Double-π fully scalable model for on-chip spiral inductors* 

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Abstract: A novel double-π equivalent circuit model for on-chip spiral inductors is presented. A hierarchical structure, similar to that of MOS models is introduced. This enables a strict partition of the geometry scaling in the global model and the model equations in the local model. The major parasitic effects, including the skin effect, the proximity effect, the inductive and capacitive loss in the substrate, and the distributed effect, are analytically calculated with geometric and process parameters in the local-level. As accurate values of the layout and process parameters are difficult to obtain, a set of model parameters is introduced to correct the errors caused by using these given inaccurate layout and process parameters at the local level. Scaling rules are defined to enable the formation of models that describe the behavior of the inductors of a variety of geometric dimensions. A series of asymmetric inductors with different geometries are fabricated on a standard 0.18-μm SiGe BiCMOS process with 100 Ω/cm substrate resistivity to verify the proposed model. Excellent agreement has been obtained between the measured results and the proposed model over a wide frequency range.

Key words: spiral inductor; double-π equivalent circuit; fully scalable
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1. Introduction

As a critical passive component, integrated spiral inductors have been widely used in CMOS RFIC design such as in RF amplifiers, voltage controlled oscillators (VCOs), mixers, filters and impedance matching circuits[1–5]. Therefore, an accurate equivalent circuit based model suitable for building a scalable spiral inductor library is essential for reliable circuit implementation and design optimization. Considerable research related to the modeling of on-chip spiral inductors has been published in recent years[6–19]. These methods are generally categorized into two types: numerical and compact circuit modeling techniques. Numerical techniques generally involve electromagnetic (EM) field solvers. Hence, they are time consuming. Therefore, Spice-format compact models (e.g. equivalent circuit models) are preferred by the IC designers.

Most of the research effort into equivalent circuit models of spiral inductors in the past years has focused on different topologies (such as the T model[6], single-π model[7–12], double-π model[13–19]) for accurate prediction of the characteristics of spiral inductors over a wide frequency range, for characterization of parasitic effects, for characterization of the capacitive and inductive coupling of the substrate, and on the model parameter extraction methods. Scalable models with scaling rules that be used to describe the behavior of spiral inductors over a whole geometric range are rarely presented.

In general, a scalable modeling procedure for inductors manufactured in a specified manufacture process is as follows. Firstly parameters are extracted for devices of various dimensions and secondly, a function is fitted to each parameter variation with the geometry. For scalable fitting, a unique parameter extraction solution and physically meaningful scaling expressions for each parameter are of the utmost importance to ensure the accuracy of the extracted scalable model. Among the reported parameter extraction approaches are the numerical optimization method, the analytic[11,12,16–18] and physical based[10,13,19] model parameter extraction techniques. The numerical optimization method is difficult to render scalable because the optimized parameters are not always unique. The availability of an analytic parameter extraction technique is strongly dependent on the complexity of the equivalent circuit used, e.g. whether every parameter can be directly determined from measurements or not. The errors introduced by the assumptions used to simplify the parameter extraction at high frequencies tend to hinder the scalability of the models. On the contrary, physically-based model parameter extraction methods are expected to be unique, because of the strictly defined calculation method based on the layout and process parameters.

Double-π type models have been widely employed in building model libraries to achieve wideband accuracy. Two different scalable models for inductors are developed in Refs. [13, 20]. In Ref. [13], a set of complicated functions are utilized to calculate $C_{ox}$, $C_{si}$ and $C_{p}$, making the parameter-extraction procedure and scalable rules intricate. The analytic parameter extraction methodology is developed in Ref. [20]. In this way, rules are arbitrary, multi-value phenomenon for parameters is more serious, and the errors of scaling rules for non-sampling geometries are hard to control. In this paper, a physics-based scalable modeling method for on-chip spiral inductors is proposed. The data included in the study cover the range from 0.1 to 22 nH, a total of 90 spiral inductors. The proposed scalable rules well reflect local level scaling of the
various component parameters. The proposed scaling rules and modeling method has strong practicality and reference, which has not been reported in previous works. The major parasitic effects, including the skin effect, the proximity effect, the vertical and lateral high frequency losses in the substrate and the distributed effect are analytically calculated with layout and process parameters. By using the complex effective thickness of the substrate that the eddy current is flowing through, novel equations for the high frequency lateral substrate losses are proposed. The overall double-$\pi$ equivalent circuit with scalable rules is firstly given in Section II. As the scaling rules for the model components are physics based, compare with the experimentally scaled double-$\pi$ model we presented early\textsuperscript{20}, the model parameter extraction becomes more clear and easier than the old model. Results from the presented model and measurements of spiral inductors fabricated on a standard 0.18 $\mu$m SiGe BiCMOS process are compared in Section 3. Finally, conclusions are drawn in Section 4.

