An empirical formula for yield estimation from singly truncated performance data of qualified semiconductor devices

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Abstract: The problem of yield estimation merely from performance test data of qualified semiconductor devices is studied. An empirical formula is presented to calculate the yield directly by the sample mean and standard deviation of singly truncated normal samples based on the theoretical relation between process capability indices and the yield. Firstly, we compare four commonly used normality tests under different conditions, and simulation results show that the Shapiro–Wilk test is the most powerful test in recognizing singly truncated normal samples. Secondly, the maximum likelihood estimation method and the empirical formula are compared by Monte Carlo simulation. The results show that the simple empirical formulas can achieve almost the same accuracy as the maximum likelihood estimation method but with a much lower amount of calculations when estimating yield from singly truncated normal samples. In addition, the empirical formula can also be used for doubly truncated normal samples when some specific conditions are met. Practical examples of yield estimation from academic and IC test data are given to verify the effectiveness of the proposed method.

Key words: yield estimation; one-sided specification; truncated normal; empirical formula

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1. Introduction

In the microelectronics manufacturing industry, the supplier usually only ships qualified semiconductor devices to the customer after weeding out unqualified devices according to the customer's specification. How to calculate the yield of the products solely from the performance data of the qualified ones is an important issue to be resolved during quality estimation of semiconductor devices. Typically, if the concerned performance data of all produced devices follow the normal distribution, the data of the qualified devices will then follow the truncated normal distribution[1−3]. Traditional yield estimation methods suitable for normally distributed performances may bring about big mistakes when dealing with truncated test data[1−3]. As a result, to estimate the yield correctly, one should first check if the original normally distributed sample is truncated aided by normality tests, and then choose the appropriate estimation method.

In this paper, we mainly discuss the yield estimation from sample data of qualified products for normally distributed performances with one-sided specification limits. An empirical formula is presented to calculate the yield conveniently according to the theoretical relation between process capability indices (PCI) and the yield of the singly truncated normal distribution. Some basic definitions and the maximum likelihood estimation (MLE) method of the truncated normal distribution are briefly reviewed. Four normality tests are compared via Monte Carlo simulation under different conditions to find which test has the biggest power in detecting truncation of the normal samples. The empirical formula is found and compared with the MLE method. Simulation results show that these two methods have very close accuracy in yield estimation from singly truncated normal samples and some special doubly truncated normal samples. Three practical examples are given to verify the simulation results.

2. Truncated normal distribution and parameter estimation methods

2.1. Truncated normal distribution

Suppose product performance data follow a normal distribution $N(\mu, \sigma^2)$ with unknown mean and variance, a random sample $x_1, x_2, \cdots, x_n$ was initially generated from this normal distribution, but all observations less than x_L and (or) bigger than x_U have been discarded, so that $x_L < x_i < x_U, i = 1, 2, \cdots, n$. x_L and x_U are truncation points corresponding to the specification limits, which are known a priori. Then the performance data X of the qualified products follow a truncated normal distribution $N_{TR}(\mu, \sigma^2)$ in essence. The probability density function of a doubly truncated normal distribution $N_{TR,d}(\mu, \sigma^2)$ is defined as[4]

$$f_{TR,d}(x) = \begin{cases} \dfrac{\phi\left(\dfrac{x-\mu}{\sigma}\right)}{\Phi\left(\dfrac{x_U-\mu}{\sigma}\right) - \Phi\left(\dfrac{x_L-\mu}{\sigma}\right)}, & x_L < x < x_U, \\ 0, \quad \text{otherwise}, \end{cases} \tag{1}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the probability density function (PDF) and cumulative distribution function (CDF) of the standard normal distribution, respectively. As for the singly truncated normal distribution, $N_{TR,s}(\mu, \sigma^2)$, the PDF is obtained by substituting ∞ for x_U or $-\infty$ for x_L in Eq. (1). Figure 1 shows the

