Refinement of an Analytical Approximation of the Surface Potential in MOSFETs *

Lu Jingxue[1], Huang Fengyi[1,†], Wang Zhigong[1], and Wu Wengang[2]

(1 Institute of RF & OEIC, Southeast University, Nanjing 210096, China)
(2 Institute of Microelectronics, Peking University, Beijing 100871, China)

Abstract: A refinement of an analytical approximation of the surface potential in MOSFETs is proposed by introducing a high-order term. As compared to the conventional treatment with accuracy between 1nV and 0.03mV in the cases with an oxide thickness $t_{ox} = 1 \sim 10$nm and substrate doping concentration $N_a = 10^{15} \sim 10^{18}\,cm^{-3}$, this method yields an accuracy within about 1pV in all cases. This is comparable to numerical simulations, but does not require trading off much computation efficiency. More importantly, the spikes in the error curve associated with the traditional treatment are eliminated.

Key words: MOSFET; surface potential; analytical approximation; device modeling
EEACC: 2560B; 2570D; 0290
CLC number: TN43 **Document code:** A **Article ID:** 0253-4177(2006)07-1155-04

1 Introduction

Thus far, the surface-potential-based MOSFET compact model is considered to be one of the best alternatives to the threshold-voltage based model due to its clear physical meaning in the transition region, i.e. between the sub-threshold and strong inversion regions. However, the surface potential can only be measured indirectly by *I-V* or *C-V* characteristics[1], which is not accurate enough for the compact model analysis. Therefore, one of the fundamental issues associated with surface potential based models for MOSFETs is the need for the elaborate computation of the surface potential, which is usually obtained by the iterative solution of a well-known transcendental equation[2,3]. However, the iteration procedure is very time-consuming and constitutes a key obstacle for implementing the surface-potential-based model. In order to avoid the iteration process, various analytical solutions for the surface potential have been investigated.

The accuracy of the analytical approximations in Refs. [4,5] is on the order of 2～3mV, which does not allow one to accurately reproduce derivatives of current and charge in the weak inversion region[2]. Chen *et al*. proposed a new closed-form analytical approximation for the surface potential that is computationally efficient and exhibits the best absolute error of about 1nV, but it still has an absolute error of about 0.03mV for some cases, e.g., when the oxide thickness is $t_{ox} = 1 \sim 10$nm and the substrate doping concentration is $N_a = 10^{15} \sim 10^{18}\,cm^{-3}$[2]. Therefore, the accuracy under the boundary condition ($t_{ox} = 10$nm, $N_a = 10^{18}\,cm^{-3}$) as shown in Fig. 1(b) is still insufficient to carry out derivatives for the current and charge. In addition, under the condition of $t_{ox} = 2.5$nm and $N_a = 5 \times 10^{17}\,cm^{-3}$, the absolute error produced by Ref. [2] exhibits 'spikes' in certain regions as shown in Fig. 1(a), which may produce problems during the calculation of trans-capacitance and trans-conductance in such regions.

In this paper, a refined analytical approach to calculating the surface potential is proposed for improving the accuracy without trading off much computation efficiency. With the present refined model, the spikes in the traditional approach have also been eliminated.

* Project supported by the National Natural Science Foundation of China (No. 60476012), the Fund of Southeast University (No. XJ0404142), and the National Doctoral Research Fund (No. 20040286022)
† Corresponding author. Email: fyhuang@seu.edu.cn
Received 5 December 2005, revised manuscript received 25 March 2006

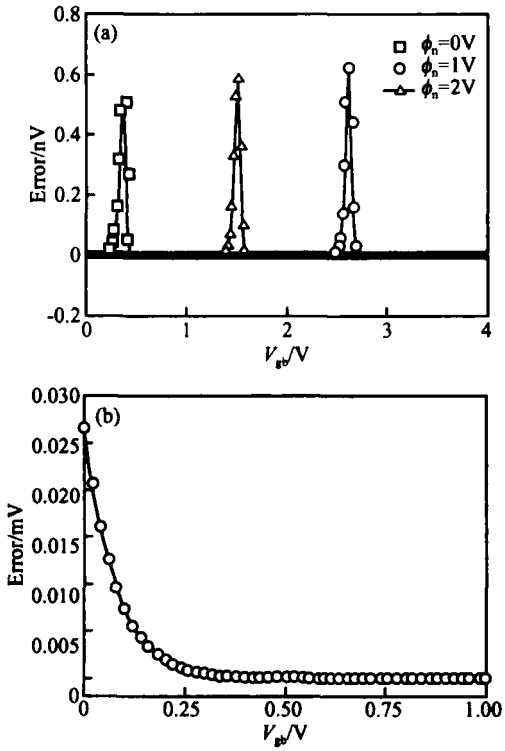

Fig. 1 Absolute error of the analytical approximation of surface potential in Ref. [2] compared to numerical solutions of Eq. (1) (a) Oxide thickness t_{ox} = 2.5nm, V_{fb} = - 1V and bulk doping concentration $N_a = 5 \times 10^{17}$ cm^{-3}, T = 300K. It is easy to see the spikes in the error curves; (b) t_{ox} = 10nm, V_{fb} = - 1V, $N_a = 10^{18} cm^{-3}$, T = 300K, and $\phi_n = 0$

