Stochastic Analysis of Interconnect Delay in the Presence of Process Variations

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Abstract: Process variations can reduce the accuracy in estimation of interconnect performance. This work presents a process variation based stochastic model and proposes an effective analytical method to estimate interconnect delay. The technique decouples the stochastic interconnect segments by an improved decoupling method. Combined with a polynomial chaos expression (PCE), this paper applies the stochastic Galerkin method (SGM) to analyze the system response. A finite representation of interconnect delay is then obtained with the complex approximation method and the bisection method. Results from the analysis match well with those from SPICE. Moreover, the method shows good computational efficiency, as the running time is much less than the SPICE simulation's.

Key words: coupled interconnects; process variations; stochastic modeling; delay estimation; stochastic Galerkin method; polynomial chaos expression
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1　Introduction

Since the advent of deep sub-micron (DSM) technology, interconnect delay has become more significant in the delay of integrated circuit (IC) systems. Specifically, beginning with 250nm technology, interconnect delay has exceeded gate delay to be the dominant part of circuit delay[1]. Thus, alleviating the impact of interconnect delay is critical in high performance system design.

The delay models for interconnects have been researched for several years. In Ref. [2] the Elmore model was proposed to estimate the delay of RC interconnects. Kahng and Muddu presented the equivalent Elmore model based on RLC interconnects by considering the impact of coupling inductances[3,4]. However, these models are all based on fixed parameters. Due to the sensitivity of circuit performance[5], the impact of process variations must be considered when the delay model for interconnects is constructed.

This paper proposes a stochastic model in the presence of process variations, which treats electrical parameters as random variables. Following this model and combining SGM with PCE, our method analyzes interconnect delay and models the stochastic response in terms of orthogonal polynomial expansions. As a result, in comparison with SPICE simulation and Mont Carlo analysis, the technique not only matches well but improves the computational efficiency.

2　Stochastic model for interconnects

The coupled interconnects can be modeled as RLC circuits consisting of many interconnect segments. Each segment is modeled as a multiple π section. Figure 1 shows the coupled RLC lumped model, where line 1 is the aggressor and line 2 is the victim. V_{in1}, V_{in2}, R_s, and C_l are the input signals, excitation resistance, and load capacitance, respectively. Part II is the lumped circuit of interconnects, which is composed of n interconnect segments.

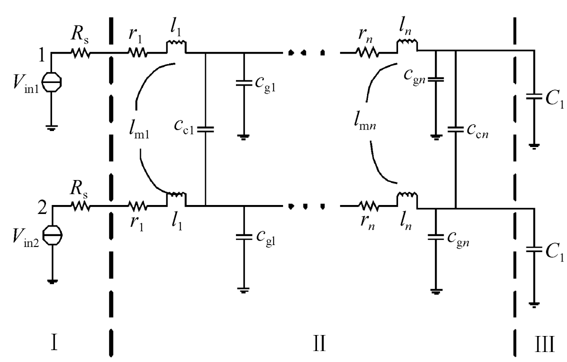

Fig.1　Lumped model for interconnects with process variations

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The electrical parameters of each segment depend on interconnect geometries, such as metal height (H), metal width (W), and ILD thickness (T). These interconnect geometries are affected by process variations, which exist because of non-ideal conditions during manufacturing. Thus, the geometric characteristics of interconnects, and consequently, their electrical characteristics, should be modeled as random variables.

Let Ω denote the sample space of experiments of manufacturing outcomes. For $z \in \Omega$, let a stochastic sequence $\{\varepsilon_1(z), \varepsilon_2(z), \cdots, \varepsilon_u(z)\}$ represent u geometric characteristics of interest. The space of all such stochastic sequences is denoted by $\Theta: \Omega \rightarrow R^u$. In general, we can assume that the elements of the sequence have mean zero. This is easily achieved by subtracting the mean from each random variable. Based on the experience of engineering practice, we know that the Gaussian distribution can describe the geometric characteristics of interconnects very well[6~8]. Thus, in this paper, the elements of stochastic sequence are modeled to be Gaussian. The fact that $\{\varepsilon_i\}_{i=1}^{u}$ are Gaussian is not a constraint. The original random variables $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_u\}$ representing the interconnect variations can be Gaussian or non-Gaussian. Other common distributions were mentioned in Ref. [9].

