

# Theoretical Analysis of Gain and Threshold Current Density for Long Wavelength GaAs-Based Quantum-Dot Lasers \*

Deng Shengling, Huang Yongzhen, Jin Chaoyuan, and Yu Lijuan

(State Key Laboratory on Integrated Optoelectronics, Institute of Semiconductors,  
Chinese Academy of Sciences, Beijing 100083, China)

**Abstract:** Quantum dot gain spectra based on harmonic oscillator model are calculated including and excluding excitons. The effects of non-equilibrium distributions are considered at low temperatures. The variations of threshold current density in a wide temperature range are analyzed and the negative characteristic temperature and oscillatory characteristic temperature appearing in that temperature range are discussed. Also, the improvement of quantum dot lasers' performance is investigated through vertical stacking and p-type doping and the optimal dot density, which corresponds to minimal threshold current density, is calculated.

**Key words:** quantum dot lasers; multiple energy levels; gain spectrum; temperature dependence

**EEACC:** 4320J; 4250; 2520

**CLC number:** TN302      **Document code:** A      **Article ID:** 0253-4177(2005)10-1898-07

## 1 Introduction

For QDs lasers, gain characteristic is very important. It decides which and how many states participate in lasing and how threshold current changes with temperature. In previous calculation, carriers were generally treated either as free ones<sup>[1]</sup> or as excitons<sup>[2]</sup>. However, free carriers and excitons coexist all the time and neglecting either one may result in inaccurate modeling. To address this problem, a model was introduced that included both free carriers and excitons<sup>[3]</sup>, and with which the author modeled  $T_0$  (characteristic temperature) oscillation with cavity length. However, that work did not explain the  $T_0$  oscillation of a given QDs laser with temperature, which was observed experimentally<sup>[4]</sup>

and is much more meaningful. In this work, we will investigate the characteristics of gain and threshold currents of long wavelength QDs lasers in a wide temperature range. The effects of vertical stacking and p-doping on threshold currents are also considered.

## 2 Harmonic oscillator model

To obtain QD energy levels, we employed the harmonic oscillator model<sup>[5]</sup>, as is illustrated in Fig. 1. This model takes into account QD's size variation and yields equally separated states. According to the model, electron energy spectrum is described as

$$\hbar \epsilon_{e,n} = \hbar \epsilon_{e,e} (n_x + n_y + 1) + \hbar \epsilon_{e,z,e} (n_z + \frac{1}{2}) \quad (1)$$

\* Project supported by the National High Technology Research and Development Program of China (No. 2003AA311070) and the National Natural Science Foundation of China (No. 60225011)

Deng Shengling male, was born in 1979, master candidate. He is working in QD lasers.

Huang Yongzhen male, was born in 1963, professor. He is interested in research on semiconductor optoelectronics.

Yu Lijuan female, was born in 1963, associated professor. She is working in semiconductor devices.

Received 10 October 2004, revised manuscript received 17 March 2005

© 2005 Chinese Institute of Electronics

where  $n_x, n_y,$  and  $n_z$  denote the quantum numbers along  $x, y,$  and  $z$  directions, respectively, and  $n_x$  is equivalent to  $n_y$ .  $\hbar \omega_{x,e}$  is the energy separation due to lateral confinement and  $\hbar \omega_{z,e}$  is that due to vertical confinement. The hole energy spectrum  $\hbar \omega_{h,h}$  has a similar form. For  $1.3\mu\text{m}$  QDs, the average

size is as large as  $30\text{nm}$  in diameter and  $11\text{nm}$  in height after covering<sup>[6]</sup>. The corresponding energy separations are  $72\text{meV}$  and  $11\text{meV}$  in conduction and valence band, which fit experimental data<sup>[7]</sup> well. There exist 3 states in conduction band and 6 in valence band.