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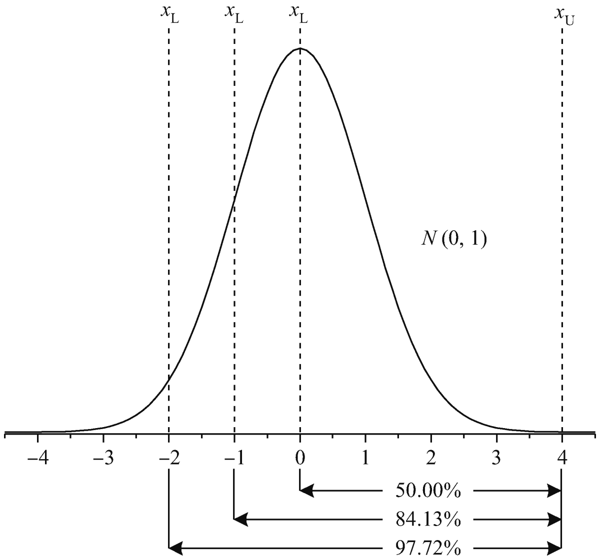

Fig. 1. Yield values of normally distributed performance with different truncation points.

normal PDF curve when the performance follows $N(0, 1)$ with different values of x_L and the corresponding yield values. Apparently $N_{TR}(\mu, \sigma^2)$ is defined based on $N(\mu, \sigma^2)$, and they share the same parameters, μ and σ, in their respective PDFs.

2.2. Parameter estimation methods

In order to calculate the yield $Y = \Phi((x_U - \mu)/\sigma) - \Phi((x_L - \mu)/\sigma)$, one needs to estimate the unknown mean μ and standard deviation σ of $N(\mu, \sigma^2)$ from the truncated normal sample $x_1, x_2, \cdots, x_n$. Since the normality assumption does not hold for the sample, μ and σ certainly cannot be estimated using the sample mean $\bar{x} = \sum_{i=1}^{n} x_i/n$ and the sample standard deviation $s = [(n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2]^{1/2}$ directly. In fact, $\bar{x}$ and s are the moment estimators of the mean μ_{TR} and standard deviation σ_{TR} of $N_{TR}(\mu, \sigma^2)$ that the sample actually follows.

In practice, people frequently use the MLE method to estimate the unknown parameters via the statistical properties of $N_{TR}(\mu, \sigma^2)$[4–6]. Given a doubly truncated normal sample $x_1, x_2, \cdots, x_n$ for example, the likelihood function is $L(\mu, \sigma) = \prod_{i=1}^{n} f_{TR,d}(x_i; \mu, \sigma)$, and the log-likelihood is

$$-\ln L(\mu, \sigma) = n \ln\left[\Phi\left(\frac{x_U - \mu}{\sigma}\right) - \Phi\left(\frac{x_L - \mu}{\sigma}\right)\right] + n\ln(\sqrt{2\pi}\sigma) + \sum_{i=1}^{n}\frac{(x_i - \mu)^2}{2\sigma^2}. \quad (2)$$

The gradient (denoted as G) of Eq. (2) with respect to μ and σ is given in Refs. [4, 6]. Since G contains the integration of unknown parameters, there is no closed-form solution to the equation $G = 0$. Thus the parameters' maximum likelihood estimators $\hat{\mu}$ and $\hat{\sigma}$ can not be easily obtained through elementary calculation. Commonly, they are obtained by minimizing the log-likelihood function using iterative algorithms such as the Newton–Raphson method[6].

The χ^2 method was proposed to estimate the parameters for doubly or singly truncated distributions by minimizing the classic statistic $\chi^2 = \sum_{i=1}^{k}(v_i - np_i)^2/np_i$[7]. v_i is the number of data points falling into the interval $[a_i, a_{i+1}]$ when the truncated data are grouped properly, and $p_i(\mu, \sigma) = \int_{a_i}^{a_{i+1}} f_{TR}(\mu, \sigma)$. Since this method is based on the theorem of large numbers, it is only valid for very large sample size[7].

Besides the above methods, the method of moments estimation can also be used in estimating the distribution parameters[8, 9]. However, it has been verified that even under large sample sizes, the moment estimators will cause large biases[6].

3. Power comparison of different normality tests in recognizing singly truncated normal samples

As mentioned before, it is a fundamental step to check whether the normal sample is truncated or not before proceeding with any relevant yield estimation procedures. This can be fulfilled by drawing the histogram or by using a more formal method such as normality tests. In normality tests, the null and alternative hypotheses are chosen as: H0: $X \sim N(\mu, \sigma^2)$; H1: $X \nsim N(\mu, \sigma^2)$. The power of a test is the probability of rejecting the null hypothesis of normality when it is actually false, and it varies with the significance level, sample size and alternative distributions. Although various normality tests were compared under different settings in Refs. [10, 11], there are still no power comparisons available for the case when the alternative distributions are specified to be truncated normal distributions.

