第27卷 第3期 半 导 体 学 报 Vol.27 No.3
2006年3月 CHINESE JOURNAL OF SEMICONDUCTORS Mar.,2006

# Modified Reynolds Equation for Squeeze-Film Air Damping of Slotted Plates in MEMS Devices*

Sun Yuancheng[1], Bao Minhang[1,†], Yang Heng[2], and Huang Yiping[1]

(1 State Key Laboratory of ASIC and System, Department of Micro Electronics, Fudan University, Shanghai 200433, China)
(2 State Key Laboratory of Transducer Technologies, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, Shanghai 200050, China)

**Abstract:** A differential equation for calculating squeeze-film air damping in slotted plates is developed by modifying the Reynolds equation. A term is added to account for the effect of airflow through the slots on the air damping of the plate. The end effect of the airflow in the slots is also treated by substituting an effective channel length for the geometric channel length (i.e. the thickness of the plate). The damping pressure distribution, damping force, and damping force coefficient of the slotted plates can be found by solving the equation under appropriate boundary conditions. With restrictions on the thickness and the lateral dimensions of the slotted plate removed, the equation provides a useful tool for analysing the squeeze-film air damping effect of slotted plates with finite thickness and finite lateral dimensions. For a typical slotted plate structure, the damping force coefficient obtained by this equation agrees well with that generated by ANSYS.

**Key words:** squeeze-film air damping; MEMS; slotted plate; Reynolds equation
**EEACC:** 2575 **PACC:** 0320; 4630M; 6240
**CLC number:** TN603.5 **Document code:** A **Article ID:** 0253-4177(2006)03-0473-05

## 1 Introduction

Squeeze-film air damping is a crucial factor in MEMS sensors and actuators such as micro accelerometers, microphones, and micro switches. A large damping effect is one of the most common factors that lead to the deterioration of MEMS device performance. To reduce squeeze-film air damping, plates in MEMS devices are often perforated or slotted[1] so that air can flow through the holes or slots as well as the peripheral borders, reducing the damping effect. As a process measure, holes or slots are opened in large-area plates by surface micro machining to reduce the etching time of the sacrificial layer under the plates. Recently, grated or slotted plates have been used in MEMS devices such as micro mechanical gyroscopes[2], electric field sensors[3], and optical displays[4]. Therefore, research on the air damping of perforated or slotted plates is important to the design of MEMS devices.

The squeeze-film air damping of MEMS devices is usually analyzed using the Reynolds equation[5~7]. For a solid plate, the squeeze-film air damping can be found by applying the Reynolds equation with appropriate boundary conditions. For perforated or slotted plates, however, the Reynolds equation can only be used if the plates are assumed to have infinitesimal thickness and infinite lateral dimensions[7]. Numerical calculations must be performed for perforated or slotted plates with finite thickness and finite lateral dimensions. Since numerical calculation is time-consuming and not transparent, an analytical method is desirable for analysis and design.

We have developed a modified Reynolds equation for the analysis of perforated plates with finite thickness and finite lateral dimensions[8]. We also show a modified Reynolds equation for slotted plates. The distribution of damping pressure, the damping force on the plate, and the damping force coefficient can be found using the equation with appropriate boundary conditions. In principle, the equation can be used to analyze damping for a slot-

* Project supported by the State Key Development Program for Basic Research of China (No. G1999033101) and the National Natural Science Foundation of China (No. 69876009)
† Corresponding author. Email: mhbao@fudan.edu.cn
Received 28 October 2005, revised manuscript received 9 December 2005 © 2006 Chinese Institute of Electronics

ted plate with finite thickness and finite lateral dimensions.

To introduce the application of the traditional Reynolds equation, and for further comparison with the damping effect of slotted plates, an analysis of the squeeze-film air damping of a long rectangular solid plate is given first before proceeding to the case of a slotted plate.