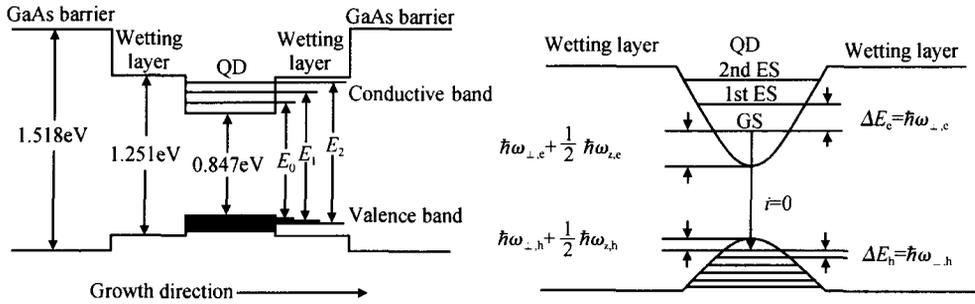


Fig. 1 Schematic illustration of the bands structure and transition in a QD

### 3 Carrier distribution and model gain

At relatively high temperatures, carriers in different QDs can couple through thermal emission into the wetting layer and the re-capture process. This coupling will establish a quasi-Fermi distribution and we call it equilibrium distribution here. At low temperatures, however, thermal escape is strongly suppressed and states population is only determined by capture probability of different-sized dots, and we call it non-equilibrium distribution. Since thermal emission relies heavily on temperature, it is reasonable to assume a boundary temperature  $T_B$ , which separates these two cases. Based on the calculation above,  $k_B T$  will be smaller than the hole energy separation when  $T < 130\text{K}$ , so we choose  $T_B = 130\text{K}$  in this work.

The linear modal gain of QDs ensemble is given in Ref. [8] and it is commonly used in modeling. We modify it to adapt to the multiple-level situation as

$$g_{\text{modal}}(E) = N_1 \frac{e^2 \hbar n_{\text{QD}}}{m_0^2 c n_r z} \sum_{i=1}^3 \frac{2S_i}{E} |M_b|^2 / M_{\text{env}}^2 \times G(E, E_i) (f_e^c(E) - f_e^v(E)) L(E, E) dE \quad (2)$$

where  $M_b$  is the bulk matrix element and  $M_{\text{env}}$  is the wavefunction overlap.  $S_i$  is the  $i$ th transition de-

generacy and  $N_1$  is the QDs layer number.  $f_e^c(E)$  and  $f_e^v(E)$  denote electrons' occupation probability in conduction and valence band. Gaussian distribution  $G(E, E_i)$  represents inhomogeneous broadening resulting from QDs inequality with accounting for size and shape fluctuation.

$$G(E, E_i) = \frac{1}{\sqrt{2}} \times \frac{1}{-2} (E - E_i)^2 \quad (3)$$

The Lorentzian

$$L(E, E) = \frac{1}{\Gamma} \times \frac{\text{in}}{(E - E)^2 + (\text{in})^2} \quad (4)$$

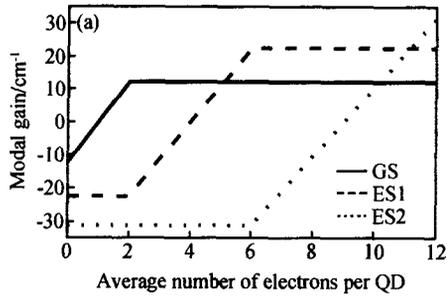
describes the effect of homogeneous broadening.  $n_{\text{QD}}$  is the QDs' surface density and  $n_r$  is the refractive index.  $\Gamma/z$  denotes optical confinement factor normalized by active layer thickness.

In the following calculation, we choose  $N_1 = 2$ ,  $\Gamma = 20\text{meV}$ ,  $n_{\text{QD}} = 3 \times 10^{10} \text{cm}^{-2}$ ,  $n_r = 3.3$ , and  $\Gamma/z = 3.2 \times 10^6 \text{m}^{-1}$ . As for Eq. (4), we replace it with a  $\delta$ -function in the non-equilibrium case<sup>[9]</sup> and choose  $\text{in} = 6\text{meV}$  in the equilibrium case.

