第 19 卷第 3 期 半 导 体 学 报 Vol. 19, No. 3
1998 年 3 月 CHINESE JOURNAL OF SEMICONDUCTORS Mar., 1998

An Improved Analytical Model for Minority Carrier Transport in Quasi-Neutral Semiconductor Regions

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Abstract In order to derive analytical models of minority carrier transport in quasi-neutral base region of advanced bipolar transistors, a new initial condition is proposed for achieving a reasonable accuracy by the first iteration. The biggest relative error up to e-b junction bias of 0.96V reaches 4.9% for collector current density and 1.75% for base transit time in a wider range of base width (such as 50~200nm) and peak base doping concentration (such as 6×10^{17} ~ $8\times10^{18}cm^{-3}$). This model is valid at any injection level before the onset of Kirk effect, and for arbitrary base doping profiles.

EEACC: 2560B, 2560J, 2530N

1 Introduction

A successive scaling down of feature processing dimension has given rise to a serial of variations in manufacturing technologies[1], device and circuit structure[2], and minority carrier transport mechanisms of bipolar transistors[3]. A further improved model for the minority carrier transport in quasi-neutral base region is essential to design and fabrication of the advanced bipolar devices and circuits. Analytical model can present two advantages over numerical solutions. The first is placed on improving computational efficiency meanwhile maintaining a higher modeling accuracy. Another is focused on facilitating physical insight into the minority carrier transport. For CAD purpose, a simplified compact model

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Received 18 July 1997, revised manuscript received 22 October 1997

should be established, so that the former must be emphasized. In fact, a considerable effort has been made to obtain closed-form analytical expressions for minority carrier current density and base transit time in the base region[4~7].

Within the framework of drift and diffusion approach, the adopted assumptions have concretely been demonstrated[6]. Worthy to be stressed, as device structural dimension is further scaled down, the dependence of minority carrier mobility on electrical field in the base region becomes little negligible[8]. This effect has been taken into account in the present analysis.

What this subject concerns concentrates on how to treat a tradeoff between the modeling accuracy and the computational efficiency so as to derive accurate, yet relative simple analytical solutions by means of iteration procedure. In this paper, a new initial condition for the iteration is proposed. Compared with the previous work, an accurate, more simplified analytical solution is obtained. This attempt can largely improve the modeling computational efficiency.

2 Review of Previous Work

For P type base region, the electrical field $E(x)$, the current density J_n and the electron profile $n(x)$ can respectively be expressed as[5]:

$$E(x) = FE[n(x), J_n, D_n] = V_T \frac{d}{dx}\left[\ln\left(\frac{n + N_B}{n_{ie}^2}\right)\right] = \frac{V_T}{2n + N_B}\left(\frac{dN_B}{dx} - \frac{n + N_B}{n_{ie}^2} \times \frac{dn_{ie}^2}{dx} - \frac{J_n}{qD_n}\right) \quad (1)$$

$$J_n = FJ_n[n(x), D_n] = [qn_i^2 \exp(V_{BE}/V_T)] \Bigg/ \left\{\sqrt{\frac{n_i^4 \exp(V_{BE}/V_T)}{v_s^2 n_{ie}^2(W_b)} + \frac{1}{4}\left[\frac{n_i^2 N_B(W_b)}{v_s n_{ie}^2(W_b)} + \int^{W_b} \frac{n_i^2}{n_{ie}^2 D_n}(n + N_B)dx\right]^2} + \frac{1}{2}\left[\frac{n_i^2 N_B(W_b)}{v_s n_{ie}^2(W_b)} + \int^{W_b} \frac{n_i^2}{n_{ie}^2 D_n}(n + N_B)dx\right]\right\} \quad (2)$$

$$n(x) = Fn[n(x), J_n, D_n] = [J_n n_{ie}^2 \int^{W_b} \frac{n + N_B}{qn_{ie}^2 D_n}dx + \frac{n_{ie}^2}{n_{ie}^2(W_b)} \times \frac{J_n}{qv_s}[\frac{J_n}{qv_s} + N_B(W_b)] + \frac{N_B^2}{4}]^{1/2} - \frac{N_B}{2} \quad (3)$$

where the boundary condition $n(W_b) = J_n/qv_s$ is taken, W_b is the base width, v_s is the saturate velocity at b-c junction and q is the electron charge; V_T is the thermal voltage; D_n is the electron diffusive coefficient; n_{ie} is the effective intrinsic carrier concentration; $N_B(x)$ is the base doping profile; V_{BE} is the e-b junction voltage drop. Taking the electron mobility as the function of N_B and E[9,10], the iterations among eqns (1)~(3) can begin at a suitable initial condition for achieving the accurate, yet relative simple iterative solution.