The Reynolds equation[5~7] used for the squeeze-film air damping of a solid plate is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{h^3} \times \frac{dh}{dt} \tag{1}$$

where $p(x)$ is the damping pressure caused by the squeeze-film effect, $\mu$ is the viscosity coefficient of air, and $h$ is the distance between the plate and the nearby substrate against which the plate moves. For the details of the derivation of Eq. (1), readers are referred to Ref. [8].

For a rectangular plate with a length much greater than its width, Equation (1) becomes one-dimensional

$$\frac{\partial^2 p}{\partial x^2} = \frac{12\mu}{h^3} \times \frac{dh}{dt} \tag{2}$$

If the width of the plate is $2c$, as shown in Fig. 1, then by using the boundary conditions $p(\pm c) = 0$, we obtain the damping pressure

$$p(x) = -\frac{6\mu}{h^3}\dot{h}(c^2 - x^2) \tag{3}$$

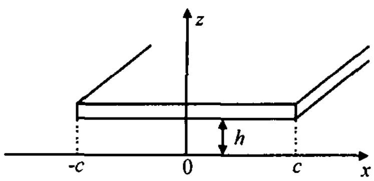


Fig. 1 Squeeze-film air damping of a long rectangular solid plate

Having briefly introduced squeeze-film air damping, we will proceed to develop a modified Reynolds equation for slotted plates with finite thickness and finite lateral dimensions. Then we will obtain the distribution of damping pressure, the damping force on the plate, and the damping force coefficient of a long rectangular slotted plate using this equation. The results agree with those for a solid plate with the same dimensions and with those for a slotted plate with the same dimensions but infinitesimal thickness.

## 2 Modified Reynolds equation for squeeze-film air damping of slotted plates

Consider a thin slotted plate as shown in Fig. 2 (e. g. a structure fabricated by surface micro-machining). If the plate width is much greater than a cell width, the air will be squeezed out or sucked in primarily through the slots when the plate oscillates against the substrate, and the air flow under the plate can be considered lateral in the direction normal to the slots. Therefore, the damping pressure on the plate is determined by Eq. (2) and the boundary conditions in Eq. (4):

$$\left.\frac{\partial p}{\partial x}\right|_{x=a} = 0, \quad p(\pm b) = 0 \tag{4}$$

The damping pressure from the airflow under a cell is

$$p(x) = \frac{6\mu}{h^3}\dot{h}(x - b)(x + b - 2a) \tag{5}$$

The average pressure from the airflow under a cell is

$$\bar{p}_L = -\frac{4\mu a^2}{h^3}\dot{h}(1 - \beta)^3 \tag{6}$$

where $\beta$ is the ratio between the slot and cell widths so that $\beta = b/a$.

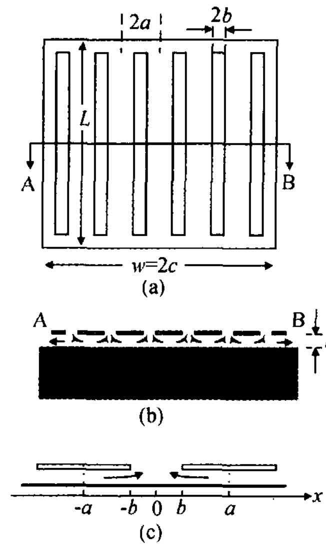


Fig. 2 Squeeze-film air damping in a thin slotted plate (a) Top view; (b) Cross-sectional view; (c) A cell with coordinates

The main problem with the above treatment is that the damping effect of the airflow through the slots, between neighboring cells, through the borders is not considered. This is an acceptable approximation for thin slotted plates. However, if the plate is thick enough, the pressure build-up from the airflow through the slot can no longer be neglected. In this case, a new analysis method for the squeeze-film air damping is needed for design considerations.

To find the differential equation for the squeeze-film air damping of a slotted plate, let us first take the case with a finite thickness as shown in Fig. 3. The vertical airflow through the slot will cause an extra resistive force on the plate.

