

# Comparison and Analysis of Two Microwave Equivalent-Circuit Models for Resonant Tunneling Diode\*

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**Abstract:** The distinction between two microwave equivalent-circuit models, quasi Esaki tunneling model (QETM) and quantum well injection transit model (QWITM), for the resonant tunneling diode (RTD) is discussed in details, and two groups of circuit parameters are extracted from experiment data by the least square fit method. Both theory analysis and the comparison of fit results demonstrate that QWITM is much more precise than QETM. In addition, the rationality of QWITM circuit's parameters confirms it too. On this basis, the resistive frequency is calculated, whose influence factors and improvement method are simply discussed as well.

**Key words:** resonant tunneling diode; microwave equivalent-circuit; quantum well injection transit; resistive frequency

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## 1 Introduction

Resonant tunneling diode (RTD), a two-terminal negative resistance device based on quantum resonant tunneling, is one of those devices with highest operation frequency and fastest response speed. The most interesting characters of RTD are high speed and special folded  $I$ - $V$  characteristics, which has made RTD useful for the implementation of ultra-fast logic gates, latches, quantizers, and static random memory element.

For its high speed, RTD also has great application prospect in the fields of microwave, because of which a precise equivalent circuit model needs to be established to get the performance parameters, on the basis of all-around theory analysis. Until now there are two kinds of primary microwave e-

quivalent-circuit models, quasi Esaki tunneling model (QETM)<sup>[1]</sup> and quantum well injection transit model (QWITM)<sup>[2]</sup>, whose difference will be discussed below by means of the circuits parameters extracted from the experiment data. Then the resistive frequency ( $f_R$ ) of each model will be calculated and the influence factors of  $f_R$  will be discussed.

## 2 Experiment

The RTD used in this paper was prepared by molecular beam epitaxy (MBE) for double barrier-single well structure and then was fabricated by traditional III-V compound semiconductor technology to gain the investigated device. In the material structure design, an  $\text{In}_{0.25}\text{Ga}_{0.75}\text{As}$  sub-well has been put into GaAs quantum well to reduce the en-

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ergy level and decrease the set-on voltage. Figures 1 and 2 show the material structure and  $I$ - $V$  characteristic of RTD,  $5\mu\text{m} \times 5\mu\text{m}$ , respectively<sup>[3]</sup>.

|   |           |                                   |
|---|-----------|-----------------------------------|
| $\text{n}^+ \text{-GaAs}$                   | 500nm     | $1 \times 10^{18} \text{cm}^{-3}$ |
| $\text{n-GaAs}$                             | 50nm      | $2 \times 10^{17} \text{cm}^{-3}$ |
| $\text{n-GaAs}$                             | 50nm      | $2 \times 10^{16} \text{cm}^{-3}$ |
| GaAs  | 3nm       | Undoped(UD)                       |
| AlAs  | 2nm       | UD                                |
| GaAs  | 1.5nm     | UD                                |
| $\text{In}_{0.25}\text{Ga}_{0.75}\text{As}$ | 2.5nm     | UD                                |
| GaAs  | 1.5nm     | UD                                |
| AlAs  | 2nm       | UD                                |
| GaAs  | 3nm       | UD                                |
| $\text{n-GaAs}$                             | 50nm      | $2 \times 10^{16} \text{cm}^{-3}$ |
| $\text{n-GaAs}$                             | 50nm      | $2 \times 10^{17} \text{cm}^{-3}$ |
| $\text{n}^+ \text{-GaAs}$                   | 1000nm    | $1 \times 10^{18} \text{cm}^{-3}$ |
| SI-GaAs                                     | Substrate |                                   |

Fig. 1 Material structure of RTD

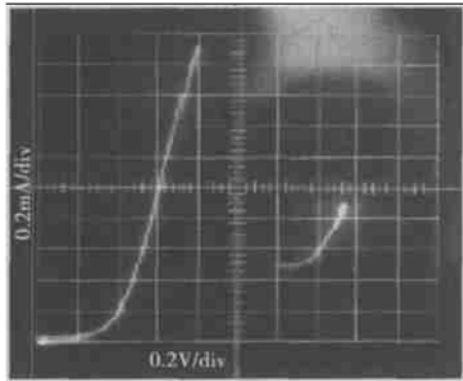


Fig. 2  $I$ - $V$  characteristic of RTD

In experiment, RTD could be considered as a network of two terminals, and its microwave performance was assessed by bias-dependent scattering parameters measurement on chip from 45MHz to 26.5GHz with network analyzer HP8510(C)<sup>[4]</sup>. In the four  $S$ -parameters of RTD obtained,  $S_{22}$  can be expressed as a function of  $Z_{in}$ :  $S_{22} = (Z_{in} - Z_0)/(Z_{in} + Z_0)$ . In this equation,  $Z_0 = 50\Omega$ , is the matching load of transmission line terminal, and  $S_{22}$  and  $Z_{in}$  are both complex numbers. In the measurement frequency range, 201 frequency points were set under the bias of 0.7V in the negative resistance region and  $S_{22}$  of each point could be transformed to  $Z_{in}$  from above equation. With so many

points of frequency, we can determine the optimal values of these parameters through a least square fit method to get the real and imaginary part of  $Z_{in}$ .