Here the alternative distributions chosen are $N_{TR,s}(0, 1)$ with three different truncation points $x_L = 0$, –1 and –2. Four commonly used normality tests are under study, including the chi-squared (χ^2) test, the Kolmogorov–Smirnov (KS) test, the Anderson–Darling (AD) test and the Shapiro–Wilk (SW) test. Most of them have been applied in identifying truncated samples in Refs. [3, 5, 7]. In this study, the SW test adopts the AS R94 algorithm proposed by Royston[12], which can be used for sample sizes between 3 and 5000. The details of the algorithm and other normality tests can be found in the relevant literatures[3, 5, 7, 10–12]. Two level of significance, $\alpha = 5\%$ and $\alpha = 10\%$, and four sample sizes $n = 50$, 100, 200 and 500 are considered. During our Monte Carlo simulation, for each sample size n, $N = 200$ samples are generated repeatedly from each alternative distribution. Then the power of each test is the proportion of samples with which the test rejects the null hypothesis of normality.

The power comparison results are listed in Table 1. The yield values corresponding to different x_L are also given, which may serve as an indicator of the degree of truncation. It is clear that the power of each test increases with n, α, and the degree of truncation. When the sample is heavily truncated (low yield), most tests gain fairly high power even under a relatively small sample size, except for the χ^2 test. When the sample is lightly truncated (high yield), only the SW test is adequate to achieve high power when $n = 500$. Overall, generally for singly truncated normal distributions, the SW test is the most powerful test. However, it is still very difficult even for the SW test to achieve high power in recognizing very lightly truncated normal samples, for example, in recognizing samples from $N_{TR,s}(0, 1)$ when $x_L = -3$ ($Y = 99.87\%$), $n = 5000$, and $\alpha = 10\%$, the power of the SW test is only 0.675.

Table 1. Comparison of power for different normality tests against singly truncated normal distributions.

x_{L} (yield)		0 (50.00%)				–1 (84.13%)				–2 (97.72%)			
α level		0.05	0.05	0.1	0.1	0.05	0.05	0.1	0.1	0.05	0.05	0.1	0.1
Sample size		50	100	50	100	100	200	100	200	200	500	200	500
Methods	χ^2	0.19	0.46	0.29	0.61	0.17	0.36	0.29	0.48	0.12	0.11	0.17	0.21
	KS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.04	0.07	0.08	0.11
	AD	0.80	0.99	0.88	1.00	0.64	0.99	0.77	1.00	0.17	0.35	0.24	0.55
	SW	0.93	1.00	0.97	1.00	0.88	1.00	0.95	1.00	0.30	0.93	0.50	0.97

4. The empirical formula for yield estimation

When the performance data is judged to be truncated, one can use the MLE method to estimate the parameters of the distribution and then calculate the yield of the devices. However, due to the adoption of iterative algorithms in MLE, the yield cannot be calculated as easily as in the normal distribution case. In this section, a very simple to use empirical formula is proposed to calculate the yield from singly truncated normal samples, and it is also suitable for doubly truncated normal samples when some specific conditions are met. This formula is based on the theoretical relation between the PCI and the yield of the singly truncated normal distribution. The accuracies of the two yield estimation methods are then compared on by Monte Carlo simulation.

4.1. Relation between PCI and yield

Process capability indices have been widely used in the microelectronics manufacturing industry to measure the quality of products and processes. For performances following the normal distribution $N(\mu,\sigma^2)$ with one-sided specification limits, x_{L} or x_{U}, the capability indices are defined as $C_{\mathrm{PL}} = (\mu - x_{\mathrm{L}})/3\sigma$ and $C_{\mathrm{PU}} = (x_{\mathrm{U}} - \mu)/3\sigma$. The relation between the yield and C_{PL} or C_{PU} can be expressed as[13]