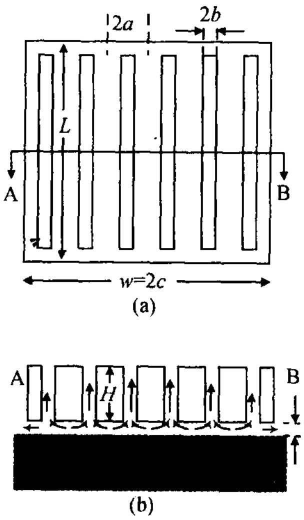


Fig. 3　Squeeze-film air damping in a thick slotted plate
(a) Top view; (b) Cross-sectional view ("cell and coordinates" see Fig. 2(c))

According to fundamental hydrodynamics, the air volume per unit time $\dot{V}_1$ passing through a slot with width $L$ and channel length $H$, and the pressure difference, $p_H$, between the two ends of the channel have the following relationship:

$$\dot{V}_1 = \frac{dV_1}{dt} = \frac{2Lb^3}{3\mu H} p_H \tag{7}$$

Under the condition of small squeeze number, the pressure variation caused by the air damping is much smaller than the original pressure, so that air is considered "incompressible". Since the reduction of volume under the cell is equal to the volume of air flowing out of the slot, we have

$$\dot{V}_1 = \frac{2Lb^3}{3\mu H} p_H = -2aL\dot{h}$$

From this equation, the pressure caused by the airflow through the slot is

$$p_H = -\frac{3\mu Ha}{b^3}\dot{h} \tag{8}$$

Therefore, the total average damping pressure in the cell is

$$p = p_H + \overline{p_L} = p_H\left(1 + \frac{4ab^3}{3h^3H}(1-\beta)^3\right) \equiv p_H\xi(\beta) \tag{9}$$

where $\xi(\beta) = \left(1 + \frac{4ab^3}{3h^3H}(1-\beta)^3\right)$.

Generally, the width of a slot is much smaller than that of the plate. Thus the pressure $p$ in Eq. (9) can be taken as the pressure at the center of the cell and further treated as a continuous function $p(x)$ of the whole plate. For the same reason, the airflow through the slot can be approximated as a uniform flow through the cell area, and the flow rate (per unit area per unit time) is

$$Q_z = \frac{\dot{V}_1}{2aL} = \frac{b^3}{3\mu aH} \times \frac{p}{\xi(\beta)} \tag{10}$$

Thus, the balance of mass flow can be expressed as (refer to Eq. (3.2.1) of Ref. [7])

$$\frac{\partial(\rho q_x)}{\partial x} + \frac{\partial(\rho q_y)}{\partial y} + \frac{b^3}{3\mu aH} \times \frac{1}{\xi(\beta)}\rho p + \frac{\partial(\rho h)}{\partial t} = 0$$

where $q_x = -\frac{h^3}{12\mu} \times \frac{\partial p}{\partial x}$ and $q_y = -\frac{h^3}{12\mu} \times \frac{\partial p}{\partial y}$. Following the derivation process in Ref. [7], the modified Reynolds equation for the squeeze-film air damping of a slotted plate is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{4b^3}{ah^3H} \times \frac{1}{\xi(\beta)} p = \frac{12\mu}{h^3}\dot{h} \tag{11}$$

Equation (11) is a generalized version of the conventional Reynolds equation. In the derivation, we assumed that the airflow in the slots is a fully developed Poiseuille flow. This generalized Reynolds equation differs from the conventional Reynolds equation by an additional term, the third term in the equation. This term represents the damping effect of vertical airflow in the slot. Thus, Equation (11) can be used generally in dealing with the air damping of slotted plates with finite thickness and finite dimensions. With appropriate boundary conditions, the pressure distribution, the damping force on the plate, and the damping force coefficient can be found by solving this modified Reynolds equation.

For a slotted plate with a finite thickness (the length of channel), the effect caused by the ends of the channel must be considered. According to Eq. (5.18) in Ref. [9], the end effect can be considered by replacing the geometric height, $H$, with an effective height, $H_{eff}$, in the equation. The expression of the effective height is

$$H_{eff} = H + \frac{16}{3\pi} b \tag{12}$$

The end effect might be significant if the thickness of the plate is small.

