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Analytical Calculation of Avalanche Breakdown Voltage of the Single-Diffused Junction Based on Double-sided Asymmetric Linearly Graded Approximation *

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Abstract By using the definition of effective doping concentration gradient and depletion approximation, an analytical expression for avalanche breakdown voltage of double-sided asymmetric linearly graded junction, which is a proper approximation to the practical single-diffused junction, has been derived. Based on the semiempirical expressions of the substrate side concentration gradient of the diffused junction, this effective doping gradient parameter, being determined by the concentration gradients of both sides of the junction, together with the published breakdown voltage format for common symmetric linearly graded junctions, immediately gives the breakdown voltage of the diffused junction. All results coincide well with the previous conclusion obtained by numerical method. Meanwhile, the analytical solution of the breakdown voltage of the single-sided linearly graded junction has been also obtained.

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1 Introduction

It is well known that the doping profile of the diffused junction (DJ) is a Gaussian or a complementary error function, and it is impossible to obtain closed-form analytical solutions of breakdown voltage for these complex profiles[1]. In order to calculate the avalanche breakdown critical quantities for DJ, in addition to the full numerical techniques[2~4], analytical methods such as the single-sided abrupt junction (SSAJ) and the symmetric linearly-graded junction (SLGJ) approximations have been widely used. Their deficiencies are obvious due to the fact that SSAJ approximation only provides a description for shallow-DJ and the classical SLGJ approximation is only reasonable for deep-DJ. However, the characteristics of the DJ depend on both the diffused side doping gradient and the substrate concentration in many cases. As a result, this kind of junction is commonly considered to be an

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asymmetrical double-sided p-n junction; the first analytical solution was derived by Brook[5] based on the main assuonption of double-sided abrupt p-n junction. In fact, this important assumption is also in doubt due to the very strong dependence of the impurity distribution on the property of the diffusion impurity and the fabricated process. On the other hand, the concept of the single-sided linearly graded junction (SSLGJ) is also used in some papers[6,7], in which breakdown voltage is always believed to be one half of the ALGJ on the basis of the intuitive judgement. However, there has not been developed any theory that could support the above view.

This work is a theoretical analysis of the breakdown voltages of the asymmetric diffused junction (ADJ) based on the assumption of double-side asymmetric linearly graded junction (DSALGJ) which is a proper approximation to the above typical DJ. In this paper, it will be shown that the breakdown voltage of such a DSALGJ may be obtained accurately from published format of breakdown voltage against the impurity gradient for SSLGJ by using the definition of the effective doping gradient and the depletion approximation. All results are verified by previous reports given by the numerical analysis. Meanwhile, the conclusion of the SSLGJ from intuition will be proved incorrect.

2 Theory

Figure 1 shows the schematic, the electric field profile, the charge density distribution, and the electrostatic potential at breakdown in an ideal DSALGJ with uniform gradient G_A (in cm^{-4}) and G_D (in cm^{-4}) and depletion region width w_1 (in cm) and w_2 (in cm). The maximum field E_M (in V/cm) at breakdown occurs at $x = 0$.

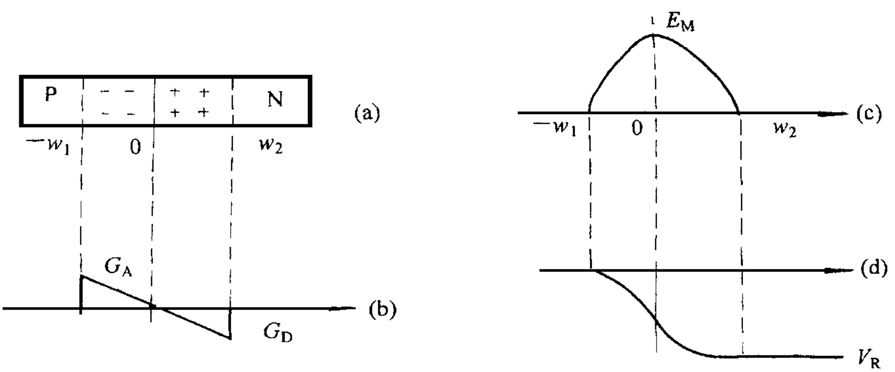

Fig. 1. DSALGJ (a) Schematic (b) Charge density (c) Electric Field (d) Electrostatic potential

In Fig. 1, the built-in potential Φ_0 (in V) is augmented by the applied reverse bias V_R, and the total voltage across the junction is $(\Phi_0 + V_R)$. The built-in potential Φ_0 can be given in the following form

$$\Phi_0 = V_T \ln (G_A G_D w_{10} w_{20} / n_i^2), \qquad (1)$$

where $V_T = kT/q \approx 26$mV at 300K, the quantity n_i is the intrinsic carrier concentration in a pure sample of the semiconductor and $n_i \approx 1.5\times10^{10}\mathrm{cm}^{-3}$ at 300K for silicon, w_{10}, w_{20} are depletion layers at thermal equilibrium case, Φ_0 is commonly neglected for simplicity.

