# A kind of magnetron cavity used in rubidium atomic frequency standards

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**Abstract:** Research on the magnetron cavity used in the rubidium atomic frequency standards is developed, through which the main characteristic parameters of the magnetron cavity are studied, mainly including the resonant frequency, quality factor and oscillation mode. The resonant frequency and quality factor of the magnetron cavity were calculated, and the test results of the resonant frequency agreed well with the calculation theory. The test results also show that the resonant frequency of the magnetron cavity can be attenuated to 6.835 GHz, which is the resonant frequency of the rubidium atoms, and the *Q*-factor can be attenuated to 500-1000. The oscillation mode is a typical TE<sub>011</sub> mode and is the correct mode needed for the rubidium atomic frequency standard. Therefore these derivative magnetron cavities meet the requirements of the rubidium atomic frequency standards well.

Key words: magnetron cavity; resonant frequency; quality factor; oscillation mode; rubidium atomic frequency standard

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## 1. Introduction

The rubidium atomic frequency standard is an outstanding combination of quantum theory and modern electronics. In the rubidium atomic frequency standards, the microwave cavity is an essential and important component, and most structural transformation of the rubidium atomic frequency standard is based on microwave cavities<sup>[1,2]</sup>. There are several microwave cavities adopted in rubidium atomic frequency standards and the magnetron cavity is the preferential choice due to its high quality factor, smaller size and better magnetic mode<sup>[3]</sup>. However, the magnetron cavity has a more complicated structure, and it is difficult to manufacture. Up to now, some approximate expressions that show relations of resonant frequency and quality factor to the dimensional parameters of the cavity have been processed, but they do not tally well with the practice. Therefore, when the magnetron cavity is used in the rubidium atomic frequency standards, its resonant frequency and quality factor need to be studied too and so we have developed a derivative method of the expressions in detail and provide the essential derivation procedure.

### 2. Structure of the magnetron cavity

The structure of the magnetron cavity used in the rubidium frequency standard is shown in Fig. 1. The outer conductive shield made of copper or aluminum is designed mainly to shield the electromagnetic field of the electrodes. It also serves as a supporter of the electrodes; usually four electrodes are used. The resonant frequency of the cavity depends on the dimensions of the cavity and in particular on the electrodes, the Q-factor is related to dimensions of the cavities and the energy loss of the cavities' inner surfaces.

There is a screw thread on the top of the cavity. Its function is to adjust the resonant frequency of the cavity by using a glass bulb, which serves as a container of rubidium atoms and adjusts the resonant frequency of these cavities. In addition, the resonant frequency of rubidium atoms is 6.835 GHz, and the attenuation of the glass bulb is about 0.2–0.8 GHz, so the resonant frequencies of the designed empty magnetron cavities should be a little bigger than 6.835 GHz.

### 3. Basic parameters of the magnetron cavity

As a kind of microwave cavity used in the rubidium atomic frequency standards, it should meet the following requirements: the resonant frequency can be accommodated to the resonant frequency of rubidium atoms (6.835 GHz); the direction of the microwave magnetic field is parallel to the direction of the cavity axis to improve the equality of microwave; and a low loss mode is needed for the signal-to-noise of rubidium atomic frequency standards. So the basic parameters should include the following: resonant frequency, quality factor, and oscillation mode. Now we develop the formulas for the basic parameters of the resonant cavity using the distribution of the electromagnetic field.



Fig. 1. Structure of the magnetron cavity.

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#### 3.1. Resonant frequency

Referring particularly to the dimensions in Fig. 1, the magnetron cavity could be considered as a capacitance–inductance device, so the basic resonant frequency of magnetron cavities is as follows<sup>[3]</sup>:

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}},\tag{1}$$

where L is inductance value in the cavity, and  $L = \frac{\mu \pi r^2}{h}$ , C is capacitance value between the gaps in the conductive loop,  $\frac{1}{C} = N \frac{t}{\varepsilon w h}$ ,  $\varepsilon$  and  $\mu$  are dielectric constant and magnetic inductivity, respectively, N is the number of identical gaps in the conductive loop or the number of the electrodes, so we could get:

$$f = \frac{1}{2\pi r} \sqrt{N \frac{t}{w} \frac{1}{\pi \mu \varepsilon}}.$$
 (2)