Without loss of generality, we assume that $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_u\}$ is subject to Gaussian distributions. These are modeled as normal Gaussian random variables. In general, $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_u$ may be correlated. This implies that the parameters are not orthogonal to each other. Here we define an orthonormal stochastic sequence $\boldsymbol{\varepsilon}' = \{\varepsilon'_1, \varepsilon'_2, \cdots, \varepsilon'_u\}$, which can be obtained from $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_u\}$ by a linear transformation like PCA[10]. The electrical parameters are functions of all the random variables. We attempt to capture the effects of the process variations on electrical parameters by expressing them as linear functions. This is in accordance with the models developed in much of the contemporary literature[11,12]. However, we highlight that there are no limitations in choosing any particular form of the expansion for electrical parameters in term of $\boldsymbol{\varepsilon}'$. Thus, we have:

$$\lambda = \bar{\lambda} + \lambda_1 \varepsilon'_1 + \lambda_2 \varepsilon'_2 + \cdots + \lambda_u \varepsilon'_u \tag{1}$$

where λ is the electrical parameter such as resistance r, self inductance l, mutual inductance l_m, ground capacitance c_g or coupling capacitance c_c, $\bar{\lambda}$ is the mean value of the electrical parameters, and λ_i is the perturbation in λ due to the variation in ε'_i. Because process variations affect these electrical parameters simultaneously, all the electrical parameters should change as Eq. (1) at the same time.

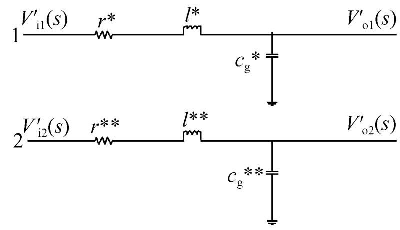

Fig. 2　Decoupled model for RLC interconnect segment

3　Analysis of delay for a stochastic model

A interconnect segment with a multiple π model is illustrated in Fig. 1. Let $\varphi_i = [V_{i1}, V_{i2}, I_{i1}, I_{i2}]^T$ be the input voltage and current of the aggressor and victim, and $\varphi_o = [V_{o1}, V_{o2}, I_{o1}, I_{o2}]^T$ be the output voltage and current of the aggressor and victim. The Kirchhoff equation in a complex frequency domain for a interconnect segment is given by

$$\varphi_o = \begin{bmatrix} \Gamma & R + sL \\ sC & \Gamma \end{bmatrix} \varphi_i \tag{2}$$

where Γ is the identity matrix, R, L, and C are the resistance matrix, inductance matrix, and capacitance matrix considering the parasitic coupling effect between interconnects.

In order to decouple the coupled interconnects, we introduce an orthogonal matrix M. Let φ'_i, and φ'_o be the voltages and currents after transformation. The relationships between φ'_i and φ'_o, and φ_i and φ_o are expressed as Eqs. (4) and (5).

$$M = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \tag{3}$$

$$\varphi'_i = \begin{pmatrix} M^{-1} & 0 \\ 0 & M^{-1} \end{pmatrix} \varphi_i \tag{4}$$

$$\varphi'_o = \begin{pmatrix} M^{-1} & 0 \\ 0 & M^{-1} \end{pmatrix} \varphi_o \tag{5}$$

Defining R', L', and C' as the resistance matrix, inductance matrix, and capacitance matrix after transformation, we have:

$$R' = R \tag{6}$$

$$L' = M^{-1} L M \tag{7}$$

$$C' = M^{-1} C M \tag{8}$$

From Eqs. (4)～(8), we can obtain a new system equation of a decoupled interconnect segment in a complex frequency domain as follows:

$$\varphi'_o = \begin{bmatrix} \Gamma & R' + sL' \\ sC' & \Gamma \end{bmatrix} \varphi'_i \tag{9}$$

Letting φ'_i and φ'_o be the input and output signals, a new model for the interconnect segment can be proposed, as shown in Fig. 2. It is clear that the coupled interconnect segment is decoupled into two independent segments.