If the depletion region penetrates a distance w_1 into the p-type region and w_2 into the n-type region, then we require, based on the Gaussian' law,

$$E_M = qw_1^2G_A/2\epsilon = qw_2^2G_D/2\epsilon, \tag{2}$$

where q is the electron charge (1.6×10^{-19}C), and ϵ is the permittivity of silicon (1.04×10^{-12}F/cm).

Transforming Eq. (2), one can obtain

$$w_1^2G_A = w_2^2G_D. \tag{3}$$

It shows that the total charge per unit area on either side of the junction must be equal in magnitude but opposite in sign.

Poisson's equation in one dimension requires that

$$d^2V/dx^2 = -\rho/\epsilon = qG_Ax/\epsilon \qquad \text{for } -w_1 < x < 0. \tag{4}$$

Integration of Eq. (4) gives

$$dV/dx = qG_Ax^2/2\epsilon + C_1, \tag{5}$$

where C_1 is a constant. However, the distribution of electric field E is given by

$$E(x) = -dV/dx = -(qG_Ax^2/2\epsilon + C_1). \tag{6}$$

Since there is zero electric field outside the depletion region, a boundary condition is

$$E = 0, \text{ for } x = -w_1,$$

and using this condition in Eq. (6) gives

$$E(x) = -qG_A/2\epsilon(w_1^2 - x^2) = -dV/dx \qquad \text{for } -w_1 < x < 0. \tag{7}$$

Thus the dipole of charge existing at the junction gives rise to an electric field that varies squarely with distance.

Integration of Eq. (7) gives

$$V(x) = qG_Aw_1^2x/2\epsilon - qG_Ax^3/6\epsilon + C_2. \tag{8}$$

If the zero potential is arbitrarily taken to be the potential of the neutral p-type region, then the second boundary condition is

$$V = 0, \text{ for } x = -w_1,$$

and using it in Eq. (8) gives

$$V(x) = qG_A/\epsilon(w_1^3/3 + w_1^2x/2 - x^3/6), \qquad \text{for } -w_1 < x < 0. \tag{9}$$

At $x = 0$, we define $V = V_1$ and then Eq. (9) gives

$$V_1 = qG_Aw_1^3/3\epsilon \tag{10}$$

Combining Eqs (10) and (2), E_M can be rewritten as

$$E_M = qG_Aw_1^2/2\epsilon \tag{11}$$

The above equations are the relationship of the single-side linearly graded junction with field strength, electrostatic and depletion width.

If the potential difference between $x = 0$ and $x = w_2$ is V_2, similarily, it follows that

$$V_2 = qG_Dw_2^3/3\epsilon, \tag{12}$$

and then the total voltage across the junction is

$$\phi_0 + V_R = V_1 + V_2 = q(G_A w_1^3 + G_D w_2^3)/3\epsilon \tag{13}$$

Substitution of Eq. (3) in Eq. (13) gives

$$\phi_0 + V_R = \frac{q w_1^3 G_A}{3\epsilon}\left(1 + \sqrt{\frac{G_A}{G_D}}\right). \tag{14}$$

From Eq. (14), the penetration of the depletion layer into the p-type region is

$$w_1 = \left[\frac{3\epsilon(\varphi_0 + V_R)}{qG_A\left(1 + \sqrt{\frac{G_A}{G_D}}\right)}\right]^{1/3}. \tag{15}$$

Similarly,

$$w_2 = \left[\frac{3\epsilon(\varphi_0 + V_R)}{qG_D\left(1 + \sqrt{\frac{G_D}{G_A}}\right)}\right]^{1/3}. \tag{16}$$

The total depletion width w_T is

$$w_T = w_1 + w_2 = \left[\frac{3\epsilon(\varphi_0 + V_R)}{q}\right]^{1/3}\left\{\left[G_A\left(1 + \sqrt{\frac{G_A}{G_D}}\right)\right]^{-1/3} + \left[G_D\left(1 + \sqrt{\frac{G_D}{G_A}}\right)\right]^{-1/3}\right\}. \tag{17}$$