$$Y = P(X > x_{\mathrm{L}}) = \Phi(3C_{\mathrm{PL}}), \quad (3)$$

$$Y = P(X < x_{\mathrm{U}}) = \Phi(3C_{\mathrm{PU}}). \quad (4)$$

Similarly, for performance data of qualified products following the singly truncated normal distribution $N_{\mathrm{TR,s}}(\mu,\sigma^2)$, we can define the PCI $C_{\mathrm{PL,TR}} = (\mu_{\mathrm{TR}} - x_{\mathrm{L}})/3\sigma_{\mathrm{TR}}$ and $C_{\mathrm{PU,TR}} = (x_{\mathrm{U}} - \mu_{\mathrm{TR}})/3\sigma_{\mathrm{TR}}$. Then the yield also can be expressed as the function of $C_{\mathrm{PL,TR}}$ or $C_{\mathrm{PU,TR}}$ because μ_{TR} and σ_{TR} are the functions of μ and σ. Taking the lower truncated distribution as an example, the mean and standard deviation of the two distributions $N(\mu,\sigma^2)$ and $N_{\mathrm{TR,s}}(\mu,\sigma^2)$ have the following relation[14]

$$\mu_{\mathrm{TR}} = \mu + \frac{\sigma}{1-\Phi(x'_L)}\phi(x'_L), \quad (5)$$

$$\sigma_{\mathrm{TR}} = \sigma\left(1 - \frac{\phi(x'_L)\{\phi(x'_L) - x'_L[1-\Phi(x'_L)]\}}{[1-\Phi(x'_L)]^2}\right)^{1/2}, \quad (6)$$

where $x'_L = (x_{\mathrm{L}} - \mu)/\sigma$ is the standardized value of x_{L}.

The curve of C_{PL} versus the reject rate $1 - Y$ for $N(0,1)$ and the curve of $C_{\mathrm{PL,TR}}$ versus $1 - Y$ for $N_{\mathrm{TR,s}}(0,1)$ are drawn in Fig. 2 sharing the same x axis when x_{L} changes from 0 to –3 according to Eqs. (3)–(6). Obviously the curve of $C_{\mathrm{PL,TR}}$ versus $1 - Y$ (in logarithmic coordinates) is very close to a straight line. In order to calculate the yield via $C_{\mathrm{PL,TR}}$ from singly truncated normal samples, we can fit a linear equation to this curve as

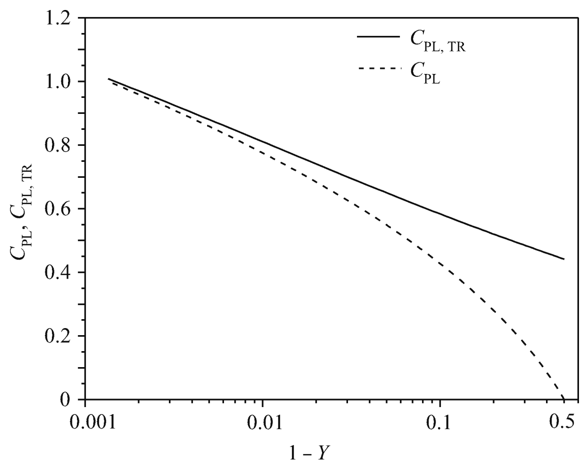

Fig. 2. The relation of $1 - Y$ with C_{PL} and $C_{\mathrm{PL,TR}}$.

$$\log_{10}(1-Y) = 1.76 - 4.71C_{\mathrm{PL,TR}}. \quad (7)$$

Since recognizing very lightly truncated normal samples is difficult, it is more meaningful for us to accurately estimate the yield values when the yield is not too high. Therefore, in fitting the above linear equation, we use the curve data with x_{L} ranging from 0 to –2 (Y = 50%–97.72%). As will be seen in the next section, fitting an equation of higher degree with a wider range of curve data will not apparently improve the estimation accuracy. Likewise, the equation above is also valid for $1 - Y$ and $C_{\mathrm{PU,TR}}$. Overall, the empirical formulas for yield estimation from singly truncated normal samples $x_1, x_2, \cdots, x_n$ can be expressed as following

$$\hat{Y} = 1 - 10^{1.76 - 4.71\frac{\bar{x} - x_{\mathrm{L}}}{3s}}, \quad (8)$$

$$\hat{Y} = 1 - 10^{1.76 - 4.71\frac{x_{\mathrm{U}} - \bar{x}}{3s}}, \quad (9)$$

where $\bar{x}$ and s are used as the estimators of μ_{TR} and σ_{TR}, respectively. Unlike the MLE method, the yield is calculated directly by $\bar{x}$ and s using the empirical formula and the procedure is quite straightforward. To the best of our knowledge, it is the first time that this method has been proposed.