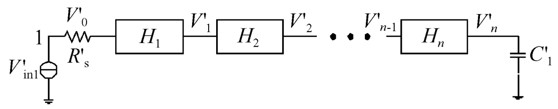

Fig.3 Decoupled aggressor line

The remaining $n-1$ interconnect segments can also be decoupled in a similar manner. By cascading the aggressor and victim of every interconnect segment, the lumped model shown in Fig. 1 is decoupled into two independent interconnects.

In this example, we analyze the decoupled aggressor line, which is shown in Fig. 3. For the driver and receiver ends, the boundary conditions are shown as follows:

$$V'_0 = V'_{in1} - R'_s I'_0 \quad (10)$$

$$I'_n = sC'_1 V'_n \quad (11)$$

Define the variables g_i and c_i as

$$g_1 = R'_s + r_1^* + sl_1^* = R'_s + \bar{r}_1^* + s\bar{l}_1^* + \sum_{j=1}^{u}(r_{1j}^* + sl_{1j}^*)\varepsilon'_j \quad (12)$$

$$g_i = r_i^* + sl_i^* = \bar{r}_i^* + s\bar{l}_i^* + \sum_{j=1}^{u}(r_{ij}^* + sl_{ij}^*)\varepsilon'_j, \quad i = 2,3,\cdots,n \quad (13)$$

$$c_i = sc_{gi}^* = s\bar{c}_{gi}^* + \sum_{j=1}^{u} sc_{gij}^*\varepsilon'_j, \quad i = 1,2,\cdots,n-1 \quad (14)$$

$$c_n = s(c_{gn}^* + C'_1) = s(\bar{c}_{gn}^* + C'_1) + \sum_{j=1}^{u} sc_{gnj}^*\varepsilon'_j \quad (15)$$

The aggressor line can be represented in complex frequency domain by Kirchhoff law based nodal analysis equations.

$$\begin{cases} V'_{in1} g_2 = V'_1(g_1 + g_2 + c_1 g_1 g_2) - V'_2 g_1 \\ 0 = -g_3 V'_1 + V'_2(g_2 + g_3 + c_2 g_2 g_3) - V'_3 g_2 \\ 0 = -g_4 V'_2 + V'_3(g_3 + g_4 + c_3 g_3 g_4) - V'_4 g_3 \\ \vdots \\ 0 = -V'_{n-1} + V'_n(1 + c_n g_n) \end{cases} \quad (16)$$

We introduce a concept called the polynomial chaos expression (PCE)[13], whose fundamental functions are Hermite polynomials. The definition of Hermite polynomials is given in Ref. [13]. The Hermite polynomials form a set of orthogonal bases in the Hilbert space with Gaussian measure. The inner product is defined as

$$\langle f(\boldsymbol{\varepsilon}'), g(\boldsymbol{\varepsilon}')\rangle = \int_{D(\boldsymbol{\varepsilon}')} f(\boldsymbol{\varepsilon}'), g(\boldsymbol{\varepsilon}')W(\boldsymbol{\varepsilon}')\mathrm{d}\boldsymbol{\varepsilon}' \quad (17)$$

where $W(\boldsymbol{\varepsilon}')$ is the probability density function (PDF), and $D(\boldsymbol{\varepsilon}')$ is the integral interval of $\boldsymbol{\varepsilon}'$.