2. Physics-based double-$\pi$ equivalent circuit model and model parameter extraction method

2.1. Model structure

The proposed spiral inductor model has a hierarchical structure, which is similar to that of standard transistor models such as BSIM3v3.2 and PSP. A strict separation of the geometry scaling in the global model and the model equations in the local model is introduced. Consequently, the model can be used at either one of the two levels. The described model structure is schematically depicted in Fig. 1.

2.2. Model topology

The topology of the proposed double-$\pi$ equivalent circuit model is shown in Fig. 3. In the circuit, $L_{si}$ and $R_{si} (i = 1, 2)$ are the DC inductance and resistance, respectively. The $L_{sij}$ – $R_{sij} (i = 1, 2; j = 1, 2, 3)$ ladders with the mutual inductors $M_{12i}, M_{13i}$, and $M_{23i}, (i = 1, 2)$ are used to capture the skin and proximity effects. $C_p$ is the forward capacitance, which includes the overlap capacitance $C_{mu}$ and the coupling capacitance $C_{mm}$ between the neighboring turns. $C_{oij} (i = 1, 2; j = 1, 2)$ represents the oxide-capacitance between the inductor and the substrate\textsuperscript{[7]}. $R_{subij}$ and $C_{subij} (i = 1, 2; j = 1, 2)$ are the vertical substrate resistance and capacitance of the substrate, respectively. $R_{lossj}$ and $L_{lossj} (j = 1, 2)$ are introduced to represent the lateral resistive and inductive losses caused by the eddy current in the substrate.

2.3. Model parameters

As accurate values of the layout and process parameters are difficult to obtain, a set of model parameters is introduced to correct the errors caused by using these given inaccurate layout and process parameters at the local level. For one specific instance of an inductor, a local parameter set is internally generated using the relevant geometric (as given in Table 1) and process parameters (as given in Table 2). The local parameter set gives a complete and accurate description of the electrical properties of a specific device with a particular geometry.

Since most of these local parameters scale with geometry,
Table 2. Process parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>Default</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{TP}$ ($\Omega/\square$)</td>
<td>0.015</td>
<td>Sheet resistance of spiral</td>
</tr>
<tr>
<td>$T_{TP}$ ($\mu$m)</td>
<td>2.9</td>
<td>Thickness of spiral</td>
</tr>
<tr>
<td>$R_{UD}$ ($\Omega/\square$)</td>
<td>0.028</td>
<td>Sheet resistance of underpass</td>
</tr>
<tr>
<td>$T_{UD}$ ($\mu$m)</td>
<td>0.85</td>
<td>Thickness of underpass</td>
</tr>
<tr>
<td>$C_{MU}$ ($\mu$F/m$^2$)</td>
<td>34.2</td>
<td>Spiral to underpass capacitance</td>
</tr>
<tr>
<td>$C_{MUF}$ (pF/m)</td>
<td>21</td>
<td>Spiral to underpass fringing capacitance</td>
</tr>
<tr>
<td>$C_{MM}$ (pF/m)</td>
<td>89.8</td>
<td>Spiral turn to turn capacitance</td>
</tr>
<tr>
<td>$C_{MUS}$ ($\mu$F/m$^2$)</td>
<td>4.307</td>
<td>Spiral to substrate capacitance</td>
</tr>
<tr>
<td>$C_{MS}$ ($\mu$F/m$^2$)</td>
<td>5.357</td>
<td>Underpass to substrate capacitance</td>
</tr>
<tr>
<td>$C_{MSF}$ (pF/m)</td>
<td>6.25</td>
<td>Underpass to substrate fringing capacitance</td>
</tr>
<tr>
<td>$C_{MSSF}$ (pF/m)</td>
<td>3.91</td>
<td>Metal windings to substrate fringing capacitance for embedded lines</td>
</tr>
<tr>
<td>$T_{SUB}$ ($\mu$m)</td>
<td>700</td>
<td>Substrate thickness</td>
</tr>
<tr>
<td>$R_{SUB}$ (ohm-cm)</td>
<td>100</td>
<td>Substrate resistivity</td>
</tr>
<tr>
<td>$T_{OX}$ ($\mu$m)</td>
<td>6.664</td>
<td>Oxide layer thickness</td>
</tr>
</tbody>
</table>

all inductors of a particular process can be described by a set of parameters, called the global parameter set. A set of scaling rules relates the local and global parameter set. By applying the set of scaling rules, a local parameter set can be obtained from a global parameter set. An overview of the local and global parameters in the model is given in the first and second column of Table 3.

2.4. Model equations for local level

As seen from Fig. 3, there are more than 22 elements in each single-$\pi$ topology, giving a total of 45 elements for the proposed double-$\pi$ equivalent model. To the best of the authors’ knowledge, it is hard to extract such a model by analytic parameter extraction techniques or by the method proposed in Ref. [16], which employs simple single-$\pi$ model parameters to determine a double-$\pi$ model. Physics-based equations are carefully investigated and employed for the local model determination in this section. The set of local parameters are employed in the equations for correction of the errors arising from inaccurate process and layout parameters.