4.2. Accuracy comparison of different yield estimation methods

In this section, the accuracy of the proposed empirical formula is compared with the MLE method by Monte Carlo simulation. To reflect the influence of different fitting methods on

Table 2. Relative errors of the yield estimation with singly truncated normal samples under various cases (unit: %).

x_L (yield)		0 (50.00%)			–1 (84.13%)			–2 (97.72%)		
Sample size		100	500	1000	100	500	1000	100	500	1000
Methods	MLE	33.75	17.00	11.56	8.23	3.62	2.39	1.34	0.57	0.37
	Eq. (8)	32.15	15.71	10.79	8.28	3.72	2.51	1.45	0.59	0.40
	Eq. (10)	33.54	16.23	10.99	8.28	3.68	2.46	1.45	0.58	0.38

Table 3. Relative errors of the yield estimation with doubly truncated normal samples under various cases ($x_U = 3$, unit: %).

x_L (yield)		0 (49.87%)			–1 (84.00%)			–2 (97.59%)		
Sample size		100	500	1000	100	500	1000	100	500	1000
Methods	MLE	37.17	18.58	13.24	9.71	3.91	2.80	1.55	0.62	0.48
	Eq. (8)	33.87	17.46	14.32	8.27	3.53	2.58	1.40	0.64	0.54

the accuracy, a slightly more complicated empirical formula is generated and also compared with the other two methods. A quadratic equation is fitted to the curve data with x_L ranging from 0 to –3 and the new empirical formula is

$$\hat{Y} = 1 - 10^{1.96 - 5.45\frac{\bar{x} - x_L}{3s} + 0.68(\frac{\bar{x} - x_L}{3s})^2}. \quad (10)$$

The singly truncated normal distribution $N_{TR,s}(0, 1)$ with three different truncation points $x_L = 0, –1, –2$ and three sample sizes $n = 100, 500, 1000$ are under consideration. Just like the procedures described in Section 3, the truncated samples are generated repeatedly for $N = 200$ times for each combination of x_L and n, and the accuracies of different methods can be measured by the N yield estimates $\hat{Y}_i, i = 1, 2, \cdots, N$. In this paper, the relative error between $\hat{Y}$ and Y is used to measure the accuracies of different yield estimation methods, which is defined as

$$\text{rmse}(Y) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\hat{Y}_i - Y}{Y}\right)^2} \times 100\%. \quad (11)$$

The relative errors are given in Table 2 under various cases. Clearly the accuracy of each method is improved when increasing the sample size and decreasing the degree of truncation, and there is no significant difference of accuracy between the three methods. As a result, the simple empirical formulas in Eqs. (8) and (9) can meet the accuracy requirement of yield estimation with the least amount calculation. Besides, in Fig. 2, the curve of $C_{PL,TR}$ versus $1 - Y$ is almost coincident with the curve of C_{PL} versus $1 - Y$ when the yield is sufficiently high (the sample is truncated very lightly), which means that we can then use C_{PL} to calculate the yield from $x_1, x_2, \cdots, x_n$ according to Eq. (3). When the sample is very lightly truncated, $N_{TR,s}(\mu, \sigma^2)$ resembles $N(\mu, \sigma^2)$, $C_{PL,TR} \approx C_{PL}$, $\bar{x}$ and s can be used to estimate μ and σ approximately. For example, when estimating the yield for $N_{TR,s}(0, 1)$ with $x_L = -3$ and $n = 100$, rmse(Y) by using the following traditional method is 0.098% and rmse(Y) by using Eq. (8) is 0.097%.

$$\hat{Y} = \Phi\left(\frac{\bar{x} - x_L}{s}\right). \quad (12)$$

In summary, for moderately truncated normal samples which can be recognized easily, Eqs. (8) and (9) are preferred; for very lightly truncated normal samples which can not be recognized easily, using the traditional method like in Eq. (12) will not produce big errors. In addition, to achieve satisfactory accuracy when dealing with heavily truncated data, the sample size should not be too small.