### 3 Equivalent circuits

#### 3.1 Quasi Esaki tunneling model

This model quotes the Esaki tunneling diode theory to explain the high frequency performance of RTD. The circuit model is shown in Fig. 3, in which  $R_s$  is the series resistance, containing the contact and bulk resistance;  $L_s$  is the series inductance;  $R_w$  stands for the intrinsic negative resistance of quantum well whose value approximately equals to the differential resistance under the bias of 0.7V; and  $C_w$  represents the total capacitance of device, including the parallel-plate capacitances of each layer, the active capacitance in the quantum well, and the depleted region capacitance owing to depletion of charge. This model has been used to estimate  $f_R$  and used in the research for large signal response in microwave operation<sup>[5]</sup>.

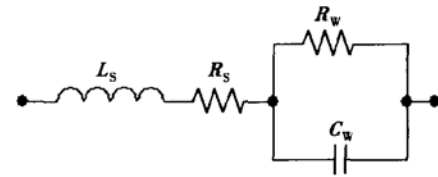


Fig. 3 Equivalent circuit of QETM for RTD

But the limitation of this model is obvious. Because it integrates the capacitances in and out of the quantum well, we apply this model only for the situation when demand of calculation precision is not very strict.

The expression of  $Z_{in}$  of this model is shown below, where  $\omega$  is the angle frequency,  $\omega = 2\pi f$ .

$$Z_{in} = \left[ R_s + \frac{R_w}{1 + (\omega R_w C_w)^2} \right] + j \left[ \omega L_s - \frac{\omega R_w^2 C_w}{1 + (\omega R_w C_w)^2} \right] \quad (1)$$

### 3.2 Quantum well injection transit model

Although above-mentioned QETM is widely applied, it is still a relatively approximate method. In fact, in most of RTDs' structures, the collector layer of RTD is usually designed not too heavy doping and there is an intrinsic spacer layer between well and collector which is inevitable to improve the quantum transmitting efficiency<sup>[6]</sup>. On this condition, a wide depleted region will emerge in the collector side of quantum well, so the electrons should traverse the space charge region before reaching the collector electrode. On the other hand, the quantum well is designed to be very thin to improve the quantum effect, which makes the transit time in the quantum well less than that in the depleted region sometimes. So a more precise model is necessary for calculating the performance parameters more exactly. And the impedances in the depleted region and quantum well should be separated too, that is to apply the quantum well injection transit model. Former studies in other transit time devices<sup>[7-9]</sup> have indicated that impedance has a form similar to the quantum well. The total impedance could be expressed like  $Z = Z_w + Z_d$ , and equivalent circuit is shown in Fig. 4.

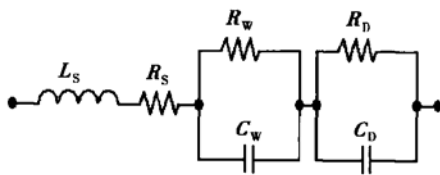


Fig. 4 Equivalent circuit of QWITM for RTD

In Fig. 4,  $R_s$  and  $L_s$  have same meaning with Fig. 3, while  $C_w$  represents the intrinsic capacitance in the quantum well only;  $C_d$  is the depleted region capacitance that comes out of the delay of electrons in depleted region, and  $R_w$  and  $R_d$  are the negative resistances of the quantum well and space charge region, respectively. In terms of the physics meaning, this model expresses, more close to reality, the operation status on the condition of high frequency and a given bias more exactly. The expression of

$Z_{in}$  of QWITM is shown below.

$$Z_{in} = \left[ R_s + \frac{R_w}{1 + (\omega R_w C_w)^2} + \frac{R_d}{1 + (\omega R_d C_d)^2} \right] + j \left[ \omega L_s - \frac{\omega R_w^2 C_w}{1 + (\omega R_w C_w)^2} - \frac{\omega R_d^2 C_d}{1 + (\omega R_d C_d)^2} \right] \quad (2)$$

The following discussion is concerned with the difference of two models in terms of data fitting and parameters extraction.