2 Theoretical analysis

The MOSFET surface potential ϕ_s satisfies the implicit function of gate bias can be expressed as[3]

$$(x_g - x)^2 = G^2[(e^{-x} + x - 1) + \Delta_n(e^x - x - 1)] \quad (1)$$

where $x = \phi_s/\phi_t$, $x_g = (V_{gb} - V_{fb})/\phi_t$, $G = \gamma/\sqrt{\phi_t}$, and $\Delta_n = \exp[-(2\phi_b + \phi_n)/\phi_t]$. V_{fb} is a flat band voltage, γ denotes the body factor, ϕ_t is the thermal voltage with constant value 0.026V at T = 300K, ϕ_b is the difference between the Fermi potential and the intrinsic potential, and ϕ_n is the voltage between channel and bulk and is equal to V_{sb} at the source end of the channel and to $V_{sb} + V_{ds}$ at the drain end. It is assumed that $\phi_s \geq 3\phi_t$ in Ref. [2], hence $x - 1 \gg e^{-x}$. So that Equation (1) can be approximated by[2]

$$(x_g - x)^2 = G^2(x - 1 + \Delta_n e^x) \quad (2)$$

However, when $\phi_s = \phi_b + \phi_n/2$, the term $\Delta_n e^x = e^{-x}$, so that the term e^{-x} cannot be ignored. Hence, to further improve the accuracy of the analytical approximation of MOSFET surface potential, here we keep the term e^{-x} in our derivation, and the normalized implicit function becomes[6]

$$(x_g - x)^2 = G^2(e^{-x} + x - 1 + \Delta_n e^x) \quad (3)$$

We shall denote

$$x = x_0 + \varepsilon \quad (4)$$

where x_0 is obtained by using the analytical approximation method in Ref. [2]. Since $\varepsilon \ll 1$, $e^{\varepsilon} \approx 1 + \varepsilon$, and $e^{-\varepsilon} \approx 1 - \varepsilon$, Equation (3) can be written as

$$\varepsilon^2 - \varepsilon[2(x_g - x_0) + G^2(1 + \Delta_0 - \Delta_1)] + [(x_g - x_0)^2 - G^2(x_0 - 1 + \Delta_0 + \Delta_1)] = 0 \quad (5)$$

where

$$\Delta_0 = \exp(x_0) \times \Delta_n, \Delta_1 = \exp(-x_0)$$

Assign

$$p = 2(x_g - x_0) + G^2(1 + \Delta_0 - \Delta_1) \quad (6)$$

$$q = (x_g - x_0)^2 - G^2(x_0 - 1 + \Delta_0 + \Delta_1) \quad (7)$$

Hence, ε can be solved as

$$\varepsilon = \frac{p - \sqrt{p^2 - 4q}}{2} \quad (8)$$

Eventually, we get ϕ_s from Eq. (4)

$$\phi_s = \phi_t x \quad (9)$$

Our analytical approach only introduced one additional term (square root) in Eq. (8), which does not require significant computing time for simulation by modern computers.

3 Results and conclusion

Figure 2 shows the comparison of our new approach with the results of the numerical solutions of Eq. (1) by iterative methods with an accuracy of

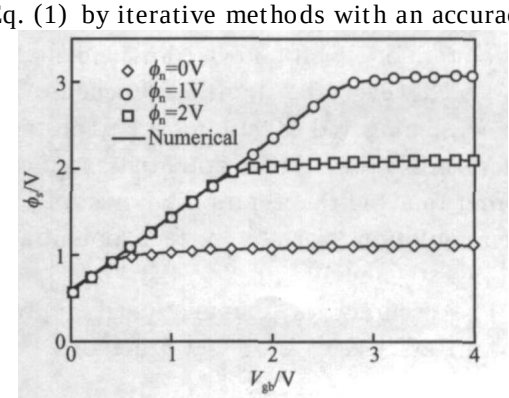

Fig. 2 Comparison of surface potential at source end by our method with numerical solutions of Eq. (1) Oxide thickness t_{ox} = 2.5nm, V_{fb} = - 1V and bulk doping concentration $N_a = 5 \times 10^{17} cm^{-3}$, T = 300K

10^{-15} V, and Figure 3 shows the absolute error of our new approximation. As shown in Fig. 2, excellent accuracy is achieved by our approach for the surface potential at the source end in both the weak and the strong inversion regions, with $\phi_n = 0, 1, 2$V respectively. Moreover, as Fig. 3(a) shows, after the refinement of our method, the 'spikes' in the error curve in the certain region are nearly eliminated, and the accuracy reaches the order of 1fV for reasonable parameter selection. That is, our refined approach can produce a surface potential comparable to the numerical solution by iterative methods with an accuracy of 10^{-15} V. In addition, even for the boundary case as demonstrated by Fig. 3(b), the absolute error is under 14pV, which is negligible compared to 0.03mV in Fig. 1(b), and sufficient for the calculation of trans-capacitance and trans-conductance in compact modeling.