We rewrite the nodal analysis equations into matrix formulation $X(s,\boldsymbol{\varepsilon}') = M(s,\boldsymbol{\varepsilon}')Y(s,\boldsymbol{\varepsilon}')$. $Y(s,\boldsymbol{\varepsilon}') = (V'_1(s,\boldsymbol{\varepsilon}'),\cdots,V'_n(s,\boldsymbol{\varepsilon}'))^{\mathrm{T}}$ is the response vector of Eq. (16). According to the orthogonal identity of Hermite polynomials[14], $Y(s,\boldsymbol{\varepsilon}')$ can be approximated by an infinite series of Hermite polynomials.

$$Y(s,\boldsymbol{\varepsilon}') = \sum_{i=1}^{\infty}\boldsymbol{\alpha}_i(s)H_i(\boldsymbol{\varepsilon}') = \left(\sum_{i=1}^{\infty}\alpha_{1i}(s)H_i(\boldsymbol{\varepsilon}'),\cdots,\sum_{i=1}^{\infty}\alpha_{ni}(s)H_i(\boldsymbol{\varepsilon}')\right)^{\mathrm{T}} \quad (18)$$

where $H_i(\boldsymbol{\varepsilon}')$ is the i-order Hermite polynomial, and $\alpha_{1i}(s),\cdots,\alpha_{ni}(s)$ are the approximation coefficients.

To calculate the approximation coefficients in Eq. (18), $Y(s,\boldsymbol{\varepsilon}')$ has to be truncated after a finite number of terms.

According to the stochastic Galerkin method (SGM)[15], one of the most important components is the test functions. They are used to make the residual error given in Eq. (19), which is the error produced by substituting the truncated expansion for the accurate solution $Y(s,\boldsymbol{\varepsilon}')$, as small as possible in the norm sense.

$$\Delta_p(s,\boldsymbol{\varepsilon}') = \sum_{i=1}^{m} M(s,\boldsymbol{\varepsilon}')H_i(\boldsymbol{\varepsilon}')\boldsymbol{\alpha}_i(s) - X(s,\boldsymbol{\varepsilon}') \quad (19)$$

In the stochastic Galerkin method (SGM), test functions are set to be the same as the fundamental functions, which means that for Eqs. (16) and (20), they should satisfy:

$$\langle\Delta_p(s,\boldsymbol{\varepsilon}'), H_j(\boldsymbol{\varepsilon}')\rangle = 0 \quad (20)$$

where $j = 0,1,2,\cdots,m$.

We thus have

$$\left\langle\sum_{i=1}^{m} M(s,\boldsymbol{\varepsilon}')H_i(\boldsymbol{\varepsilon}')\boldsymbol{\alpha}_i(s), H_j(\boldsymbol{\varepsilon}')\right\rangle = \langle X(s,\boldsymbol{\varepsilon}'), H_j(\boldsymbol{\varepsilon}')\rangle \quad (21)$$

Based on the definition of inner product, the equation above can be rewritten as

$$\sum_{i=1}^{m}\left[\int_{D(\boldsymbol{\varepsilon}')} M(s,\boldsymbol{\varepsilon}')H_i(\boldsymbol{\varepsilon}')H_j(\boldsymbol{\varepsilon}')W\mathrm{d}\boldsymbol{\varepsilon}'\right]\boldsymbol{\alpha}_i(s) = \left(\int_{D(\boldsymbol{\varepsilon}')} V'_{in1} g_2 H_j W\mathrm{d}\boldsymbol{\varepsilon}', 0\cdots 0\right)^{\mathrm{T}} \quad (22)$$

The term in the square brackets on the L. H. S of Eq. (22) is an $n\times n$ matrix. $\boldsymbol{\alpha}_i(s)$ is an $n\times 1$ coefficient vector to be determined. The integration on the R. H. S is a constant vector. Considering the situations of $j = 1,\cdots,m$ and integrating them into a matrix formulation, we can obtain a new matrix equation about s:

$$M''(s)Y''(s) = X''(s) \quad (23)$$

where

$$X''(s) = \left(\int_{D(\boldsymbol{\varepsilon}')} V'_{in1} g_2 H_1 W\mathrm{d}\boldsymbol{\varepsilon}', \cdots, \int_{D(\boldsymbol{\varepsilon}')} V'_{in1} g_2 H_m W\mathrm{d}\boldsymbol{\varepsilon}', 0\cdots 0\right)^{\mathrm{T}}$$

$$Y''(s) = (\alpha_{11}(s),\cdots,\alpha_{1m}(s),\cdots,\alpha_{n1}(s),\cdots,\alpha_{nm}(s))^{\mathrm{T}}$$

$M''(s)$ is the coefficient matrix, whose elements are integral matrixes of order m.