Several typical papers are reviewed so as to select the most suitable initial condition for the iteration procedure.

(A) Considering the dependence of the minority carrier mobility on the field, the iterative analytical solutions for the collector current density and the base transit time are pro-

posed at low injection as the initial condition[5]. According to eqns (1), (2) and (3), the initial condition can be written as[5]

$$E_L(x) = FE[0, 0, D_{nL}] \quad (4)$$

$$J_{nL} = FJ_n[0, D_{nL}] \quad (5)$$

$$n_L(x) = Fn[0, J_{nL}, D_{nL}] \quad (6)$$

The third iteration arrives at a higher accuracy.

(B) Based on the perturbation theory, also including the field effect on the minority carrier mobility, an accurate analytical model is obtained at the initial condition[8]:

$$J_{n2} = J'_{nL} f_W \quad (7)$$

$$n_2 = n'_L f_W \quad (8)$$

where, J'_{nL} and n'_L is respectively slightly different from J_{nL} and n_L, and can be obtained by literature[8]; f_W can be expressed as

$$f_W = \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{n_{ie}^2(0)}{N_B^2(0)} \exp\left(\frac{V_{BE}}{V_T}\right)}} \quad (9)$$

Although the derived results are too complex for practical application, the initial condition is still used in the present analysis for comparing with the new proposed initial condition.

(C) Recently, another analytical model is proposed by using the following initial condition as the accurate solutions[6]:

$$J_{n3} = FJ_n[n(0), D_{nL}] \quad (10)$$

$$n_3(x) = Fn[n(0), J_{n3}, D_{nL}] \quad (11)$$

The major lack results from neglecting the dependence of the minority carrier mobility on the field. In the present analysis, this initial condition is also employed so as to compare with the new initial condition.

(D) For uniform doping profile, neglecting the field effect on the minority carrier mobility, the accurate solutions are presented by using the simplified initial condition[7]:

$$J_{n4} = \frac{qn(0)}{\frac{W_b}{D_{nL}} + \frac{1}{v_s}} \left[2 - \frac{N_B}{n(0)} \ln\left(1 + \frac{n(0)}{N_B}\right)\right] \quad (12)$$

$$n_4(x) = n(0) - \frac{n(0) - \frac{J_{n4}}{qv_s}}{W_b} x \quad (13)$$

In the present analysis, this initial condition is developed to fit to arbitrary doping profiles in the base region and include the field effect on the minority carrier mobility.

3 Improved Analytical Model

A vital aim to this paper is to obtain accurate, yet relative simple analytical solutions by means of finding the most suitable initial condition for the iterative orders as low as possible among eqns (1), (2) and (3). In order to extend section (D) of part 2 into arbitrary base doping profiles, eqns (12) and (13) are re-written by

$$J_{n4} = \frac{qn(W_b/e)}{\frac{W_b}{D_{nL}(W_b/e)} + \frac{1}{v_s}} \left[2 - \frac{N_B(W_b/e)}{n(W_b/e)} \ln\left(1 + \frac{n(W_b/e)}{N_B(W_b/e)}\right)\right] \quad (14)$$

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$$n_4(x) = n(W_b/e) - \frac{n(W_b/e) - \frac{J_{n4}}{qv_s}}{W_b}x \tag{15}$$

where,

$$n(W_e/e) = [n_{ie}^2(W_b/e)\exp(\frac{V_{BE}}{V_T}) + \frac{N_B^2(W_b/e)}{4}]^{1/2} - \frac{N_B(W_b/e)}{2} \tag{16}$$

and e is the base of natural logarithms. Instead of eqns (12) and (13), the arbitrary base doping profiles are empirically handled by using an effective doping concentration at W_b/e in eqns (14) and (15). Eqns(7) and (8) are obtained by improving the low injection solutions. Eqns (14) and (15) are gotten by modifying the uniform doping concentration. In general, the influence of the doping profiles on the accurate solutions is smaller than the bias variations under the high injection. Therefore, eqns (14) and (15) should have a higher calculation efficiency than (7) and (8). This can be certified in the comparison with the numerical calculation.