4.3. Application of the empirical formula for doubly truncated normal samples

If $N(\mu, \sigma^2)$ is moderately truncated from one side and very lightly truncated from the other side, then the characteristics of $N_{TR,d}(\mu, \sigma^2)$ are similar to those of $N_{TR,s}(\mu, \sigma^2)$. For example, in Fig. 1, x_U is far from $\mu = 0$, and then $Y = P(X < x_U) - P(X < x_L) \approx 1 - P(X < x_L) = P(X > x_L)$. Therefore, the empirical formulas can also be used to estimate the yield from doubly truncated normal samples of that kind. The accuracies of the empirical formula (8) and the MLE method are compared for $N_{TR,d}(0, 1)$ under various conditions with a fixed upper truncation point $x_U = 3$. From Table 3, there is no significant difference between the two methods and the accuracies are close to the singly truncated case.

5. Practical application

5.1. Singly truncated case

5.1.1. Using the data in Ref. [5]

In this section, the singly truncated normal sample of Ref. [5] is adopted to compare different methods, although the sample is not used for yield estimation purpose. We define the probability $Y = P(X > x_L)$ for the sample where x_L is known to be 277.5. The data are listed in Table 4, and the histogram is also given in Fig. 3 from which the truncation is distinguishable. However, the SW test fails to recognize this singly truncated normal sample with $\alpha = 10\%$ because the power is low for the sample size ($n = 102$) and the degree of truncation ($Y = 95\%$). According to Ref. [5], the MLE estimators of μ and σ are 279.24 and 1.057, respectively, and then the yield estimated by MLE method is 95.01%. Using the empirical formula in Eq. (8), the yield is then 94.98%. Therefore the two methods perform equally well. If we use $\bar{x}$ and s to estimate μ and σ directly, as in the traditional method, the yield turns out to be 97.43% by using Eq. (12). In fact, the calculated PCI is $\hat{C}_{PL,TR}$ not $\hat{C}_{PL}$ by using $\bar{x}$ and s directly. According to Fig. 2, the yield calculated by C_{PL} is always bigger than by the same value of $C_{PL,TR}$, which can explain the calculation results.

Table 4. Singly truncated normal data in Ref. [5].

281.821	279.445	278.642	281.165	280.098	280.521	279.093	281.046	279.967	280.890	279.174	281.401
281.974	280.257	279.862	280.387	278.790	279.728	279.569	279.967	278.678	279.624	279.834	279.223
280.319	279.274	279.383	279.543	279.834	279.675	278.821	279.409	280.884	278.155	277.664	280.419
280.207	279.333	278.006	279.285	279.862	278.481	280.728	280.178	278.852	279.702	278.547	279.862
279.731	279.702	279.254	279.384	278.953	279.445	278.139	280.098	280.649	279.970	279.624	279.333
279.414	280.521	278.517	278.251	277.546	279.333	280.151	279.223	279.463	279.143	278.901	279.754
278.221	279.494	277.828	279.383	278.284	277.892	278.926	280.413	277.710	277.701	278.771	277.874
278.965	278.528	279.046	279.675	277.892	277.946	279.705	278.958	278.981	278.738	280.239	280.623
279.304	277.781	278.938	277.505	278.575	279.282						

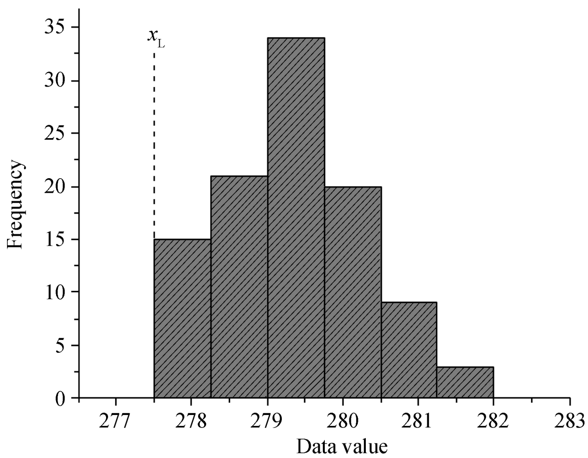

Fig. 3. Histogram of 102 singly truncated normal data.