The model equations for all of the elements depicted in Fig. 3 are given as follows.

2.4.1. Skin and proximity effects model: $L_{spij}, R_{spij}$ and mutual inductances, $M_{i12}, M_{i13}$, and $M_{i23}$, ($i = 1, 2, j = 1, 2, 3$)

With an increase in the frequency, the skin effect causes current crowding towards the surface of the conductor. The current density decreases from the surface to the center of the conductor. If the cross-section of the conductor is partitioned into many smaller subsections (in this work, three subsections are used), the current distribution in each subsection can be taken as uniform[21]. A simplified partitioning and modeling method as seen in Fig. 4 is used to accurately capture the skin and proximity effects. The thickness and width for the partitioned three subsections of a metal line with a thickness $T_h$, width $W_i$ and length $l_e$ are defined as $h_1s, h_2s$ and $h_3s$, and $w_1s$, $w_2s$ and $w_3s$, respectively.

At the frequency $f_{\text{max}}$ (the highest frequency used in this work is 20 GHz), the skin depth of metal winding line, $\delta_{\text{max}}$ can be calculated as

$$\delta_{\text{max}} = \sqrt{\frac{1}{\pi f_{\text{max}} \mu \sigma_m}},$$

where $\mu$ and $\sigma_m$ are the permeability and conductivity of metal line, respectively. $\sigma_m$ is defined as

$$\sigma_m = (R_{\text{film}} T_h)^{-1}.$$

where $R_{\text{film}}$ is the metal sheet resistivity.

In this work, experimental relationships between $h_{js}$ ($j = 1, 2, 3$) and $\delta_{\text{max}}$ defined as given in Eqs. (3)–(5) are used to determine the thicknesses ($h_{1s}$, $h_{2s}$ and $h_{3s}$) of the three subsections, respectively.
Table 3. Model parameter set and the geometry scaling rules. A total of 17 local parameters and 68 global model parameters are used for asymmetric on-chip spiral inductors. Extracted results of the global parameter set for devices listed in Table 4 are given.

<table>
<thead>
<tr>
<th>Local parameter set</th>
<th>Global parameter set</th>
<th>Geometry scaling for local parameter set</th>
<th>Extracted results of the global parameter set for devices listed in Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{spcorr}$</td>
<td>$l_{sp1}$, $l_{sp2}$, $l_{sp3}$, $l_{sp4}$</td>
<td>$k_{spcorr} = k_{sp1}h_{lsp}^{0.2}N_{lop}D_{lop}^{0.4}$</td>
<td>0.7945, −0.9205, 0.6433, 0.3471</td>
</tr>
<tr>
<td>$r_{spcorr}$</td>
<td>$r_{sp1}$, $r_{sp2}$, $r_{sp3}$, $r_{sp4}$</td>
<td>$r_{spcorr} = r_{sp1}h_{lsp}^{0.2}N_{rop}D_{rop}^{0.4}$</td>
<td>0.0218, −1.368, −0.1977, 0.0967</td>
</tr>
<tr>
<td>$c_{pcorr}$</td>
<td>$c_{p1}$, $c_{p2}$, $c_{p3}$, $c_{p4}$</td>
<td>$c_{pcorr} = c_{p1}h_{lsp}^{0.4}N_{csp}D_{osp}^{0.6}$</td>
<td>1.438, −0.9049, −1.299, 0.1428</td>
</tr>
<tr>
<td>$c_{concorr}$</td>
<td>$C_{ox1}$, $C_{ox2}$, $C_{ox3}$, $C_{ox4}$, $C_{ox5}$</td>
<td>$C_{concorr} = C_{ox1}h_{lsp}^{N_{con}D_{con}^{N_{con}D_{con}}}W_{s_{con}}^{N_{con}}W_{s_{con}}^{N_{con}}$</td>
<td>0.005237, −0.5084, −0.3639, 0.215, −0.562</td>
</tr>
<tr>
<td>$l_{concorr}$</td>
<td>$C_{ox1}h_{lsp}^{N_{con}D_{con}^{N_{con}D_{con}}}W_{s_{con}}^{N_{con}}W_{s_{con}}^{N_{con}}$</td>
<td>$l_{concorr} = l_{con12}h_{lsp}^{N_{con}D_{con}^{N_{con}D_{con}}}N_{con12}D_{con12}^{N_{con12}D_{con12}}W_{s_{con12}}^{N_{con12}}W_{s_{con12}}^{N_{con12}}$</td>
<td>0.2437, −0.2584, −0.7202, 0.05421, −0.05775</td>
</tr>
</tbody>
</table>