#### 3.1 Non-equilibrium distribution ( $T < T_B$ )

In this situation, it is a good approximation that carriers fill from the lowest energy level in each dot. Figure 2 shows the non-equilibrium gain spectrum. As a result of this unique filling pattern,

the first excited state (ES1) modal gain remains minimum until the ground state (GS) saturates,



and the same is true for ES2 and ES1. The GS saturated modal gain is  $12.2\text{cm}^{-1}$  here.

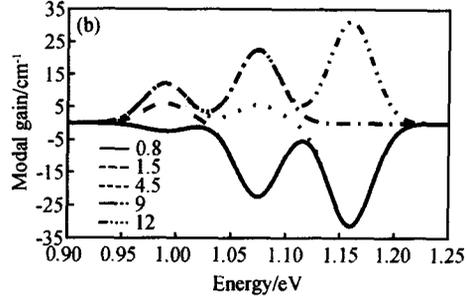


Fig. 2 (a) Non-equilibrium modal gain at center energy; (b) Non-equilibrium modal gain versus injection intensity

### 3.2 Equilibrium distribution ( $T > T_B$ )

In this situation, electrons' density relates to QDs' surface density through

$$n_e = \int_{i=1}^3 2S_i n_{\text{QD}} G(E, E_i) f_e(E) dE \quad (5)$$

According to the relation of  $n_e = \int f_e(E) dE$ , electrons' effective density of states is

$$g_e = \frac{2n_{\text{QD}}}{\sqrt{2}} \int_{i=1}^3 S_i \exp\left(-\frac{1}{2}(E - E_i)^2\right) dE \quad (6)$$

As Fig. 3 shows, with increasing  $\sigma$ , overlaps between different states become stronger, which will prevent carriers from concentrating into certain energy positions<sup>[10]</sup> and degrade device performance. Therefore, QDs with less size fluctuation is crucially important. Moreover, since  $n_e$  is proportional to  $n_{\text{QD}}$ , higher  $n_{\text{QD}}$  is also desired.

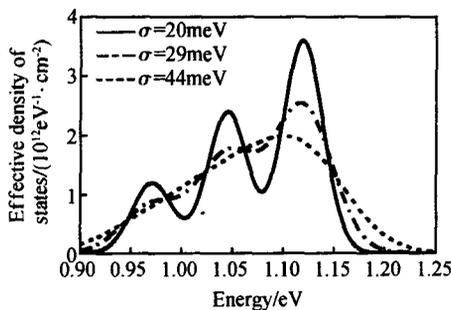


Fig. 3 QDs effective density of states versus different size fluctuations

Comparing Fig. 4 with Fig. 2, we find that the equilibrium saturated modal gain of each state is much lower than its counterpart in the non-equilibrium

case. This is because lots of holes are emitted into higher states in the equilibrium case and the inversion factor  $f_e^c(E) - f_e^v(E)$  is reduced.

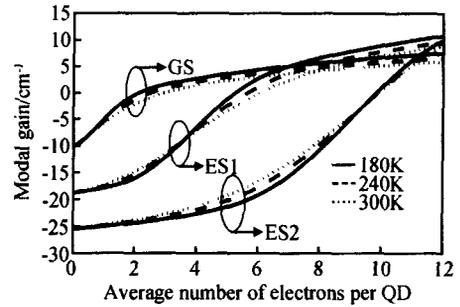


Fig. 4 Equilibrium modal gain at center energy

### 3.3 Excitons effect on gain spectrum

The above discussion is based on a free carriers' assumption. However, excitons and free carriers in fact exist in the QDs simultaneously. Because of strong QDs confinement and strong Coulomb interaction, excitons have much larger wavefunction overlaps. Moreover, in contrast to separate electron and hole quasi-Fermi levels, we assume excitons follow a uniform exciton Fermi level. These differences may have a remarkable influence on modal gain. A model<sup>[3]</sup> incorporating excitons influence can be shown as

$$g_{\text{modal}}(E) = g_{\text{free}}(E) n_{\text{free}} + g_{\text{exciton}}(E) n_{\text{exciton}} \quad (7)$$

