}
 i_{nu} &= i_n - i_{nc} = i_n - Y_c v_n \\
 &= -\frac{sC_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) i_{nd} - i_{ngu} + Y_c \left\{ \frac{1}{g_m} \left[s^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + 1 \right] i_{nd} + sL_s i_{ngu} \right\} \\
 &= \left\{ \frac{Y_c}{g_m} \left[s^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + 1 \right] - \frac{sC_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right\} i_{nd} + (Y_c sL_s - 1) i_{ngu}.
 \end{aligned}$$

$$\begin{aligned}
 G_u &= \frac{\overline{i_{nu}^2}}{4kT\Delta f} \\
 &= \left| \frac{Y_c}{g_m} \left[s^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + 1 \right] - \frac{sC_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right|^2 \frac{\overline{i_{nd}^2}}{4kT\Delta f} + |Y_c sL_s - 1|^2 \frac{\overline{i_{ngu}^2}}{4kT\Delta f} \\
 &= \left| \frac{Y_c}{g_m} \left[1 - \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right] - \frac{j\omega C_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right|^2 \gamma g_{d0} + |j\omega Y_c L_s - 1|^2 \delta g_g (1 - |c|^2) \\
 &= \left| \frac{jB_c}{g_m} \left[1 - \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right] - \frac{j\omega C_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right|^2 \gamma g_{d0} + |\omega B_c L_s + 1|^2 \delta g_g (1 - |c|^2)
 \end{aligned}$$

$$\begin{aligned}
 &= \left| \frac{B_c}{g_m} \left[1 - \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right] - \frac{\omega C_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right|^2 \gamma g_{d0} + |\omega B_c L_s + 1|^2 \delta g_g (1 - |c|^2) \\
 &= \frac{\gamma g_{d0} B_c^2}{g_m^2} \left[1 - \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right]^2 + \frac{\gamma g_{d0} \omega^2 C_{gs}^2}{g_m^2} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right)^2 \\
 &\quad - 2 \frac{\gamma g_{d0} \omega C_{gs}}{g_m} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \frac{B_c}{g_m} \left[1 - \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right] \\
 &\quad + (\omega^2 B_c^2 L_s^2 + 1 + 2\omega B_c L_s) \delta \frac{\omega^2 C_{gs}^2}{5g_{d0}} (1 - |c|^2). \\
 \\
 G_u &= \frac{\gamma g_{d0} B_c^2}{g_m^2} - 2 \frac{\gamma g_{d0} B_c^2}{g_m^2} \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + \frac{\gamma g_{d0} B_c^2}{g_m^2} \omega^4 C_{gs}^2 L_s^2 \left(\alpha^2 |c|^2 \frac{\delta}{5\gamma} + 2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \\
 &\quad + \frac{\gamma g_{d0} \omega^2 C_{gs}^2}{g_m^2} \left(\alpha^2 |c|^2 \frac{\delta}{5\gamma} + 2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \\
 &\quad - 2 \frac{\gamma g_{d0} \omega C_{gs}}{g_m^2} B_c \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \left[1 - \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right] \\
 &\quad + [\omega^2 B_c^2 L_s^2 + 1 + 2\omega B_c L_s] \delta \frac{\omega^2 C_{gs}^2}{5g_{d0}} (1 - |c|^2) \\
 &= \frac{\gamma B_c^2}{\alpha g_m} - \frac{1}{g_m} \frac{2\gamma}{\alpha} B_c^2 \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \\
 &\quad + B_c^2 \omega^4 C_{gs}^2 L_s^2 \left(\alpha^2 |c|^2 \frac{\delta}{5\gamma} \frac{\gamma}{\alpha g_m} + 2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} \frac{\gamma}{\alpha g_m} + \frac{\gamma}{\alpha g_m} \right) \\
 &\quad + \frac{\gamma \omega^2 C_{gs}^2}{\alpha g_m} \left(\alpha^2 |c|^2 \frac{\delta}{5\gamma} + 2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) - 2 \frac{\gamma \omega C_{gs}}{\alpha g_m} B_c \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \\
 &\quad + 2 \frac{\gamma}{\alpha g_m} B_c \omega^3 C_{gs}^2 L_s \left(\alpha^2 |c|^2 \frac{\delta}{5\gamma} + 2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \\
 &\quad + B_c^2 \omega^4 C_{gs}^2 L_s^2 \left(\frac{\delta}{5g_{d0}} - |c|^2 \frac{\delta}{5g_{d0}} \right) + \frac{\gamma}{\alpha g_m} \omega^2 C_{gs}^2 \left(\frac{\alpha^2 \delta}{5\gamma} - |c|^2 \frac{\alpha^2 \delta}{5\gamma} \right) + B_c \omega^3 C_{gs}^2 L_s \left(2 \frac{\delta}{5g_{d0}} - |c|^2 2 \frac{\delta}{5g_{d0}} \right) \\
 &= \frac{1}{g_m} B_c^2 \omega^4 C_{gs}^2 L_s^2 \left(2 |c| \sqrt{\frac{\delta\gamma}{5}} + \frac{\alpha\delta}{5} + \frac{\gamma}{\alpha} \right) + \frac{1}{g_m} B_c \omega^3 C_{gs}^2 L_s \left(4 |c| \sqrt{\frac{\delta\gamma}{5}} + \frac{2\gamma}{\alpha} + \frac{2\alpha\delta}{5} \right) \\
 &\quad - \frac{1}{g_m} \frac{2\gamma}{\alpha} B_c^2 \omega^2 C_{gs} L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + \frac{\gamma}{\alpha g_m} \omega^2 C_{gs}^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \\
 &\quad - 2 \frac{\gamma}{\alpha g_m} B_c \omega C_{gs} \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + \frac{\gamma B_c^2}{\alpha g_m}.
 \end{aligned}$$

So, Y_{opt} can be worked out.

$$\begin{aligned}
G_{\text{opt}} &= \sqrt{G_c^2 + \frac{G_u}{R_n}} = \sqrt{0 + \frac{G_u}{R_n}} = \sqrt{\frac{G_u}{R_n}} \\
&= \left[B_c^2 \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + \frac{\alpha^2 \delta}{5\gamma} + 1 \right) + 2B_c \omega^3 C_t^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \right. \\
&\quad - 2B_c^2 \omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + \omega^2 C_t^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) \\
&\quad \left. - 2B_c \omega C_t \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) + B_c^2 \right]^{1/2} \\
&\quad \times \left[1 + \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + \frac{\alpha^2 \delta}{5\gamma} + 1 \right) - 2\omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) \right]^{-1/2}.
\end{aligned}$$

$$B_{\text{opt}} = -B_c$$

$$\begin{aligned}
&= - \frac{\omega C_t \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right) - \omega^3 C_t^2 L_s \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right)}{1 + \omega^4 C_t^2 L_s^2 \left(2\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 + \frac{\alpha^2 \delta}{5\gamma} \right) - 2\omega^2 C_t L_s \left(\alpha |c| \sqrt{\frac{\delta}{5\gamma}} + 1 \right)}.
\end{aligned}$$

$$Y_{\text{opt}} = G_{\text{opt}} + jB_{\text{opt}}.$$

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