By applying the DC resistance and self-inductance calculation method\(^{[22]}\) to rectangular conductors, the six equivalent circuit model parameters as shown in the upper right corner of Fig. 4, $R_{1s}$, $R_{2s}$, $R_{3s}$, $L_{1s}$, $L_{2s}$, and $L_{3s}$ can be determined as follows:

$$R_{js} = \frac{1}{\sigma_{m} \frac{I_{e}}{h_{js}w_{js}}}, \quad j = 1, 2, 3. \quad (9)$$

$$L_{js} = 2I_{e} \left( \ln \left( \frac{2I_{e}}{w_{js} + h_{js}} \right) + \frac{w_{js} + h_{js}}{3I_{e}} \right) \cdot 10^{-7}. \quad (10)$$

The equivalent radius of the three subsections is given by

$$r_{d1s} = 0.2235 \left( W_{i} + T_{h} \right) / 0.7788, \quad (11)$$

$$r_{d2s} = 0.2235 \left( W_{i} + T_{h} - 4h_{1s} \right) / 0.7788, \quad (12)$$

$$r_{d3s} = 0.2235 \left[ W_{i} + T_{h} - 4 \left( h_{1s} + h_{2s} \right) \right] / 0.7788. \quad (13)$$

The geometric mean distance $g_{mds}$ between the three parts are given by

$$g_{md_{1s}} = \exp \left[ \frac{r_{d1s} \cdot \ln (r_{d1s}) - r_{d2s} \cdot \ln (r_{d2s})}{r_{d1s} - r_{d2s}} - 0.5 \right]. \quad (14)$$

$$g_{md_{3s}} = \exp \left[ \frac{r_{d3s}}{r_{d2s}} \cdot \ln (r_{d3s}) - r_{d2s} \cdot \ln (r_{d2s})}{r_{d2s} - r_{d3s}} - 0.5 \right]. \quad (15)$$

![Fig. 4. Schematic of the skin effect modeling method.](image-url)
The mutual inductance between the three parts is,

$$M_{12s} = 2l_{e} \left[ \ln \left( \frac{l_{e}}{gmd_{12s}} + \sqrt{1 + \frac{l_{e}^2}{gmd_{12s}^2}} \right) - \sqrt{1 + \frac{gmd_{12s}^2}{l_{e}^2} + \frac{gmd_{12s}}{l_{e}}} \right] \cdot 10^{-7}, \quad (17)$$

$$M_{13s} = 2l_{e} \left[ \ln \left( \frac{l_{e}}{gmd_{13s}} + \sqrt{1 + \frac{l_{e}^2}{gmd_{13s}^2}} \right) - \sqrt{1 + \frac{gmd_{13s}^2}{l_{e}^2} + \frac{gmd_{13s}}{l_{e}}} \right] \cdot 10^{-7}, \quad (18)$$

$$M_{23s} = 2l_{e} \left[ \ln \left( \frac{l_{e}}{gmd_{23s}} + \sqrt{1 + \frac{l_{e}^2}{gmd_{23s}^2}} \right) - \sqrt{1 + \frac{gmd_{23s}^2}{l_{e}^2} + \frac{gmd_{23s}}{l_{e}}} \right] \cdot 10^{-7}. \quad (19)$$

The proximity effect may be considered simultaneously by introducing the mutual inductances, $M_{12s}$, $M_{13s}$, and $M_{23s}$ between the three different inductances $L_{spij}$ ($i = 1, 2; j = 1, 2, 3$). These are calculated with mutual coefficients $K_{1i2}$. $K_{1i3}$, and $K_{123}$, ($i = 1, 2$) which are obtained by empirical method.

$$K_{1i2} = \min \left[ \frac{M_{1i2}}{\sqrt{L_{sp12}L_{sp1}}}, \quad i = 1, 2, \quad (26) \right]$$

$$K_{1i3} = \min \left[ \frac{M_{1i3}}{\sqrt{L_{sp13}L_{sp1}}}, \quad i = 1, 2, \quad (27) \right]$$

$$K_{23i} = \min \left[ \frac{M_{23i}}{\sqrt{L_{sp13}L_{sp12}}}, \quad i = 1, 2. \quad (28) \right]$$

2.4.2. DC inductance and resistance: $L_{dc}$ and $R_{dc}$ ($i = 1, 2$)

The DC resistance $R_{dc}$ of a spiral inductor can be calculated as

$$R_{dc} = R_{UD} \frac{l_{sp}}{W_{U}} + R_{TP} \frac{l_{total}}{W_{S}}. \quad (29)$$