Accounting for fringe effect of capacitance between the electrodes and the coaxial effect of the shielding, the resonant frequency of the magnetron cavity is expressed as follows<sup>[5]</sup>:

$$f = \frac{a}{2\pi r} \sqrt{\left(1 + \frac{A_1}{A_2}\right) \frac{Nt}{\mu \varepsilon \pi w}},$$
 (3)

where  $A_1 = \pi r^2$ ,  $A_2 = \pi [R^2 - (r + w)^2]$ , and  $c = \frac{1}{\sqrt{\mu\varepsilon}}$  is the light velocity in free space, it is about  $3 \times 10^8$  m/s. *a* is the correction coefficient for fringe effect of the capacitance<sup>[4, 5]</sup>, it should be  $\sqrt{\frac{1}{1+2.5\frac{t}{w}}} \le a \le \sqrt{\frac{1}{1+\frac{t}{w}}}$  according to different structures, substituting these expressions into Eq. (1), then we can get the calculation formula for f:

$$f = \frac{ac}{2\pi r} \sqrt{\left(1 + \frac{A_1}{A_2}\right) \frac{Nt}{\pi w}}.$$
 (4)

#### **3.2.** Calculation of *Q*-factor

According to the definition of Q-factor of the cavity, we have the following formula:

$$Q = \frac{\omega W_0}{P_0},\tag{5}$$

where  $P_0$  is the power loss in the resonating cavity,  $W_0$  is the total energy stored in the resonating cavity and  $\omega = 2\pi f$  is angular frequency. The energy loss in the cavity mainly refers to the loss on the metal surfaces. If the medium loss can be neglected, we can get the approximation formula to calculate  $P_0$  as follows:

$$P_0 = \frac{R_s}{2} \int |\boldsymbol{H}|^2 \mathrm{d}s, \qquad (6)$$

where  $H_1$  represents the tangential component of the magnetic density on the metal surfaces in the cavity, S is the total area of metal surface in the cavity,  $R_s$  is expressed by the following formula:

$$R_{\rm s} = \frac{\delta}{2}\omega\mu,\tag{7}$$

where  $\delta$  is the skin effect depth of electromagnetic field in the material. We suppose that the skin effect depths are the same



Fig. 2. Distribution of microwave field simulated by high frequency structure simulator software.

in all the metal surfaces, all  $R_s$  are regarded as the same value when calculating the energy loss on metal surfaces.

When calculating the energy loss on the electrodes surfaces, we also deal with the N pieces of electrodes as an approximately cylindrical tube. Let  $S_1$  represent the inner surface area of the outer cavity,  $S_2$  and  $S_3$  the outer and inner surface area of top cover and bottom plate of the outer cavity. Let  $H_{in}$  represent magnetic density in the area between the electrodes, let  $H_{ex}$  represent the magnetic density in area between the electrodes and the outer shielding, and both them are regarded as uniformly distributed in their respective areas. By using Eq. (6) we can get:

$$P_{0} = \frac{R_{s}}{2} \left( \int_{s_{1}} |\boldsymbol{H}_{ex}|^{2} ds + \int_{s2} |\boldsymbol{H}_{ex}|^{2} ds + \int_{s_{3}} |\boldsymbol{H}_{ex}|^{2} ds + \int_{s_{4}} |\boldsymbol{H}_{ex}|^{2} ds \right).$$
(8)

Substituting Eq. (7) into Eq. (8) and having  $S_4 = 2\pi R^2$ , we can get:

1

$$P_{0} = \pi R_{\rm s} h H_{0}^{2} \left[ r + \left( r + R + \frac{R^{2}}{h} \right) \left( \frac{A_{1}}{A_{2}} \right)^{2} \right].$$
(9)

By using expressions, Q can be described as follows:

$$Q = \frac{r\omega\mu}{2R_{\rm s}} \frac{1 + \frac{A_1}{A_2}}{1 + \left(1 + \frac{R}{r} + \frac{R^2}{hr}\right) \left(\frac{A_1}{A_2}\right)^2}.$$
 (10)

Then substituting Eq. (9) into this one, we can get the final expression of Q.

$$Q = \frac{r}{\delta} \frac{1 + \frac{A_1}{A_2}}{1 + \left(1 + \frac{R}{r} + \frac{R^2}{hr}\right) \left(\frac{A_1}{A_2}\right)^2}.$$
 (11)

		Table	1. Geometri	c uniterisioi	is, calculation and test les	suns for the magn	etron cavities.	
<i>t</i> (mm)	<i>r</i> (mm)	w (mm)	<i>R</i> (mm)	<i>h</i> (mm)	Evaluated $f$ (GHz)	Evaluated $Q$	Test result $f$ (GHz)	Test result $Q$
1.8	8.0	1.0	13.0	16	7.05	3197	7.04	3032
2.0	8.0	1.0	13.0	16	7.22	3197	7.23	3036
2.2	8.0	1.0	13.0	16	7.33	3197	7.35	3095
2.4	8.0	1.0	13.0	16	7.44	3197	7.48	3053
2.6	8.0	1.0	13.0	16	7.54	3197	7.58	3105

Table 1. Geometric dimensions, calculation and test results fot the magnetron cavities

Table 2. Geometric dimensions, calculation results for the magnetron cavities.