$$n_{\text{exciton}} = 1 - \frac{1}{\sqrt{1 + \frac{1}{n_{\text{exciton}}} \exp\left(\frac{E_B}{kT}\right) - \left(1 + \frac{E_B}{kT}\right)}} \quad (8)$$

where  $n_{\text{free}}$  and  $n_{\text{exciton}}$  denote the ratios of free carriers and excitons. The binding energy  $E_B$  is chosen

to be  $20\text{meV}^{[2]}$ . According to this model, GS saturated modal gain at 300 K is  $15.7\text{cm}^{-1}$ , much larger than  $5.2\text{cm}^{-1}$  of the free carriers case. Due to decreased excitons, threshold level will shift from GS to ES when temperature increases.

For non-equilibrium case, modal gain is affected by temperature similarly. This makes threshold current not completely independent of temperature, differing from predictions for ideal QDs lasers.

### 4 Characteristics of threshold current density

In the equilibrium regime, QDs couple through the wetting layer and the occupation of the wetting layer makes states population temperature-dependent. With rising temperature, more carriers will evaporate out of QDs. Therefore, we must include the wetting layer influence when considering current density.

$$J = N_1 \frac{q}{i} \times \left( \frac{n_w}{\tau_w} + \frac{n_{\text{QD}} N_a}{\tau_{\text{QD}}} \right) \quad (9)$$

where  $\tau_w$  is the recombination time in the wetting layer,  $\tau_{\text{QD}}$  is the average recombination time in

QDs,  $N_a$  is the average carriers number per QD, and  $n_w$  is carriers' surface density in the wetting layer, which should be derived from steady-state rate equation,  $i$  is the injection efficiency.

For non-equilibrium case, thermal coupling is negligible, and Equation (9) can be simplified as

$$J = N_1 \frac{q}{i} \times \frac{n_{\text{QD}} N_a}{\tau_{\text{QD}}} \quad (10)$$

Equations (11) and (12) describe the threshold condition and threshold current's temperature dependence respectively

$$g_{\text{modal}} = \frac{1}{L} \ln\left(\frac{1}{R}\right) + i n \quad (11)$$

$$J_{\text{th}} = J_0 \exp(T/T_0) \quad (12)$$

We choose  $\tau_w = 2\text{ns}$ ,  $\tau_{\text{QD}} = 0.5\text{ns}$ , and  $i = 70\%$ , the magnitude of which are widely used in modeling<sup>[3]</sup>.  $R = 0.31$  is reflectivity of facets as cleaved and  $n = 2\text{cm}^{-1}$  is after most recent experimental results. Figure 5 (a) shows the calculated  $J_{\text{th}}$  for QDs lasers with different cavity lengths. We find  $J_{\text{th}}$  changes only slightly at low temperatures while increases remarkably at high temperatures. The whole trend agrees with experimental results<sup>[5]</sup> very well.

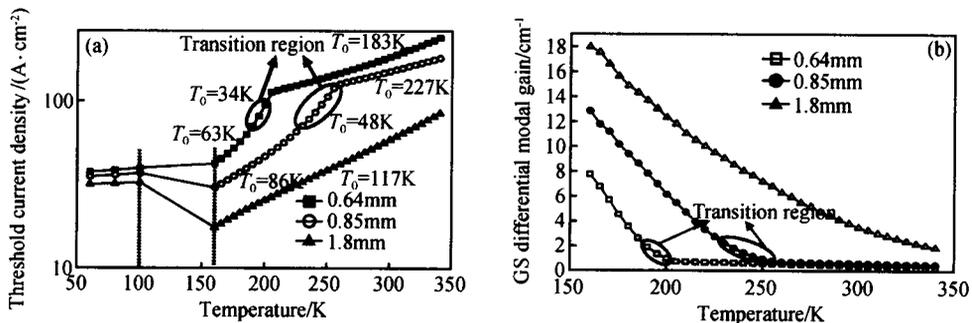


Fig. 5 (a)  $T_0$  and  $J_{\text{th}}$  for QDs lasers with different cavity lengths; (b) GS differential modal gain at threshold level versus temperature