The distributed DC inductance $L_{dc}$ of a spiral inductor can be calculated using Ref. [22] as follows,

$$L_{dc} = \beta D_{out}^{a_{1}} W_{S}^{a_{2}} D_{out}^{a_{5}} S^{a_{4}}. \quad (30)$$

where $a_{i}$ ($i = 1, 2, 3, 4, 5$) and $\beta$ are layout dependent coefficients. For octagonal inductors, $a_{i}$ ($i = 1, 2, 3, 4, 5$) and $\beta$ are $-1.21, -0.163, 2.43, 1.75, -0.149$ and $1.33 \times 10^{-3}$, respectively. The average diameter $D_{avg}$ is defined as

$$D_{avg} = \frac{1}{2} (D_{in} + D_{out}). \quad (31)$$

As the partition method used in Eqs. (3)–(5) for the skin and proximity effects modeling is experiential, this method causes differences between the total resistance of the inductances $R_{spij}, (i = 1, 2; j = 1, 2, 3)$ and the DC resistance calculated by Eq. (29) and between the total inductance of the inductances $L_{spij}, (i = 1, 2; j = 1, 2, 3)$ and the DC inductance calculated by Eq. (30) directly. $L_{dc}$ and $R_{dc}$ are introduced to account for this. The following gives the calculation method of $L_{sd}$ and $R_{sd}$.

$$L_{sd} = \frac{1}{2} l_{2} d_{corr} M_{L} L_{dc}, \quad i = 1, 2, \quad (32)$$

$$R_{sd} = \frac{1}{2} t_{corr} M_{R} R_{dc}, \quad i = 1, 2, \quad (33)$$

where

$$M_{R} = 1 - 2 \frac{R_{sp11} R_{sp12} R_{sp13}}{R_{dc}} \quad (34)$$

and

$$M_{L} = 1 - 2 \frac{L_{sp11} L_{sp12} L_{sp13}}{L_{dc}} \quad (35)$$
2.4.3. Overlap and coupling capacitance: $C_p = C_{mn} + C_{mu}$

$C_p$ represents the forward capacitance, which includes the overlap capacitance $C_{mu}$ and the coupling capacitance $C_{mn}$ between the neighboring segments. A simplified schematic overview of the capacitive parasitics in an asymmetric spiral inductor is illustrated in Fig. 5. Considering the parallel-plate capacitance and the fringing capacitance from spiral turn to underpass, the overlap capacitance including the fringing effect can be calculated as follows,

$$C_{mu} = c_{pcorr} \left[ W_{US} \left( C_{MU} W_s + 2 C_{MUF} \right) \right]$$  \hspace{1cm} (36)

The coupling capacitance between the neighboring turns is calculated as follows,

$$C_{mn} = c_{pcorr} C_{MM} l_{sp}$$  \hspace{1cm} (37)

where $c_{pcorr}$ is a local model parameter, and $l_{sp}$ is the length of turn spacing.

2.4.4. Metal–oxide capacitance: $C_{oxij}$

The metal–oxide capacitances $C_{ox11}$, $C_{ox12}$, $C_{ox21}$ and $C_{ox22}$ are defined as follows,

$$C_{ox1j} = c_{oxcorr1j} \left[ \frac{1}{2} (C_{map} + C_{msf}) + C_{usp} + C_{usf} \right], \hspace{1cm} j = 1, 2$$  \hspace{1cm} (38)

$$C_{ox2j} = \frac{1}{2} c_{oxcorr2j} (C_{map} + C_{msf}), \hspace{1cm} j = 1, 2$$  \hspace{1cm} (39)

where $C_{map}$ and $C_{msf}$ represent the parallel-plate capacitance and the fringing capacitance from the top pass to substrate, respectively. $C_{map}$ and $C_{msf}$ are the parallel-plate capacitance and the fringing capacitance from the underpass to the substrate, respectively. $C_{map}$, $C_{msf}$, $C_{map}$ and $C_{msf}$ are defined as follows,

$$C_{map} = C_{MS} l_{total} W_s$$  \hspace{1cm} (40)

$$C_{usp} = C_{US} l_{usp} W_s$$  \hspace{1cm} (41)

$$C_{msf} = 2 C_{MMSF} I_m + 4 C_{MSF} I_{ud} + (C_{MSF} - C_{MMSF}) (l_{inner} + l_{outer})$$  \hspace{1cm} (42)

$$C_{usf} = 2 l_{up} C_{USF}$$  \hspace{1cm} (43)

where $l_{inner}$ and $l_{outer}$ are the length of the inner and outer spiral turn, respectively. $c_{oxcorr1j}$ ($i = 1, 2; j = 1, 2$) are local model parameters. $c_{oxcorr12} = c_{oxcorr21}$ is used in this work.