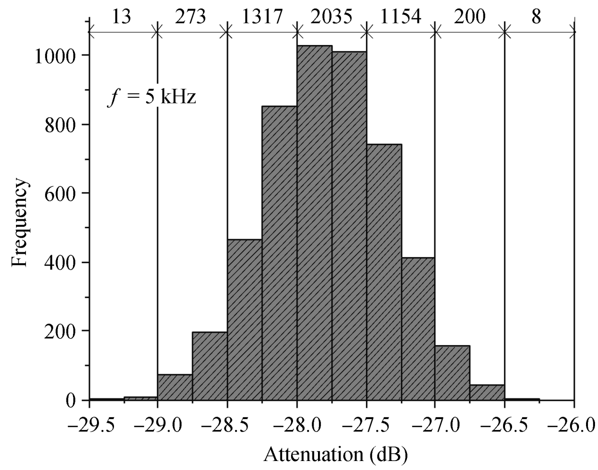

Fig. 5. Histogram of 5000 simulated performance data of the filter.

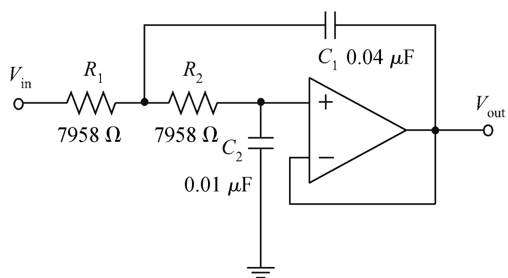

Fig. 4. Schematic of the Sallen and Key filter.

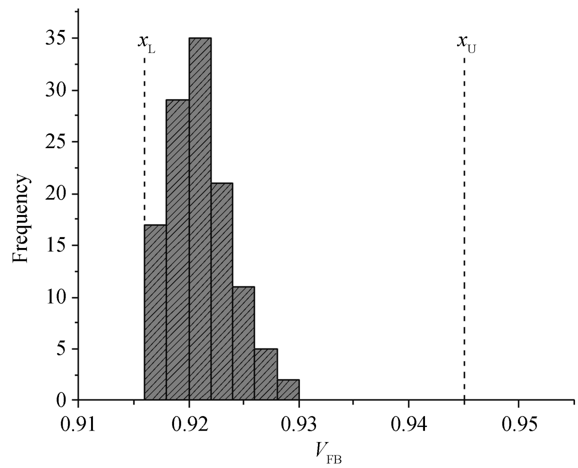

Fig. 6. Histogram of 120 voltage test data of qualified devices.

5.1.2. Yield estimation for the Sallen and Key filter[15]

In this section, different methods are compared by yield estimation of the second-order low-pass Sallen and Key active filter, as shown in Fig. 4[15]. The attenuation of the filter can be expressed mathematically as

$$A(f) = 20\log_{10}\left[\left|1 - R_1R_2C_1C_2(2\pi f)^2 + j2\pi f(R_2C_2 + R_1C_2)\right|\right]^{-1}, \text{ (in dB)}. \quad (13)$$

According to Ref. [15], the values of passive parts are normally distributed with standard deviations equal to 3.33% of the nominal values for the resistors and 1.67% for the capacitors. It can be easily verified that the attenuation when f = 5 kHz obeys the normal distribution using the nominal values of Fig. 4. The histogram of 5000 performance data obtained by Monte Carlo circuit simulations is shown in Fig. 5 and the numbers of data points falling into consecutive data intervals are also given. n is the truncated sample size after weeding out data bigger than x_U and the yield can then be calculated accurately by $Y = P(x < x_U) \approx n/5000$ when x_U taking different values as in Table 5. Since sample sizes are relatively large, all the truncated samples can be recognized by the SW test, and the χ^2 method is workable. Clearly, the traditional method overestimates the yield when it is actually low, and the χ^2 method performs unsatisfactorily as well. While the empirical formula and MLE method give accurate yield estimates under different cases.