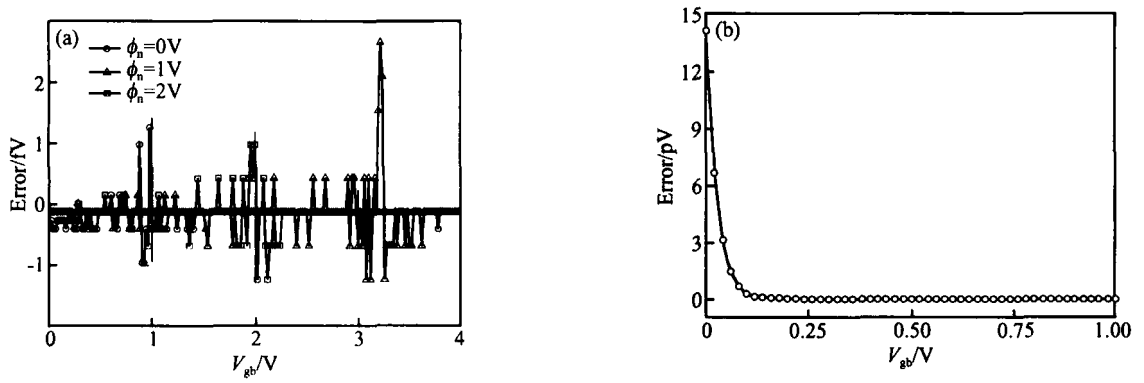

Fig. 3　Absolute error of analytical approximation of surface potential by our method compared to numerical solutions of Eq. (1)　(a) $t_{ox} = 2.5$nm, $V_{fb} = -1$V, $N_a = 5 \times 10^{17}$ cm^{-3}, T = 300 K. There are no apparent spikes in error curves; (b) $t_{ox} = 10$nm, $V_{fb} = -1$V, $N_a = 10^{18}$ cm^{-3}, T = 300 K, and $\phi_n = 0$

References

[1] Chen Zhiyun, Liu Xiaowei, Wei Jianlin, et al. New method for analysis of surface potential of MOSFET in sub-threshold zone. Chinese Journal of Semiconductors, 1997, 18(11): 844 (in Chinese) [陈之昀, 刘晓卫, 卫建林, 等. 一种新的 MOS 器件亚阈值区表面势分析方法. 半导体学报, 1997, 18(11): 844]

[2] Chen T L, Gildenblat G. Analytical approximation for the MOSFET surface potential. Solid-State Electron, 2001, 45(2): 335

[3] Pao H C, Sah C T. Effects of diffusion current on characteristics of metal-oxide (insulator)-semiconductor transistors. Solid-State Electron, 1966, 9: 927

[4] Hows R, Redman-White W, Nichols K G, et al. A charge-conserving silicon-on-sapphire SPICE MOSFET model for analog design. IEEE Int Symp Circuits and Systems, 1991, 4: 2160

[5] Van R, Langevelde F M. An explicit surface-potential-based MOSFET model for circuit simulation. Solid-State Electron, 2000, 44(3): 409

[6] Park H J, Ko P K, Hu C. A charge sheet capacitance model of short channel MOSFETs for SPICE. IEEE Trans Computer-Aided Design, 1991, 10(3): 376

MOSFET 表面势解析近似方法的改进 *

陆静学[1] 黄风义[1,†] 王志功[1] 吴文刚[2]

(1 东南大学射频与光电集成电路研究所，南京 210096)
(2 北京大学微电子学研究院，北京 100871)

摘要：通过在表面势公式中增加一高阶近似项，大大提高了传统表面势的解析近似精度. 改进前通用参数的精度一般达到 1nV 量级，某些情况下只能达到 0. 03mV. 改进后的方法在所有情况下精度都达到 1pV 量级. 同时，改进后的近似方法消除了原有方法误差曲线中的毛刺现象.

关键词：MOSFET；表面势；解析近似；器件建模
EEACC：2560B；2570D；0290
中图分类号：TN43 **文献标识码**：A **文章编号**：0253-4177(2006)07-1155-04

*国家自然科学基金(批准号:60476012)，东南大学基金(批准号:XJ0404142)及博士研究基金(批准号:20040286022)资助项目
†通信作者. Email:fyhuang@seu.edu.cn
2005-12-05 收到，2006-03-25 定稿