2.4.5. The vertical substrate loss model: $R_{subij}$ and $C_{subij}$

The displacement current loss that resulted from the capacitive coupling effect of the silicon substrate is highly dependent on the process parameters and operating frequency. For multi-turn inductors, due to the fringing electric field of the neighboring metal segments, the electric field of the embedded turns is much smaller than that of the outer turn, inner turn and the feed lines. Consequently, the effective thicknesses of the substrate relative to the embedded turns and the inner turn/outer turn/feed lines are different. The thickness of the substrate relative to the embedded turns is much smaller than the original value $T_{SUB}$. A substrate effect factor $\gamma_{sub}$ is defined here to take this effect into account. By using the modeling method for substrate capacitive and resistive coupling effects proposed in Refs. [24, 25], the substrate capacitance and conductance of the embedded turns per unit-length can be calculated by

$$C_{sub, e} = \varepsilon_0 \varepsilon_{eff, e} \left[ \frac{1}{2 \pi} \ln \left[ \frac{8 h_{r, e} + 1}{4 h_{r, e}} \right] \right]$$  \hspace{1cm} (44)

$$G_{sub, e} = \frac{\pi \sigma_{sub}}{\ln \left[ \frac{8 h_{r, e} + 1}{4 h_{r, e}} \right]} \left[ 1 + (1 + 10 h_{r, e})^{-1/2} \right]$$  \hspace{1cm} (45)

where $\sigma_{sub}$ is the conductivity of substrate, $h_{r, e}$ is the ratio of the effective thickness of the substrate relative to the embedded turns to the width of spiral turn. $\varepsilon_{sub}$ and $\varepsilon_{eff, e}$ are the dielectric constant and the effective dielectric constant of the substrate, respectively. $\sigma_{sub}$, $h_{r, e}$ and $\varepsilon_{eff, e}$ are defined as follows,

$$\sigma_{sub} = 1 / R_{SUB}$$  \hspace{1cm} (46)

$$h_{r, e} = \gamma_{sub} T_{SUB} / W_s$$  \hspace{1cm} (47)

$$\varepsilon_{eff, e} = \frac{1}{2} \left[ (\varepsilon_{sub} + 1) + \frac{\varepsilon_{sub} - 1}{\sqrt{1 + 10 h_{r, e}}} \right]$$  \hspace{1cm} (48)

The substrate capacitance and the conductance of the inner turn, the outer turn and the feed lines per unit-length can be calculated as follows,

$$C_{sub, o} = \varepsilon_0 \varepsilon_{eff, o} \left[ \frac{1}{2 \pi} \ln \left[ \frac{8 h_{r, o} + 1}{4 h_{r, o}} \right] \right]$$  \hspace{1cm} (49)

$$G_{sub, o} = \frac{\pi \sigma_{sub}}{\ln \left[ \frac{8 h_{r, o} + 1}{4 h_{r, o}} \right]} \left[ 1 + (1 + 10 h_{r, o})^{-1/2} \right]$$  \hspace{1cm} (50)

where $h_{r, o}$ is the ratio of the thickness of the substrate to the width of spiral turn, $\varepsilon_{eff, o}$ is the dielectric constant of substrate respectively. $h_{r, o}$ and $\varepsilon_{eff, o}$ are defined as

$$h_{r, o} = T_{SUB} / W_s$$  \hspace{1cm} (51)

$$\varepsilon_{eff, o} = \frac{1}{2} \left[ (\varepsilon_{sub} + 1) + \frac{\varepsilon_{sub} - 1}{\sqrt{1 + 10 h_{r, o}}} \right]$$  \hspace{1cm} (52)

The vertical substrate capacitance $C_{subij}$ and resistance $R_{subij}$, ($i = 1, 2; j = 1, 2$) are calculated by
Fig. 6. Local/Global model extraction flow.

\[ C_{\text{sub}ij} = \frac{c_{\text{subcorr}ij}}{2} \left[ l_m (1 - \text{ratio}) C_{\text{sub},c} + (l_m \text{ratio} + 2l_f) \times C_{\text{sub},\rho} \right], \quad i = 1, 2; j = 1, 2. \]

\[ R_{\text{sub}ij} = 2r_{\text{subcorr}ij} \left[ l_m (1 - \text{ratio}) G_{\text{sub},c} + (l_m \text{ratio} + 2l_f) \times G_{\text{sub},\rho} \right]^{-1} \times \left[ G_{\text{sub},\rho} \right]^{-1}, \quad i = 1, 2; j = 1, 2. \]

where \( l_{\text{ratio}} \) is defined as \( l_{\text{ratio}} = (l_{\text{inner}} + l_{\text{outer}}/l_m) \), \( r_{\text{subcorr}ij} \) and \( c_{\text{subcorr}ij} \) (\( i = 1, 2; j = 1, 2 \)) are local model parameters. For single or half turn spiral inductors, the calculation of Eqs. (51)–(54) can be omitted.