## 3 Air damping of long rectangular slotted plates

As a useful example, we consider a rectangular slotted plate with its length, $L$, much larger than its width, $2c$ ($L \gg 2c \gg 2a$, as shown in Fig. 3). Thus, Equation (11) becomes one-dimensional. By defining two constants $l = \sqrt{ah^3 H\xi(\beta)/4b^3}$ and $R = -\frac{12\mu}{h^3}\dot{h}$, Equation (11) can be further simplified to

$$\frac{d^2 p}{dx^2} - \frac{p}{l^2} + R = 0 \tag{13}$$

With the boundary conditions $p(\pm c) = 0$, the air damping pressure is

$$p(x) = Rl^2 \left( 1 - \cosh\left(\frac{x}{l}\right) / \cosh\left(\frac{c}{l}\right) \right) \tag{14}$$

The dependence of pressure distribution on the ratio $c/l$ is shown in Fig. 4, where the damping pressure has been normalized to $Rl^2$.

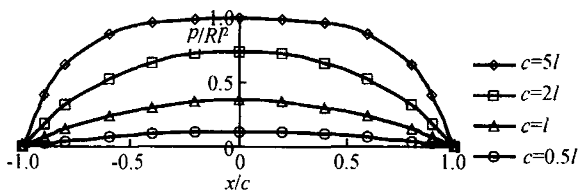


Fig. 4 Dependence of pressure distribution on $c/l$

From Eq. (14), the damping force on the long rectangular slotted plate is

$$F_d = 2cLRl^2[1 - (l/c)\tanh(c/l)] \tag{15}$$

Since $c$ is usually much larger than $l$, Equation (15) can be further simplified to

$$F_d = 2LRl^2(c - l) \tag{16}$$

and the damping force coefficient for the plate is

$$c_d = \frac{24}{h^3}\mu L l^2 (c - l) \tag{17}$$

These results imply that the damping pressure can be considered to have a uniform value of $Rl^2$ in most of the area under the plate but drops to zero at the border of the area with a width of $l$. Therefore, $l$ is sometimes referred to as the attenuation length.

If the slots are very narrow and the plate is very thick so that the attenuation length $l$ is much greater than the half-width $c$ of the plate, Equation (14) reduces to Eq. (3) for the solid plate with the same dimensions.

To another extreme, if the plate is very thin (i.e. $H \to 0$), so that the attenuation length is very small (i.e. $l \to 0$) and $\xi(\beta) \to \frac{4ab^3}{3h^3 H}(1-\beta)^3$. Equation (14) shows that the damping pressure in most of the area under the plate (except for the area very close to the borders of the plate) has a constant value

$$p = Rl^2 = -\frac{4\mu a^2}{h^3}\dot{h}(1-\beta)^3$$

This result coincides with the squeeze-film damping force of an infinite slotted plate obtained by the conventional Reynolds equation, as shown in Eq. (6).

To verify the above analysis, we compare the damping force coefficient obtained from the modified Reynolds equation with that generated by ANSYS software for a typical micro-slotted plate. The geometric parameters of the plate are $2a = 20\mu m$, $2b = 4\mu m$, $H = 20\mu m$, $h = 2\mu m$, and $L = 5mm$, and the number of slots is 19. The damping force coefficient found by the modified Reynolds equation is $c_d = 3.88 \times 10^{-3} N/(m \cdot s)$, while that found by ANSYS is $c_d = 3.99 \times 10^{-3} N/(m \cdot s)$. The difference is small for practical applications. Therefore, the modified Reynolds equation can be used to calculate the damping force coefficient for design purposes.