If the definition of effective doping concentration gradient G_{eff} is in the following form

$$\frac{1}{\sqrt{G_{eff}}} = \frac{1}{\sqrt{G_A}} + \frac{1}{\sqrt{G_D}}, \tag{18}$$

then Eq. (17) can be simplified as follows

$$w_T = \left[\frac{3\epsilon(\varphi_0 + V_R)}{qG_{eff}}\right]^{1/3}. \tag{19}$$

Similarly, combining Eqs. (1), (15) and (19) gives

$$E_M = \frac{qG_A w_1^2}{2\epsilon} = \frac{q w_T^2 G_{eff}}{2\epsilon}. \tag{20}$$

Then transforming Eq. (18), one can obtain

$$G_{eff} = \frac{G_A G_D}{G_A + G_D + 2\sqrt{G_A G_D}}. \tag{21}$$

It has been shown that the relationships between G_{eff} and E_M and w_T, and $(\phi_0 + V_R)$ are the same as those in SSLGJ. Equivalent diagram is shown in Fig. 2. If G_D is equal to G_A, the

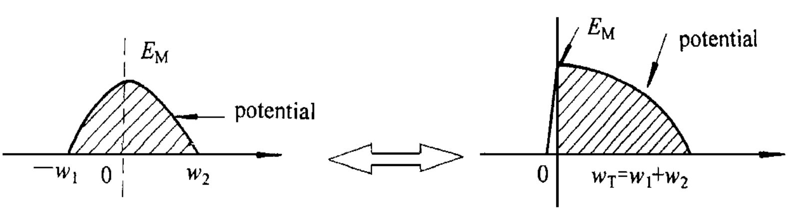

Fig. 2 Equivalent diagram from DSALGJ with gradient G_A and G_D to SSLGJ with doping gradient G_{eff}

following relationships exist

$$G_{eff} = G_A/4 = G_D/4, \tag{22}$$

$$E_M = \frac{qG_A w_T^2}{8\epsilon} = \frac{qG_D w_T^2}{8\epsilon}, \tag{23}$$

$$\phi_0 + V_R = \frac{qG_A w_T^3}{12} = \frac{qG_D w_T^3}{12}. \tag{24}$$

The above conclusion leads to the useful results that the breakdown voltage of a DSALGJ can be predicted from the published format of voltage against doping gradient for SLGJ, if we simply read off that voltage corresponding to the calculation value of G_{eff}. Firstly, the breakdown voltage of SSLGJ can be derived from SLGJ. According to Ref. [8]

$$BV = 9.2 \times 10^9 G_A^{-2/5}, \tag{25}$$

$$w_T = 9.1 \times 10^5 G_A^{-7/15}, \tag{26}$$

for SLGJ with the gradient G_A and G_D (note: $G_A = G_D$).

Transforming Eqs (26) and (25), based on Eq. (22), one can give

$$BV = 5.284 \times 10^9 G_{eff}^{-2/5}, \tag{27}$$

$$w_T = 4.8 \times 10^5 G_{eff}^{-7/15}, \tag{28}$$

for DSALGJ with the effective doping gradient G_{eff}.

Then, if there exists a SSLGJ with a concentration gradient G_s, based on the similarity to the DSALGJ, breakdown voltage of SSLGJ can be written as

$$BV = 5.284 \times 10^9 G_s^{-2/5}, \tag{29}$$

$$w_T = 4.8 \times 10^5 G_s^{-7/15}. \tag{30}$$

In fact, breakdown voltage of SSLGJ could be obtained by integrating ionization integral equation for breakdown condition based on Flop's expression for the ionization rate. However, the deduction is very difficult and complicated compared with the above methods.

It can be seen that the breakdown voltages of DSALGJ and SSLGJ can be obtained simply from the published format and graphical illustrations of SLGJ, by using $4G_{eff}$ or $4G_s$ to replace the concentration gradient, G_A or G_D of SLGJ. Furthermore, the breakdown voltage of SSLGJ with the gradient G_s is proved to be higher than one half of that of SLGJ with the concentration gradient G_s. The result from the intuitive knowledge is not suitable for the accurate calculation.