**2.4.6. The lateral substrate loss model: \( R_{\text{loss}i} \) and \( L_{\text{loss}i} \)**

As the eddy current is generally flowing in the lateral direction in the substrate, the eddy current effect can be taken as the major mechanism of the lateral substrate losses for spiral inductors manufactured in a specified process. For compact modeling, the concept of a complex effective thickness\(^{[18]} \) of the substrate is introduced to determine the lateral resistances and inductances, \( R_{\text{loss}i} \) and \( L_{\text{loss}i} \) in this work.

The complex effective thickness of the substrate in which the eddy current is flowing can be defined as,

\[ h_{\text{eff}} = T_{\text{OX}} + \frac{\delta}{2} (1 - j) \coth \left[ \frac{T_{\text{SUB}}}{\delta} (1 + j) \right]. \]

where \( T_{\text{OX}} \) and \( T_{\text{SUB}} \) are the thicknesses of oxide layer and the silicon substrate, respectively. \( \delta \) is defined as the skin depth of substrate at \( f_{\text{max}} \), which is calculated by

\[ \delta = \frac{1}{\sqrt{\pi f_{\text{max}} \mu_{\text{si}} \sigma_{\text{si}}}}. \]

where \( \mu_{\text{si}} \) and \( \sigma_{\text{si}} \) are the permeability and conductivity of the silicon substrate, respectively.

According to the method introduced in Ref. [26], the per-unit-length series impedance of the substrate with \( \delta \) is calculated as follows,

\[ L^* (\omega) = \frac{\mu_0}{4\pi} \ln \left[ 1 + 32 \left( \frac{h_{\text{eff}}}{W_S} \right)^2 \left[ 1 + \sqrt{1 + \left( \frac{\pi W_S}{h_{\text{eff}}} \right)^2} \right]^2 \right]. \]

By using the real and imaginary part of Eq. (57), \( R_{\text{loss}i} \) and \( L_{\text{loss}i} \) are determined by

\[ R_{\text{loss}i} = -l_{\text{loss}i} l_{\text{total}} \frac{\omega}{2} \text{Im} L^* (\omega), \quad i = 1, 2. \]

\[ L_{\text{loss}i} = \frac{1}{2} r_{\text{loss}i} l_{\text{loss}i} \text{Re} L^* (\omega), \quad i = 1, 2. \]

where \( l_{\text{loss}i} \) and \( r_{\text{loss}i} \) (\( i = 1, 2 \)) are local model parameters. For compact modeling, the results of \( L^* (\omega) \) at \( \omega = 2\pi f_{\text{max}} \) is used in this work.

In order to see the effect of the geometry on the strength of the eddy current, mutual inductances, \( M_{\text{ij}} (i = 1, 2; j = 1, 2, 3) \) calculated with the mutual coefficients \( K_{\text{ij}} (i = 1, 2; j = 1, 2, 3) \), between \( L_{\text{loss}i} \) and \( L_{\text{spij}} (i = 1, 2; j = 1, 2, 3) \), \( K_{\text{ij}} (i = 1, 2; j = 1, 2, 3) \) is experientially defined as

\[ K_{\text{ij}} = \min \left[ 0.99, \frac{L_{\text{spij}}}{\sqrt{L_{\text{spij}} L_{\text{loss}i}}} \right], \quad i = 1, 2; j = 1, 2, 3. \]
2.5. Geometry based scaling rules and the global model

The local parameter set can be viewed as a correction of the model determined from the local model equations and the employed process and geometric parameters. The global parameters account for geometric scaling. The complete geometry scaling rules developed for the local parameter set is listed in Table 3. All the local model parameters are considered as a function of the $N$, $D_{in}$, and $h_p$. $h_p$ is the hollowness of an inductor with a specified geometry, which is defined as

$$h_p = \frac{D_{in} + W_S}{D_{out} - W_S}.$$  (61)

3. Model extraction and verification

A simplified global/local model extraction flowchart is given in Fig. 6. Once the process and geometry parameters required in Eqs. (1)–(47) are given (the required process parameters are generally obtained from the foundry), the local parameter set given in Table 3 can be extracted using a simple optimization procedure. The local model is then determined. For global parameter set determination, a set of measurements from inductors with different geometric parameters are required to accurately determine the global parameters given in Table 3 with an optimization procedure. Here, we choose the random optimization package and employ a HSPICE simulator as the evaluation tool for the optimization.

In order to verify the accuracy of the proposed scalable model, a set of asymmetric octagonal spiral inductors with different geometric parameters fabricated on a standard 0.18-μm SiGe BiCMOS technology are modeled using the global model proposed in this paper. The geometric parameters of these inductors are outlined in Table 4. Two-port $S$-parameters were measured and de-embedded (Open + Short) for parasites introduced by the GSG PAD using an Agilent E8363B Network Analyzer and a CASCADE Summit probe station. The feed lines connected at the right and left side of the inductors are finally de-embedded from the test structures for model extraction.