Due to higher losses, shorter cavity corresponds to higher  $J_{\text{th}}$ , but this is much more evident in the equilibrium regime. Also in this regime,  $T_0$  changes not monotonously for the 0.64mm and 0.85mm lasers. Comparing Fig. 5 (a) with Fig. 5 (b), we find  $T_0$  changes when threshold level shifts

from GS linear region to ES1 linear region via GS saturated region. For the 0.64mm laser, this appears when temperature increases from 190 to 200 K; for the 0.85mm laser, this happens at a higher temperature range of 230 ~ 255 K. But for the 1.8mm one, such a transition does not appear

up to 340 K. Near GS saturated region,  $T_0$  decreases from 63 to 34 K for the 0.64 mm laser and from 86 to 48 K for the 0.85 mm one. In ES1 linear region,  $T_0$  is 183 K for the 0.64 mm laser and 227 K for the 0.85 mm one, which is much larger than that in GS due to the higher differential gain of ES1. Such variations will appear repeatedly if temperature further increases and  $T_0$  will exhibit oscillation characteristic. This phenomenon was firstly reported in Ref. [4]. We find it directly results from the excited states involvement.

At lower temperatures,  $J_{th}$  is rather insensitive to temperature. For example,  $T_0$  is 677, 813, and 1324 K for the 0.64, 0.85, and 1.8 mm lasers respectively. Such high  $T_0$  agrees with experimental results<sup>[4]</sup> well. Around  $T_B$ , lasers undergo gradual changes from non-equilibrium distribution to equilibrium distribution and  $J_{th}$  decreases during the process. In Eq. (12), this corresponds to a negative  $T_0$ , as reported in Ref. [11]. Based on our calculation and analysis, it can be explained by QDs' equal occupation probability in the non-equilibrium re-

gime, where carriers favor no particular energy, making saturated modal gain lower than that at temperatures immediately above  $T_B$ .

### 5 Improvement of QDs laser's performance

According to the above discussion, higher modal gain is desired for improving  $T_0$  and lowering  $J_{th}$ . p-type doping<sup>[7,13]</sup> and vertical stacking<sup>[12]</sup>, for instance, are feasible. The former measure increases GS inversion factor through enhancing GS hole occupation probability, while the latter increases the dot density in the active region. The results of these two approaches are shown in Fig. 6 (a) for the 0.85 mm laser. Due to the increased modal gain,  $T_0$  increases to 124 K and 145 K by stacking 4 and 8 layers respectively. Through doping 30 acceptors per dot,  $J_{th}$  reduces to be the lowest and  $J_{th}$ 's temperature stability is also ameliorated. For all the cases, the shift of threshold level from GS to ES1 is avoided.

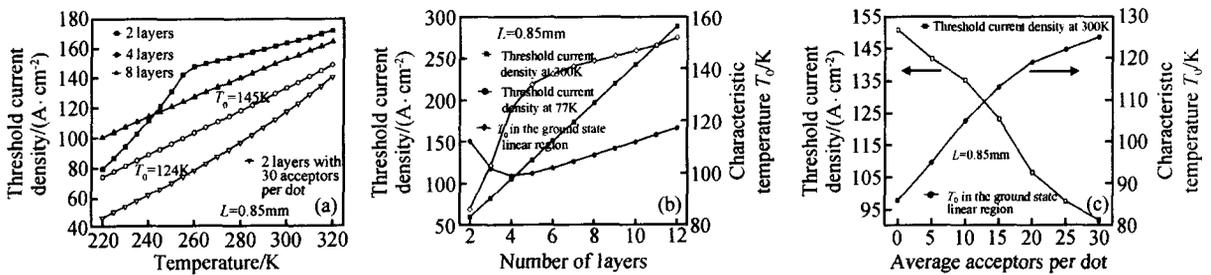


Fig. 6 (a) Variations of  $T_0$  and  $J_{th}$ ; (b) Influence of vertical stacking; (c) Influence of p-doping