In complex frequency domains, s can be expressed in the form of complex exponential function $s = |s|\exp(\mathrm{i}\theta)$, where $|s|$ is the amplitude and θ is the phase. In this paper, we acquire sample points in the complex frequency domain by increasing the amplitude and phase alternately. The sampling curve in a complex plane is like a helical curvilinear that extends infinitely from the origin. Then, the numerical solutions of $\alpha_{n1}(s), \cdots, \alpha_{nm}(s)$ can be calculated through adopting different sampling points s_i ($i = 1, 2, \cdots, P_b$), where P_b is the number of sampling. Due to the characteristic of this analytical method, it is not feasible to approximate functions in a complex domain only with the polynomials of s in real domain. Therefore, according to the Walsh law[16,17], a new approximation model is introduced as

$$\frac{1}{\sum_{v=-k}^{k} a_v s^v} \tag{24}$$

where $a_v = a_{v1} + a_{v2}\mathrm{i}$.

If we let $f(s) = 1/V_n(s)$, then the problem focuses on approximating the function $f(s)$. According to the least square method in a complex domain, the normal equation is given by

$$F_{(4k+2)\times(4k+2)} \cdot A_{(4k+2)\times 1} = f_{(4k+2)\times 1} \tag{25}$$

where $A_{(4k+2)\times 1} = [a_{-k1}, a_{-k2}, \cdots, a_{k1}, a_{k2}]^{\mathrm{T}}$ is the matrix of unknown quantities, $F_{(4k+2)\times(4k+2)}$ is the coefficient matrix, and $f_{(4k+2)\times 1}$ is the sampling matrix of $f(s)$.

After solving Eq. (25), the approximation coefficient a_v is determined. Substituting a_v into Eq. (24), we can obtain the approximate expressions of $\alpha_{n1}(s)$, $\cdots, \alpha_{nm}(s)$. Thus, the voltage response of the aggressor in a complex domain is shown as

$$V'_{o1}(s, \boldsymbol{\varepsilon}') = V'_n(s, \boldsymbol{\varepsilon}') = \sum_{i=1}^{m} \alpha_{ni}(s) H_i(\boldsymbol{\varepsilon}') \tag{26}$$

The time domain response can be obtained by the inversion of a Laplace transform.

$$v'_{o1}(t, \boldsymbol{\varepsilon}') = \sum_{i=1}^{m} (q_{1i}\mathrm{e}^{p_{1i}t} + \cdots + q_{ji}\mathrm{e}^{p_{ji}t}) H_i(\boldsymbol{\varepsilon}') \tag{27}$$

where p_{ji} and q_{ji} are the zeros and poles, respectively.

Analyzing the decoupled victim similarly, the time response $v'_{o2}(t, \boldsymbol{\varepsilon}')$ can be obtained the same way as the aggressor was.

Suppose V_{th} is the value of the output signal and τ is the interconnect delay. According to Eq. (5), the exponential equation of the aggressor's response before decoupling is taken by

$$V_{th} = v'_{o1}(\tau, \boldsymbol{\varepsilon}') + v'_{o2}(\tau, \boldsymbol{\varepsilon}') \tag{28}$$

Note Equation (28) is a transcendental equation; the analytic solution cannot be obtained directly. In the signal's rising time, the transcendental equation is monotone. Thus, we can obtain the numerical solution of τ with numerical techniques such as the bisection method. Here we define a function $Q(\tau)$ as follows:

$$Q(\tau) = v'_{o1}(\tau, \boldsymbol{\varepsilon}') + v'_{o2}(\tau, \boldsymbol{\varepsilon}') - V_{th} \tag{29}$$

Table 1　Means of lumped RLC interconnect segments

Parameter	R_s/Ω	C_1/pF	c_g/pF	c_c/pF	l/μH	l_m/μH	r/Ω
Value	250	50	0.2699	0.9582	2.1578	1.6746	25.6692

Let ζ be the midpoint between zero and T_r, where T_r is the signal's rising time. The possible situations of $Q(\zeta)$ are listed as follows.