3 Discussion

In order to make the DSALGJ approximation applicable to the calculation of breakdown voltage for the practical diffused junction, the concentration gradients of both sides of DSALGJ must be determined before G_{eff} is calculated, although the breakdown voltage of DSALGJ can be predicted simply based on the format or graphical illustration of SLGJ.

The concentration gradient G_A of the diffused side of DSALGJ can be determined in the common way. If the impurity distribution function of DSALGJ is $f(x)$, then G_A can be written as

$$G_A = \mathrm{d}f(x)/\mathrm{d}x\,\big|_{x=x_j} \tag{31}$$

Since the substrate side concentration gradient G_D (in cm^{-3}) is related to the substrate concentration N_D and the junction depth, developing a G_D model is physically too cumbersome to be practical. However, the semi-empirical expression of G_D can be obtained based on the developed equivalent theory[9]

$$G_D = G_A N_D / (8.78 \times 10^{+9} G_A + N_D). \tag{32}$$

Based on the above discussions, the breakdown voltage of DSALGJ in all cases can be obtained. Figure 3 shows the comparison between breakdown voltages of the typical diffused junction calculated from DSALGJ, of the value obtained from SLGJ and of the predicted value from SSAJ approximation.

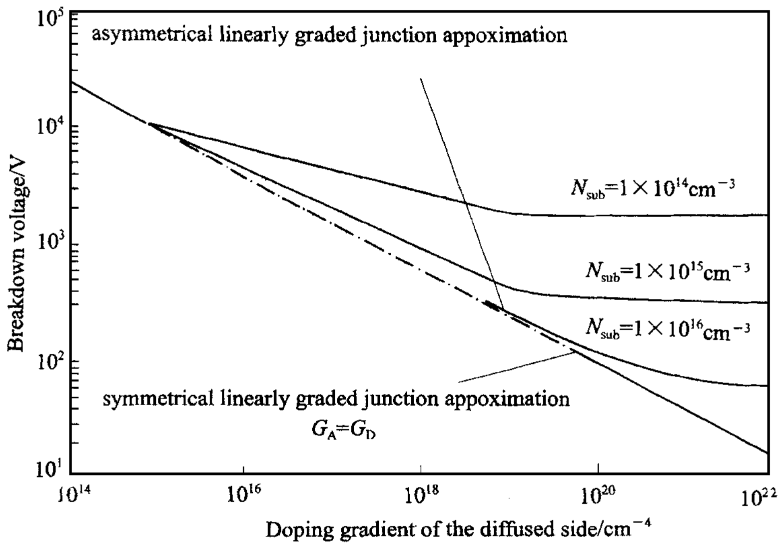

Fig. 3 Breakdown voltage versus doping concentration for the typical diffused junction case

From Fig. 3, it can be seen that not only do the DSALGJ's approximation include the results of the SLGJ and SSAJ, but also predict the breakdown voltage of diffused junction of mid-depth which could be obtained only by the numerical analysis method. In order to compare DSALGJ's breakdown voltage with the that of the SLGJ and SSAJ and numerical analysis[3,4], all results are summarized in Table 1. An excellent agreement can be observed between the results of the DSALGJ and numerical analysis[4]. Meanwhile, the SSAJ'and classical SLGJ'results depart from the breakdown voltage obtained from the numerical analysis in many practical diffused cases very far.

Table 1 Comparison between the breakdown voltages obtained for SSAJ, SLGJ, numerical and DSALGJ obtained in this paper

Impurity distribution		breakdown voltage/V			
G_A/cm^{-4}	N_{sub}/cm^{-3}	SSAJ	SLGJ	numerical	DSALGJ
1×10^{17}	1×10^{14}	1688	605.7	1938	1944
1×10^{17}	1×10^{15}	300.29	605.7	698	709
1×10^{17}	1×10^{16}	53.6	605.7	634	638
1×10^{17}	1×10^{16}	53.6	605.7	634	638
1×10^{20}	1×10^{16}	53.6	96	110	115.6
1×10^{23}	1×10^{16}	53.6	6.06	53.6	56.4

4 Conclusions

In this paper, by using the definition of effective doping gradient for DSALGJ and the common depletion approximation, the analytical solutions for the breakdown critical parameters for the double-sided asymmetric and single-sided linearly graded p-n junctions are derived. Thus the effective doping gradient, together with the published breakdown voltage format or graphical illustration of the symmetrical linearly graded junction can be used to predict the breakdown voltage of DSALGJ. The results agree well with the previous conclusion obtained by numerical analysis.

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