The RMS errors of the $S$-parameters, quality factor ($Q$) and inductance ($L$) between the measured and simulated results from the extracted global model are given in Table 5. The RMS error is defined as

$$RMS_{error} = 100 \times \sqrt{\frac{1}{n} \sum_{1}^{n} \frac{(X_{\text{mea}} - X_{\text{sim}})^2}{n}}.$$  (62)

where $n$ is the total number of data points. The RMS error cal-

![Fig. 7. Comparison between measured and double-π scalable model for asymmetric inductors with $W = 8 \mu m$, $OD = 200 \mu m$, $S = 2 \mu m$. (a) Quality factor. (b) Inductance. (c) Real parts of $S_{11}$. (d) Imagery parts of $S_{11}$. (e) Real parts of $S_{12}$. (f) Imagery parts of $S_{12}$.](image-url)

Table 5. RMS errors between the global model simulated and measured inductance ($L$), quality factor ($Q$) and $S$-parameter of the devices listed in Table 4.

<table>
<thead>
<tr>
<th>DUT</th>
<th>$L$ (%)</th>
<th>$Q$ (%)</th>
<th>$S_{11}$ (%)</th>
<th>$S_{12}$ (%)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>real</td>
<td>imag</td>
</tr>
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<td>1.475</td>
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<tr>
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<td>1.892</td>
<td>1.983</td>
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<tr>
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<tr>
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<tr>
<td>D32</td>
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<tr>
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<tr>
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<tr>
<td>D43</td>
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<tr>
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<tr>
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<td>5.636</td>
<td>4.400</td>
<td>3.743</td>
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</table>
Fig. 8. Comparison between measured and double-π scalable model for asymmetric inductors with $W = 10 \, \mu m$, $OD = 280 \, \mu m$, $S = 2 \, \mu m$. (a) Quality factor. (b) Inductance. (c) Real parts of $S_{11}$. (d) Imagery parts of $S_{11}$. (e) Real parts of $S_{12}$. (f) Imagery parts of $S_{12}$.

Fig. 9. Comparison between measured and double-π scalable model for asymmetric inductors with $N = 6.5$, $OD = 360 \, \mu m$, $S = 2 \, \mu m$. (a) Quality factor. (b) Inductance. (c) Real parts of $S_{11}$. (d) Imagery parts of $S_{11}$. (e) Real parts of $S_{12}$. (f) Imagery parts of $S_{12}$.

culation is executed over the frequency range from 50 MHz to the self-resonant frequency (SRF) of devices.

The measured and simulated $Q$, $L$, real($S_{11}$), real($S_{12}$), imag($S_{11}$) and imag($S_{12}$) characteristics of the asymmetric on-chip inductors, with $W$ fixed at 8 $\mu m$, 10 $\mu m$ and 15 $\mu m$, $S$ fixed at 2 $\mu m$ while changing $ID$ and $N$, are shown in Figs. 7–9, respectively. The RMS errors of $L$ between the measured and simulated data for the inductors are below 5%. The average RMS error of $L$ is 2.291%. For most of these devices, the RMS errors of $Q$ are below 5%, and the average error of $Q$ is 3.511%. The average RMS errors of the real and imagery parts of $S_{11}$ and $S_{12}$ are 2.339%, 2.387%, 3.429% and 2.516%, respectively. The excellent agreement between the measured and simulated results verified and validated the accuracy of the proposed on-chip spiral inductor modeling technique. The accuracy of the proposed scalable rules is verified by the excellent agreement between the extracted data and scalable data of inductors with typical geometries, as shown in Fig. 10. In this picture, the extracted data of local parameters can be obtained though a optimization procedure as follows: (1) set optimization target, usually we set target to be quality factor $Q$, inductance $L$ and the parasitic resistance $R$; (2) select the range of...
every local parameter, the optimization algorithm and times, then start optimization. In the optimization process, we need adjust the optimization values of parameters and their range according to the geometry rules of inductors, ultimately get the best value.

4. Conclusion

An industry-oriented fully scalable compact circuit model for on-chip spiral inductors has been proposed and implemented in HSPICE. This article mainly concerns the comprehensive application of the reported literature, and obtains results to meet industrial applications. This work has not been reported in previous papers. It is very useful for circuit designers to see the scaling behavior of a model library under a defined process. The model is developed with a hierarchical structure, in which a strict partition of the geometry scaling in the global model and the model equations in the local model is defined. The major parasitic effects, including the skin effect, the proximity effect, the inductive and capacitive loss in the substrate and the distributed effect are considered and calculated using physics-based equations. Once physical-based scaling rules are determined, the model has good precision for non-sampling geometries. This has also been verified in our experiments. The accuracy of the proposed method is validated through the excellent agreement observed up to the SRF between the simulated and measured results of asymmetric inductors with different geometries fabricated by a standard 0.18-μm SiGe BiCMOS technology.

References

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