### 4 Fitting and analysis

Depending on the real and imaginary parts of  $Z_{in}$  in Eqs. (1) and (2), the experiment data have been fitted with program Origin, after being transformed from  $S$ -parameters to  $Z$ -parameters. Figure 5 shows the fitting results of QETM and equivalent parameters:  $R_s = -5.54 \Omega$ ,  $L_s = -1.05 \times 10^{-11}$  H,  $R_w = -358.27 \Omega$ ,  $C_w = 1.21 \times 10^{-13}$  F, the best-

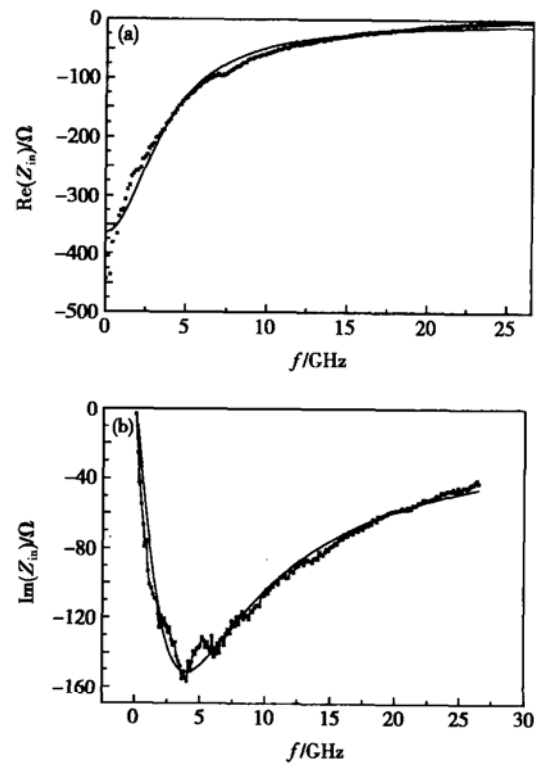


Fig. 5 (a) Curve fit result of real part impedance of QETM; (b) Curve fit result of imaginary part impedance of QETM — fit data; ..... experimental data

fit root mean square error is 160.57 of real part, and 38.57 of imaginary part. Figure 6 shows the fitting results of QWITM and equivalent parameters:  $R_s = 4.60\Omega$ ,  $L_s = 3.61 \times 10^{-11} \text{H}$ ,  $R_w = -171.11\Omega$ ,  $C_w = 1.01 \times 10^{-12} \text{F}$ ,  $R_D = -258.21\Omega$ ,  $C_D = 1.14 \times 10^{-13} \text{F}$ , the best-fit root mean square error is 19.06 of real part, and 9.65 of imaginary part.

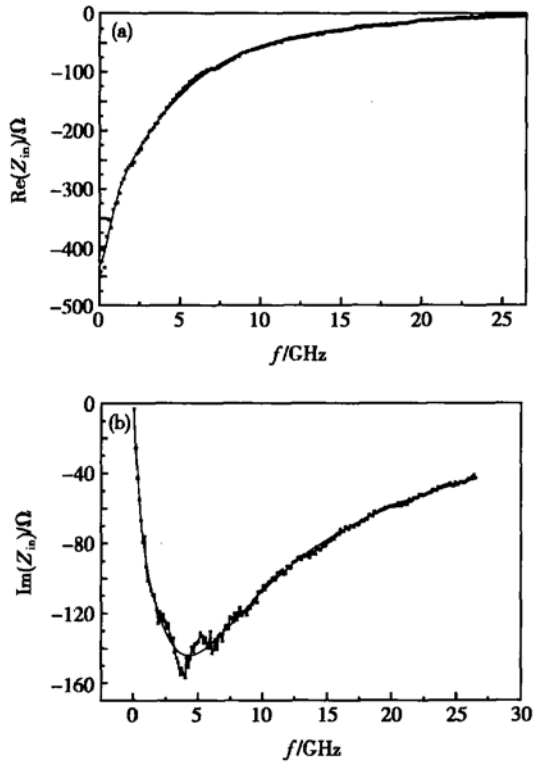


Fig. 6 (a) Curve fit result of real part impedance of QWITM; (b) Curve fit result of imaginary part impedance of QWITM — fit data; .....experimental data

Judging from the fit results, QWITM excels QETM in two sides: First, the best-fit root mean square error of each part of QWITM is much smaller than QETM, that is to say, the impedance expression of QWITM matches the experiment curve better than QETM through the optimization procedure and is closer to reality. Second, during the optimization procedure,  $R_s$  and  $L_s$  are negative numbers in QETM, while positive in QWITM. But according to the definition of these two parameters, both they should be positive; it is obvious that

QWITM embodies more precise physical mechanism. In former calculation in order to get positive  $R_s$  and  $L_s$  in QETM, consequently calculate  $f_R$ , we had to initiate them with positive numbers by estimate, which apparently is not scientific and exact.