(1) $Q(\zeta) = 0$, then $\tau = \zeta$.

(2) $Q(\zeta) \neq 0$, then $Q(\zeta)$ has the same sign as $Q(0)$ or $Q(T_r)$.

If $Q(\zeta)$ has the same sign as $Q(0)$, we obtain $\zeta < \tau < T_r$; else if $Q(\zeta)$ has the same sign as $Q(T_r)$, we have $0 < \tau < \zeta$. Define ρ as the error of Eq. (29) and repeat the arithmetic in the new interval of τ until $Q(\zeta) < \rho$. ζ is regarded as the solution of the transcendental equation under tolerance ρ.

4　Experimental results and analysis

We illustrate our method with the aid of an example calculation on the model shown in Fig. 1. Let the input excitations V_{in1} and V_{in2} be constant voltage sources, where V_{in1} is the step signal and V_{in2} is zero. Without loss of generality, we assume that the only variation of significance is in the width W, and ε_w is the geometric characteristic of W. Thus Equation (1) can be rewritten as follows:

$$\lambda = \bar{\lambda} + \lambda_w \varepsilon_w \tag{30}$$

The number of interconnect segments in the lumped model is determined by thumb criterion[18]. Let D be the length of the interconnect, T_r be the rising time of the input signal, and c be the propagation speed of electromagnetic waves in the medium. The segment number n can be given by

$$n > 10 \times \frac{D}{T_r c} \tag{31}$$

The first test case taken into consideration is the RLC model shown in Fig. 1, whose segment number is 4. Table 1 shows the mean values of the distribution parameters of interconnect segments. The percentage of process variations is the 3σ value of Gauss distribution of geometric parameters. We compare the 50% delays obtained from HSPICE simulation with those from the analytical method. The results with a maximum width variation of 20% are summarized in Table 2. The relative errors are also listed. When considering different process variations, we assume that every interconnect segment has different geometric characteristic. The experimental results demonstrate the method proposed in this work is effective to estimate the delay of interconnects with process variations.

The result of our method is an expression of the response as a multi-dimensional polynomial that can be directly evaluated. There is no need to repeatedly

Table 2 Time and relative errors of interconnect delay

	HSPICE /ns	Analysis /ns	Relative error /%
No process variations ($\varepsilon_w = 0\%$)	24.543	24.472	0.289
Same process variations ($\varepsilon_w = -2.971\%$)	23.520	23.739	0.931
Different process variations ($\varepsilon_{w1} = 2.18\%$, $\varepsilon_{w2} = 1.16\%$, $\varepsilon_{w3} = -1.24\%$, $\varepsilon_{w4} = 4.84\%$)	25.743	25.623	0.466

generate samples of the random parameters, which is required to solve the system in Monte Carlo analysis. Due to the large dimensionality of the sample space, Monte Carlo analysis can be very time consuming. Thus, the running time of our method is much less than that of Monte Carlo analysis.

In the next test, we consider new distributed RLC lines, which have different numbers of interconnect segments. The variations of parameters are also the same as above. We compare the 50% delays and running times obtained from SPICE based Monte Carlo (SPMC) simulation (500 and 1000 sampling points) with those from the analytical method in Table 3. At each of the leaf nodes, the differences between the delays obtained from the analytical method and SPMC is about 0.1% or less, while the running times of the analytical method are much less than SPMC. Figure 4 shows the comparative curves of delay and running time between the analytical method and SPMC. Figure 5 shows the histograms of the delay at node 4 produced by the analytical method and SPMC, respectively.