## 4 Conclusion

For the squeeze-film air damping of a slotted plate with a finite thickness and finite lateral dimensions, a modified Reynolds equation is developed. To the conventional Reynolds equation, an additional term accounting for the effect of air damping caused by airflow through the slots is added. The end effect of the slot for the airflow is considered by replacing the geometric thickness of the plate with an effective thickness. For the two extremes (including the conditions of finite plate thickness and infinitesimal plate thickness), the results from the modified Reynolds equation agree with the corresponding results given by the conventional Reynolds equation. This validates the modified Reynolds equation. The modified Reynolds equation will be a useful tool for the design of many MEMS devices with slotted plates.

## References


[ 1 ] Davies X, Montague S, Smith J, et al. Micromechanical structures and microelectronics for accelerometer sensing. Proc SPIE, 1997, 3223: 237

[ 2 ] Xiong B, Wang Y, Che L, et al. A model silicon micromachined gyroscope with high *Q* at atmosphere. Proc SPIE, 2001, 4601: 38

[ 3 ] Gong C, Xia S, Deng K, et al. Design and simulation of miniature vibrating electric field sensors. Proc of IEEE Sensors, Viena, Austria, 2004: 1589

[ 4 ] Zhang J, Huang S L, Yan X, et al. Optical characteristics analysis and simulation of grating moving light modulator. Chinese Mechanical Engineering, 2005, 16 (Supplement): 218 (in Chinese) [张洁, 黄尚廉, 闫许, 等. 光栅平动式光调制器的光学特性分析和仿真. 中国机械工程, 2005, 16(增刊): 218]

[ 5 ] Starr T. Squeeze-film damping in solid-state accelerometer. IEEE Workshop on Solid-State Sensor and Actuator, Hilton Head Island, SC, USA, 1990: 44

[ 6 ] Sadd M H, Stiffler A K. Squeeze film dampers: amplitude effects at low squeeze numbers. Journal of Engineering for Industry, 1975: 1366

[ 7 ] Bao M H. Analysis and design principles of MEMS devices. Elsevier, 2005

[ 8 ] Bao M H, Sun Y C, Yang H, et al. A generalized Reynolds' equation for squeeze-film air damping in MEMS. Chinese Journal of Semiconductors, 2002, 23 (12): 1245 (in Chinese) [鲍敏杭, 孙远程, 杨恒, 等. 微电子机械器件压膜空气阻尼的一般化雷诺方程. 半导体学报, 2002, 23(12): 1245]

[ 9 ] Sharipov F, Seleznev V. Data on internal rarefied gas flows. Journal of Physical and Chemical Reference Data, 1997, 27 (3): 657


# 微机械槽板结构空气压膜阻尼的修正雷诺方程 *


孙远程[1]　鲍敏杭[1,†]　杨　恒[2]　黄宜平[1]

(1 复旦大学微电子学系 专用集成电路国家重点实验室, 上海　200433)

(2 中国科学院上海微系统和信息技术研究所 传感技术国家重点实验室, 上海　200030)



**摘要**: 推导出了一个适用于槽板结构压膜空气阻尼的微分方程. 该方程与传统雷诺方程的区别是在雷诺方程的左边增加了一个用以表示气体从槽流出所引起的阻尼效应的修正项, 并考虑了槽中有限气流通道长度的端头修正. 在适当的边界条件下, 利用此方程可以求解槽板压膜阻尼的压强分布、阻尼力和阻尼力系数. 该槽板结构压膜空气阻尼的微分方程对槽板的厚度和横向尺度没有限制, 为分析有限尺寸和有限厚度槽板的压膜空气阻尼提供了一个有用的方法.



**关键词**: 压膜空气阻尼; 微电子机械系统; 槽板; 雷诺方程

**EEACC:** 2575　　**PACC:** 0320; 4630M; 6240

**中图分类号**: TN603.5　　**文献标识码**: A　　**文章编号**: 0253-4177(2006)03-0473-05

* 国家重点基础研究发展规划(批准号: G1999033101)和国家自然科学基金(批准号: 69876009)资助项目



† 通信作者. Email: mhbao@fudan.edu.cn



2005-10-28 收到, 2005-12-09 定稿



c 2006 中国电子学会