Table 5. Estimated yield by different methods when x_U taking different values.

x_U (dB)	n	yield $\approx n/5000$	Traditional method	χ^2 method	MLE method	Empirical formula
−26.5	4992	99.84%	99.80%	99.76%	99.74%	99.77%
−27	4792	95.84%	97.58%	96.47%	95.42%	95.43%
−27.5	3638	72.76%	93.13%	79.04%	73.46%	73.26%
−28	1603	32.06%	89.12%	60.15%	29.87%	33.24%

Table 6. Voltage test values of qualified devices (V).

0.91765	0.92588	0.92571	0.91887	0.92313	0.92258	0.92997	0.92089	0.91830	0.91910	0.92491	0.92340
0.91725	0.92184	0.92824	0.91653	0.92170	0.92092	0.92609	0.91729	0.92419	0.91867	0.92132	0.92003
0.91866	0.92197	0.91986	0.92212	0.92250	0.92491	0.91690	0.92059	0.92101	0.92304	0.91825	0.91655
0.92618	0.91887	0.92106	0.92286	0.91787	0.92023	0.91844	0.92417	0.92513	0.92329	0.91876	0.92403
0.91668	0.92103	0.91885	0.92096	0.92165	0.91619	0.91891	0.91989	0.91905	0.91726	0.91999	0.91933
0.92020	0.92234	0.92215	0.92001	0.92155	0.92041	0.92293	0.92039	0.91773	0.91939	0.91928	0.92033
0.92108	0.92398	0.92010	0.92004	0.92240	0.91871	0.92236	0.92560	0.91667	0.91876	0.92085	0.92139
0.92776	0.92138	0.91953	0.92191	0.92620	0.92413	0.92217	0.92371	0.91903	0.91733	0.92626	0.91999
0.92185	0.92038	0.92260	0.91990	0.91925	0.92388	0.92178	0.92543	0.92188	0.92299	0.91682	0.91695
0.91663	0.91816	0.91886	0.92126	0.92230	0.92387	0.92105	0.91778	0.91814	0.92132	0.92002	0.91896

Table 7. Advantages and disadvantages of different methods in yield estimation from singly truncated normal samples.

Methods	Advantages	Disadvantages
Traditional method	Simple calculation	Overestimate the yield especially for low yield values
MLE method	High accuracy	Calculation is complex due to iterative algorithms
χ^2 method	—	Complicated calculation, need very large sample sizes
Moments method	Simple calculation	Low accuracy, cause big biases
Empirical formula	Simple calculation, high accuracy	—

5.2. Doubly truncated case

The specification for the feedback voltage of a monolithic DC/DC converter is [0.916 V, 0.945 V]. From 2166 test data, this voltage parameter is known to follow the normal distribution $N(0.9204, 0.0033^2)$, and then the theoretical value of the yield is 90.88%.

After weeding out unqualified devices according to the specification, 120 performance data are sampled from qualified devices listed in Table 6. Both the SW test and the histogram in Fig. 6 indicate the truncation of the sample. Clearly, the influence of x_U on the yield is negligible. The yield calculated from this truncated sample by MLE, the empirical formula in Eq. (8), and the traditional method in Eq. (12) are 91.28%, 90.75%, and 96.24%, respectively. Obviously, the traditional method overestimates the yield. Therefore the proposed empirical formula and the MLE method give relatively accurate quality estimations for the voltage parameter.

Finally, the advantages and disadvantages of different methods are summarized in Table 7. Obviously, the proposed empirical formula is an attractive method in dealing with singly truncated normal samples when the samples can be recognized by using the SW test. Meanwhile the accuracies of different methods are closely related to the sample sizes especially when estimating low yield values. Tables of the relative errors in Section 4 and the powers of the SW test in Section 3 under various conditions can be useful when choosing suitable sample sizes.

6. Conclusion

In this paper, a very easy to use empirical formula is presented to calculate the yield from singly truncated normal performance data of qualified semiconductor devices based on the theoretical relation between process capability indices and yield. Four normality tests are compared under different conditions, with the Shapiro–Wilk test achieving the highest power in recognizing singly truncated normal samples. Simulation results show that MLE method and the empirical formula have very close accuracy in yield estimation from singly truncated normal samples and some special doubly truncated normal samples. However, the empirical formula is much more easier to use since the yield is calculated directly by $\bar{x}$ and s. Tables of the relative errors under various cases can be helpful for people when choosing the sample size.

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