Table 3 50% delay and running time of the analytical method and SPMC (500 and 1000 sampling points)

Node	Interconnect delay			Running time		
	Analysis/ns	SPMC_500/ns	SPMC_1000/ns	Analysis/s	SPMC_500/s	SPMC_1000/s
$n=4$	24.472	24.521	24.554	6.1410	34.57	76.74
$n=5$	27.217	27.379	27.358	7.3280	34.81	78.72
$n=6$	29.876	29.963	29.999	9.0230	35.72	79.30
$n=7$	32.148	32.399	32.382	10.5780	36.53	81.05
$n=8$	35.061	34.885	34.419	12.2190	37.39	82.72

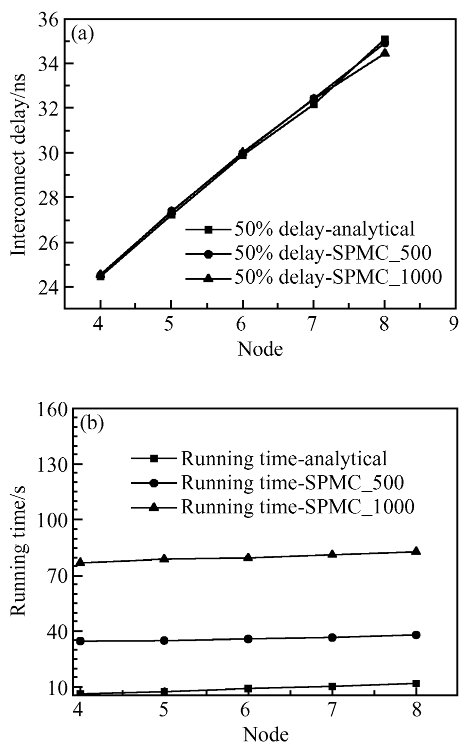

Fig.4 Comparison of 50% delay and running time from this method and those from SPMC (500 sampling points and 1000 sampling points) (a) Comparative curves of 50% delay; (b) Comparative curves of running time

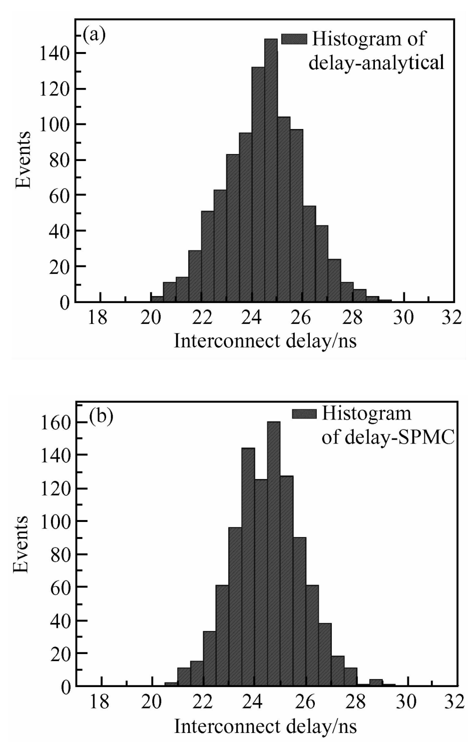

Fig.5 Delay distribution at node 4 by the analytical method and SPMC (1000 sampling points) (a) Histogram produced by this method; (b) Histogram produced by SPMC

5 Conclusion

The development of IC technology has made process variations a new challenge in IC manufacturing. In order to describe an efficient approach to estimate interconnect delay, this paper constructs a stochastic model in the presence of process variations. The proposed approach decouples the coupled interconnect segments and combines PCE with SGM to analyze the delay of stochastic coupled interconnects. Comparing of our results to SPICE simulations demonstrates an excellent match. In addition, our method consumes much less time than SPMC.

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基于工艺参数扰动的随机互连线时延分析

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摘要: 考虑工艺参数扰动对互连电路传输性能的影响,建立了基于工艺扰动的互连线随机模型. 通过改进的去耦算法对随机互连线元进行去耦,结合随机伽辽金方法(SGM)和多项式混沌展开(PCE)进行互连分析,进而利用复逼近及二分法给出工艺参数扰动下互连时延的有限维表达式. 仿真实验结果不仅与 SPICE 仿真吻合得较好,相较于 SPICE 蒙特卡洛仿真还具有更高的计算效率.

关键词: 耦合互连线; 工艺参数扰动; 随机建模; 时延估计; 随机伽辽金方法; 多项式混沌展开
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