<i>t</i> (mm)	<i>r</i> (mm)	<i>w</i> (mm)	<i>R</i> (mm)	<i>h</i> (mm)	Evaluated $f$ (GHz)	Evaluated Q
1.8	8.0	1.0	13.0	16	7.05	3197
1.8	8.0	1.0	12.5	16	7.35	2839
1.8	8.0	1.0	12.0	16	7.57	2438
1.8	8.0	1.0	11.5	16	8.05	1994
1.8	8.0	1.0	11.0	16	8.60	1625

### 4. Oscillation mode

With the values of electrical and dielectric parameters and the shapes dimension of the cavity, we have obtained the distribution of microwave magnetic field in the cavity and the cavity response through a high frequency three-dimensional numerical simulation and experiments. Figure 2 shows the simulation results using HFSS (high frequency structure simulator is a high-performance full-wave electromagnetic field simulator for arbitrary 3D volumetric passive device modeling, it can be used to calculate parameters such as parameters, resonant frequency, and fields.) software of the microwave magnetic field in the cavity. It can been seen that the distribution of the microwave magnetic field in the cavity is the typical  $TE_{011}$  $mode^{[6, 7]}$ , which is stronger in the middle than in the margin and the direction of the microwave magnetic field is parallel to the direction of the cavity axis to improve the equality of microwave. The microwave signal is injected into the cavity by a microwave coupling loop and the direction of the microwave magnetic field is almost parallel to the direction of cavity axis, which meets the requirement of the system. So the electromagnetic field structure is the TE<sub>011</sub> mode in the microwave cavity shown in Fig. 1, and it is the correct mode required for the rubidium atomic frequency standards<sup>[8-10]</sup>.

### 5. Test results

We have designed and manufactured several resonant cavities according to the cavity structure shown in Fig. 1 by using this method, their geometric dimensions (in millimeters) are listed in Table 1. Substituting these data into Eq. (4) and (11), and the material is copper, then considering that  $\delta =$  $1.4 \times 10^{-6}$  m<sup>[5]</sup>, the resonant frequencies and *Q*-factors of the empty cavities are achieved. A network analyzer was used to test the parameters for these magnetron cavities, the test results of these cavities are also shown in Table 1.

By comparing the measured values with the calculated values, we find that the resonant frequencies tally well with the calculated values, whose error scope was confined to 5%, and the Q-factors are all smaller than the calculated values, this is because of the roughness of the cavities' inner surfaces. These results validate the formulas used to evaluate parameters of



Fig. 3. Resonant frequency and quality factor test results of the magnetron cavity.



Fig. 4. Resonant frequency and quality factor test results of magnetron cavity.

magnetron cavities and we also offer another table for evaluated results of magnetron cavities shown in Table 2.

The results in Table 2 show that with a bigger volume of cavities, a higher Q-factor is achieved, which offers a wider Q-factor range for the rubidium frequency standards to select. The microwave cavity in the rubidium frequency standard is used essentially for two purposes: providing the microwave field interrogating the <sup>87</sup>R<sub>b</sub> atom in the cell and using as the band-pass

filter for the microwave frequency multiplier and mixer. Figures 3 and 4 show the resonant frequency and quality factor test results of two specimens for magnetron cavity. From these two figures we could find that the resonant frequency is attenuated to 6.835 GHz, and the *Q*-factor can be attenuated to 500-1000, which is because that the energy loss on the metal surfaces and loss in the glass bulb. The test results also show that all of the cavities resonant frequency listed in Table 1 can also be attenuated to 6.835 GHz with a glass bulb in them, which is the resonant frequency for the rubidium atoms, and the *Q*-factor can be attenuated to 500-1000, which can also meet the requirement for the rubidium atomic frequency standard. Therefore a kind of rubidium atomic frequency standard has been developed.

# 6. Conclusion

In conclusion, based on the theoretical calculations, a kind of magnetron cavity for space rubidium atomic frequency standards is developed. In the whole designed process, the resonant frequency and Q-factor cavity were deduced by this method, and the theoretical analysis agrees very well with measured characteristics and test results. The resonant frequency and Qfactor can also be accommodated to meet the requirements of the rubidium atomic frequency standards, the oscillation mode is the correct mode required for the rubidium atomic frequency standards.

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