Thus, the new initial condition is achieved by

$$E_{new} = FE[n_4(x), J_{n4}, D_{nL}] \tag{17}$$

$$J_{n\text{-}new} = FJ_n[n_4(x), D_{n\text{-}new}] \tag{18}$$

$$n_{new}(x) = Fn[n_4(x), J_{n\text{-}new}, D_{n\text{-}new}] \tag{19}$$

Substituting eqn. (19) into eqns (2) and (3), the first iteration would reach a higher accuracy in comparison with the numerical results[5].

4 Comparisons With Numerical Analysis

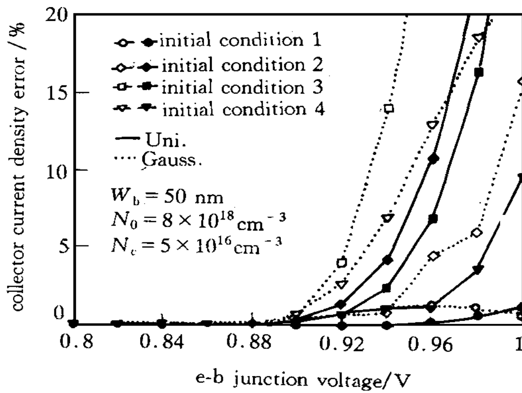
In order to testify the validity of the improved analytical model, the relative error associated with the existed and improved analytical solutions compared to numerical results is proposed by

$$\text{error\%} = 100 \times \left|\frac{\text{numerical} - \text{analytical}}{\text{numerical}}\right| \tag{20}$$

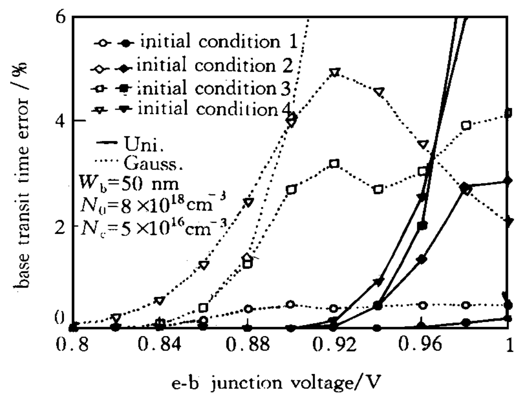
where the numerical solutions can be obtained by a convergent iteration[5].

For modern bipolar transistors, narrow base width and high peak base doping concentration can be achieved[1,2]. Thereby, a wider range of the base width (50～200nm) and the peak base doping concentration (6×10^{17}～8×10^{18}cm^{-3}) is chosen for a detailed calculation.

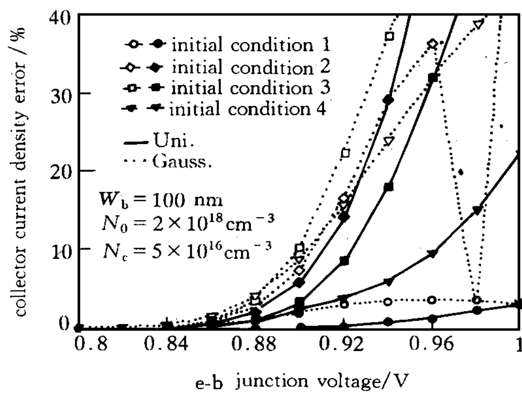
Considering the dependence of minority carrier mobility on the field and running the first iteration based on different initial conditions, Fig. 1 and Fig. 2 respectively plots the collector current density error and the base transit time error as a function of e-b junction voltage, where a serial of marks (a), (b) and (c) respectively covers the typical values of the base width and the peak base doping concentration. Besides, several combinations of the base width and the peak base doping concentration such as 50nm and 6×10^{17}cm^{-3}, and 200nm and 8×10^{18}cm^{-3} are also included in the calculation. The concerning figures are not presented in this paper for the sake of simplicity. In the figures, the ‘initial condition 1’ represents the improved analytical model, the ‘initial condition 2’ does Suzuki's results[8]