However, because more QDs have to be pumped at the same time, carriers per dot are reduced,  $J_{th}$  will not necessarily decrease with more QDs layers. It is the competition outcome between carriers number per dot and QDs number that decides whether  $J_{th}$  increases or decreases. Seeing from Fig. 6 (a), although  $J_{th}$  of the 4-layer structure remains smaller than that of the 2-layer one in the whole temperature range concerned,  $J_{th}$  of the 8-layer structure is larger than that of the 2-layer one below 240 K. Figure 6 (b) shows the relation of  $J_{th}$ ,

$T_0$ , and  $N_1$  at two typical temperature points. At 300 K,  $J_{th}$  declines as  $N_1$  increases from 2 to 4, but with still larger  $N_1$ ,  $J_{th}$  begins to rise instead. At 77 K, however,  $J_{th}$  increases sharply with increasing layer. In GS linear region,  $T_0$  improves evidently from 86 K of 2 layers to 124 K of 4 layers, but the improvement becomes much slower afterwards. Figure 6 (c) illustrates the influence of acceptors doped in the active region. Because p-doping does not increase QDs number,  $J_{th}$  will absolutely decrease. This is different from vertical stacking. Ob-

viously, when acceptors increase from 10 to 15 per QD, the threshold level shifts to GS from ES1. For  $T_0$ , it keeps rising steadily from 86 K of undoped case to 108 K of 30 acceptors' case.

However, we must be aware of the concomitant negative effects. For example, stacking too many layers will spatially separate electrons and holes, lowering the direct recombination efficiency. Also, large amounts of acceptors will degrade the injection efficiency and increase the internal losses<sup>[7]</sup>. Therefore, in QDs laser design, all factors must be carefully balanced.

Figure 7 exhibits the relationship between  $J_{th}$  and  $n_{QD}$ . Undoubtedly, higher  $n_{QD}$  makes threshold condition easier to be met by GS, as is manifested in Eq. (2). For instance, when  $N_1 = 2$ ,  $n_{QD}$  has to surpass  $4 \times 10^{10} \text{ cm}^{-2}$ , while for  $N_1 = 4$ , this value reduces to  $2 \times 10^{10} \text{ cm}^{-2}$ . However, there exists optimal  $n_{QD}$  corresponding to minimal  $J_{th}$ . Further increasing  $n_{QD}$  above this value continues to decrease carriers in a single dot but will lead to larger  $J_{th}$ . The reason for this characteristic is similar to the vertical stacking case.

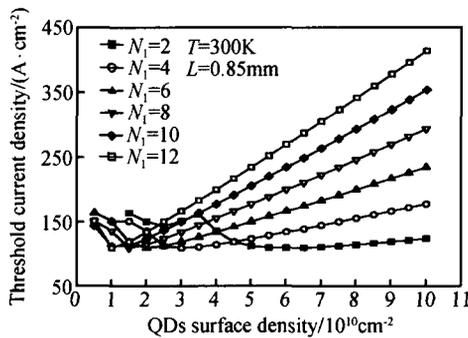


Fig. 7  $J_{th}$  versus  $n_{QD}$  as  $N_1$  increases from 2 to 12

## 6 Conclusion

Gain spectrum strongly relies on temperature in the equilibrium regime. Deeply affected by this dependence,  $T_0$  oscillates when temperature increases. Between non-equilibrium regime and equilibrium regime,  $T_0$  is negative, which results from the different carrier distributions.

The existence of excited states may lead to many negative effects on QDs lasers performance. Through vertical stacking and p-type doping, we can limit the excited states influence, realize GS operation, increase  $T_0$ , and lower  $J_{th}$ .