## 5 Resistive frequency

Equation(3) is obtained from Eq. (1) on the base of the definition of  $f_R$ , the corresponding frequency when the real part of  $Z_{in}$  equals to zero.

$$f_{R1} = \frac{1}{2\pi R_w C_w} \sqrt{-\frac{R_w}{R_s} - 1} \quad (3)$$

But for negative  $R_s$  in QETM, the initiated value of  $R_s$  is  $4.596\Omega$  in QWITM to compute  $f_R$ . And the result is  $f_{R1} = 35.57\text{GHz}$ . It could be concluded from Eq. (3) that the decrease of  $R_s$ ,  $R_w$ , and  $C_w$  would increase  $f_R$ . So in the design and fabrication of RTD, following methods have been applied to improve  $f_R$ : sufficiently alloyed and heavily doping in E region to minish  $R_s$ , thinning barrier's width and using high mobility materials to minish  $R_w$ , reducing area of E region, and adopting atmosphere bridge structure to minish  $C_w$ .

As far as the resistive frequency, it is more complex to analyze. When  $\text{Re}(Z_{in})$  equals zero, we can get a quadratic equation (4) versus  $\omega^2$  from which three coefficients could be obtained according to the circuit parameters as following.

$$(R_w C_w R_D C_D)^2 \omega^4 + \left[ 1 + \frac{R_w}{R_s} \right] (R_D C_D)^2 + \left[ 1 + \frac{R_D}{R_s} \right] (R_w C_w)^2 \omega^2 + 1 + \frac{R_w}{R_s} + \frac{R_D}{R_s} = 0 \quad (4)$$

$$a = (R_w C_w R_D C_D)^2 = 2.748 \times 10^{-41} \text{S}^4$$

$$b = \left[ 1 + \frac{R_w}{R_s} \right] (R_D C_D)^2 + \left[ 1 + \frac{R_D}{R_s} \right] (R_w C_w)^2 = -1.778 \times 10^{-18} \text{S}^2$$

$$c = 1 + \frac{R_w}{R_s} + \frac{R_D}{R_s} = -93.475$$

(5)

The positive root is  $\omega^2 = 6.474 \times 10^{22} \text{S}^{-2}$ , and  $f_{R2} = \frac{1}{2\pi} \sqrt{6.474 \times 10^{22}} = 40.49\text{GHz}$ . The relation of

$f_{R2}$  and the circuit parameters comes from the relation of  $f_{R2}$  and  $\omega^2$ . In the expression  $\omega^2 = -\frac{b + \sqrt{b^2 - 4ac}}{2a}$ , and  $b^2 = 3.160 \times 10^{-36} \gg 4ac = -1.027 \times 10^{-38}$ , so  $4ac$  could be ignored and a simple expression is obtained:

$$\omega^2 = -\frac{b}{a} = \left[ -\frac{R_w}{R_s} - 1 \right] / (R_w C_w)^2 + \left[ -\frac{R_D}{R_s} - 1 \right] / (R_D C_D)^2 \quad (6)$$

From above equation, it could be observed that  $f_{R2}$  and  $f_{R1}$  have similar forms with the only difference that QWITM has an extra RC oscillation circuit comparing with QETM. In order to increase  $f_{R2}$ , all of  $R_s$ ,  $R_w$ ,  $C_w$ ,  $R_D$ , and  $C_D$  should be minimized. So besides those methods mentioned above, the doping concentration of collector should be increased to thin the width of depleted region to increase  $C_D$ . But for that it will increase  $R_s$ , the limit of both sides in structure design should be considered.

## 6 Conclusion

This paper has analyzed the similarities and differences of QETM and QWITM in both physics theory and fitting results of data. And it is believed that QWITM suits to high frequency performance of RTD better than QETM because of the existence of depleted region, which is inevitable with the spacer layer and light doping in collector side. So QETM could be used as an approximate model only. While considering the important role that depleted region plays in performance, QWITM should

be adopted for its greater precision.

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## 谐振隧穿二极管的两种高频小信号模型比较与分析\*

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**摘要:** 讨论了谐振隧穿二极管(RTD)的两种微波等效电路模型: 类江崎隧道模型(QETM)和量子阱注入传输模型(QWITM)之间的差异, 并用最小二乘法拟合实验数据, 提取了两种模型的电路参数. 理论分析和对拟合结果的比较都说明 QWITM 比 QETM 更加精确, 而提取到的 QWITM 的电路参数的合理性也证明了这一点. 在此基础上计算了阻性频率 $f_R$ , 并简单介绍 $f_R$ 的影响因素及其提高方法.

**关键词:** RTD; 微波等效电路; 量子阱注入传输; 阻性频率

**EEACC:** 2520D; 2560X

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