(a)



(a)



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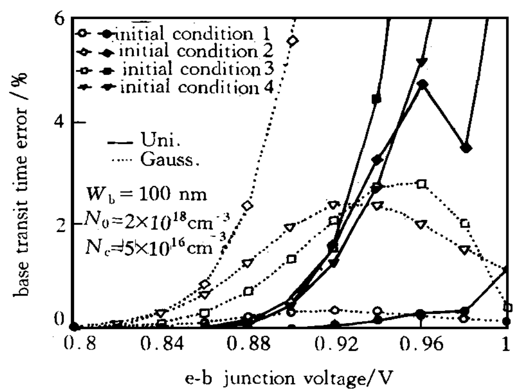

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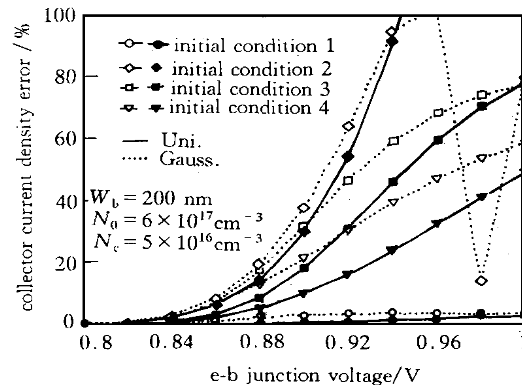

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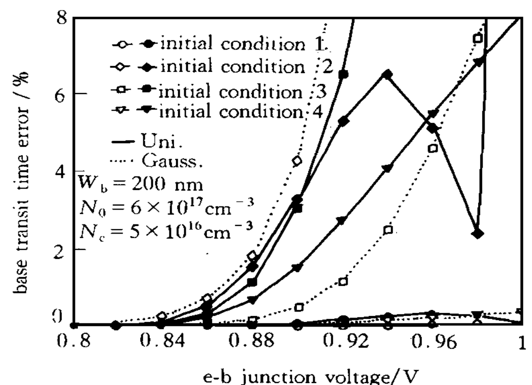

(c)

Fig. 1　Collector current density error as the function of e-b junction bias

where W_b represents the base width, N_0 does the peak base doping concentration, and N_c does the uniform collector concentration. (a), (b) and (c) respectively correspond to different W_b and N_0 values

Fig. 2　Base transit time error as the function of e-b junction bias

where W_b represents the base width, N_0 does the peak base doping concentration, and N_c does the uniform collector concentration. (a), (b) and (c) respectively correspond to different W_b and N_0 values

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as section (B) of part 2 shown, the 'initial condition 3' does Ma *et al.*'s results[5] relative to section (A) of part 2, and the 'initial condition 4' does Rinaldi's results[6] corresponding to section (C) of part 2.

Obviously, the first iteration is the simplest available solution based on the existed and improved initial conditions. For the smaller e-b junction bias, the relative error increases as V_{BE} increases, which reflects that both existed and improved analytical models are valid in the low infection region. For the bigger e-b junction bias, the relative error presents some undulating variations with V_{BE}, which could be caused by the oscillation in convergent process. Comparing with the numerical results, the improved analytical model achieves the lowest relative errors in a wider range of the base width (50～200nm) and the peak base doping concentration (6×10^{17}～$8\times10^{18}cm^{-3}$) for the uniform and the Gaussian doping profiles. The biggest relative error up to e-b junction bias of 0.96V respectively arrives at 4.9% for the collector current density and 1.75% for the base transit time, which is comparable to Rinaldi's results without considering the mobility dependence on the field[6] and Ma *et al.*'s solutions with the third iteration[5].

Including the mobility dependence on the field and maintaining a reasonable accuracy, the present solutions should be the simplest available by now. Consequently, the improved analytical model can be more suitable to device simulation and compact model for the advanced bipolar transistors.

5 Conclusion

For the advanced bipolar transistors with narrow base width (such as 50～200nm) and high peak base doping concentration (such as 6×10^{17}～$8\times10^{18}cm^{-3}$), the first iteration using the improved initial condition can achieve reasonably accurate solution for the collector current density and the base transit time. The improved analytical model applies to device simulation and compact model.

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