## References

- [ 1 ] Asryan L V, Grundmann M, Ledentsov N N, et al. Effect of excited-state transitions on the threshold characteristics of a quantum dot laser. *IEEE J Quantum Electron*, 2001, 37: 418
- [ 2 ] Bimberg D, Grundmann M, Ledentsov N N. *Quantum dot heterostructures*. Chichester, New York: John Wiley, 1999
- [ 3 ] Dikshit A A, Pikal J M. Carrier distribution, gain, and lasing in 1.3- $\mu\text{m}$  InAs-InGaAs quantum-dot lasers. *IEEE J Quantum Electron*, 2004, 40: 105
- [ 4 ] Chang C A, Hwang F C, Wu Z R, et al. Oscillatory characteristic temperature of InAs quantum-dot laser. *IEEE Photonics Technol Lett*, 2001, 13: 915
- [ 5 ] Park G, Shchekin O B, Deppe D G. Temperature dependence of gain saturation in multilevel quantum dot lasers. *IEEE J Quantum Electron*, 2000, 36: 1065
- [ 6 ] Krishna S, Zhu D, Xu J, et al. Structural and luminescence characteristics of cycled submonolayer InAs/GaAs quantum dots with room-temperature emission at 1.3  $\mu\text{m}$ . *J Appl Phys*, 1999, 86: 6135
- [ 7 ] Shchekin O B, Deppe D G. 1.3  $\mu\text{m}$  InAs quantum dot laser with  $T_0 = 161 \text{ K}$  from 0 to 80 . *Appl Phys Lett*, 2002, 80: 3277
- [ 8 ] Bimberg D, Kirstaedter N, Ledentsov N N, et al. InGaAs-GaAs quantum-dot lasers. *IEEE J Sel Topics Quantum Electron*, 1997, 3: 196
- [ 9 ] Sauvage S, Boucaud P, Brunhes T, et al. Dephasing of intersublevel polarizations in InAs/GaAs self-assembled quantum dots. *Phys Rev B*, 2002, 66: 153312-1
- [ 10 ] Ning Yongqiang, Gao Xin, Wang Lijun, et al. Spontaneous emission and optical gain in InGaAs quantum dots. *Chinese Journal of Semiconductors*, 2002, 23 (4) : 373 (in Chinese) [ 宁永强, 高欣, 王立军, 等. InGaAs 量子点的自发发射及光增益. *半导体学报*, 2002, 23 (4) : 373 ]
- [ 11 ] Zhukov A E, Ustinov V M, Egorov A Y, et al. Negative characteristic temperature of InGaAs quantum dot injection laser. *Jpn J Appl Phys*, 1997, 36: 4216
- [ 12 ] Grundmann M, Bimberg D. Gain and threshold of quantum dot lasers: theory and comparison to experiments. *Jpn J Appl Phys*, 1997, 36: 4181
- [ 13 ] Shchekin O B, Ahn J, Deppe D G. High temperature performance of self-organized quantum dot laser with stacked p-doped active region. *Electron Lett*, 2002, 38: 712

## GaAs 基长波长量子点激光器增益和阈值电流密度的理论分析\*

邓盛凌 黄永箴 金潮渊 于丽娟

(中国科学院半导体研究所 集成光电子学国家重点实验室, 北京 100083)

**摘要:** 基于谐振子模型的量子点能级, 计算了包括和排除激子影响时多能级的增益谱. 考虑了低温时非平衡载流子分布, 得出了较宽温度范围内阈值电流密度的变化, 包括负温度及振荡温度效应. 研究了垂直层叠和 p 型掺杂对量子点激光器性能的改善, 并讨论了获得极小阈值电流密度时的最佳量子点密度.

**关键词:** 量子点激光器; 多能级; 增益谱; 温度依赖

**EEACC:** 4320J; 4250; 2520

**中图分类号:** TN302      **文献标识码:** A      **文章编号:** 0253-4177(2005)10-1898-07

---

\* 国家高技术研究发展计划(批准号:2003AA311070) 和国家自然科学基金(批准号:60225011) 资助项目

邓盛凌 男, 1979 年出生, 硕士研究生, 主要从事量子点激光器研究.

黄永箴 男, 1963 年出生, 研究员, 主要从事半导体光电子学研究.

于丽娟 女, 1963 年出生, 副研究员, 主要从事半导体材料和器件研究.

2004-10-10 收到, 2005-03-17 定稿