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A Novel Statistical Delay Model Based on the Birnbaum-Saunders Distribution for RLC Interconnects in 90nm Technologies

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Abstract: For performance optimization such as placement, interconnect synthesis, and routing, an efficient and accurate interconnect delay metric is critical, even in design tools development like design for yield (DFY) and design for manufacture (DFM). In the nanometer regime, the recently proposed delay models for RLC interconnects based on statistical probability density function (PDF) interpretation such as PRIMO, H-gamma, WED and RLD bridge the gap between accuracy and efficiency. However, these models always require table look-up when operating. In this paper, a novel delay model based on the Birnbaum-Saunders distribution (BSD) is presented. BSD can accomplish interconnect delay estimation fast and accurately without table look-up operations. Furthermore, it only needs the first two moments to match. Experimental results in 90nm technology show that BSD is robust, easy to implement, efficient, and accurate.

Key words: delay model; interconnect; moment; probability distribution function
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1 Introduction

The effect of interconnect networks in signal propagation delay and transition time degradation, such as floorplanning, placement, and routing, has to be considered in various physical design tools. As the impact of manufacturing variation becomes increasingly important to performance and yield, accurate and efficient simulation for interconnects is critical to DFY and DFM research[1]. For 90nm technology nodes and below, an increase in integration density, closer proximity of adjacent wires on the same layer, a high aspect ratio, and the chip size result in an increase in coupling capacitance and interconnect delay. Improved technology nodes, such as copper and low-k technology, and the dual damascene structure, create challenges for calculating the interconnect delay accurately. On the other hand, it needs higher efficiency to simulate huge interconnect networks for high performance SoC design in 90nm and below technology. Although traditional methods (such as Elmore, AWE, D2M) can be used to metric interconnect in 90nm and below technology, the defects in accuracy, efficiency, and complicacy prevent the popularization of these methods for 90nm and below technology. A more accurate and efficient interconnect delay model is crucial for a signal integrity verification in 90nm and below technology.

The Elmore model[2] is the simplest interconnect metric, only matching the first moment ED (Elmore delay) $= u = -m_1$ which approximated the median (the desired delay) by the mean value of the impulse response. However, the disadvantage of the Elmore model is its insufficient accuracy. There are two reasons for the inaccuracy of the Elmore model: First, the Elmore model only uses the first moment to estimate the delay; Second, it approximates the median (the desired delay) by the mean value. For the first drawback, AWE[3] and other co-generic models were improved by computing and matching higher order moments of the impulse response. Nevertheless, AWE cannot be expressed by a closed-form formula. It requires the solution of a non-linear equation, which is very accurate but computationally expensive to use within a tight optimization loop. Alpert *et al*.[4] proposed the D2M model, which is a simple function of the first two circuit moments. But the D2M model can not be used to compute 10/90 slew directly.

To satisfy higher technology node requirements, many models based on PDF interpretation have been proposed recently such as PRIMO[5], H-gamma[6], WED[7] (based Weibull distribution), and RLD[8] (based Rayleigh distribution), which have advantages in both accuracy and efficient. These models directly use the median value of the distribution to estimate the RC's delay, which efficiently avoids the difference between the median and mean values. In order to

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get the median of the distribution with varying parameters, these methods require a large 2-dimension table lookup operation. A drawback of the table lookup approach is that it requires large memory storage to accommodate the 2-dimensional tables that are often necessary to obtain relatively high accuracy. So, in this paper we propose a novel delay model based on the Birnbaum-Saunders distribution[9]. For the distribution characteristic, the method still approximates the median (the desired delay) by the mean with the first two moments of the impulse response, but effectively avoids table lookup operations and retains satisfactory accuracy compared with other delay models based on PDF interpretation.

2 Background

Assume $h(t)$ is the impulse response of a node voltage in an RLC circuit. The Taylor expansion of the Laplace transformation, $H(s)$, is:

$$H(s) = \int_0^\infty h(t)\mathrm{e}^{-st}\mathrm{d}t = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}s^k\int_0^\infty t^k h(t)\mathrm{d}t \tag{1}$$

Therefore, the circuit moments of the impulse response are:

$$m_k = \frac{(-1)^k}{k!}\int_0^\infty t^k h(t)\mathrm{d}t,\quad k = 0,1,2,\cdots,n \tag{2}$$

By path tracing[10], the circuit moments m_k can be calculated efficiently as functions of the RCs. From the characteristic of the impulse response, $h(t)$ satisfies the following condition:

$$h(t) \geqslant 0 \text{ and } \int_0^\infty h(t)\mathrm{d}t = 1 \tag{3}$$

The impulse response is a PDF based on probability theory, but with an unknown underlying statistical distribution. Consequently, the step response of the same circuit is a cumulative density function (CDF). Based on the characteristic, the impulse response of the RLC circuit can be approximately interpreted by a statistical probability method. The mean of the impulse response is defined as:

$$u = \int_0^\infty th(t)\mathrm{d}t \tag{4}$$

The kth central moments or moments about the mean are defined as:

$$u_k = \int_0^\infty (t-u)h(t)\mathrm{d}t \tag{5}$$

The relationship between the central moments and the circuit moments listed in Eq. (2) is:

$$\begin{aligned}\mu_1 &= -m_1\\ \mu_2 &= 2m_2 - m_{21}\\ \mu_3 &= -6m_3 + 6m_1m_2 - 2m_{31}\end{aligned} \tag{6}$$

According to probability theory, the variance and the skewness of the impulse response can be expressed in terms of the central moments and the circuit moments[11]:

$$\begin{aligned}\sigma^2 &= \mu_2 = 2m_2 - m_{21}\\ \gamma &= \frac{u_3}{\sigma^3} = \frac{-6m_3 + 6m_1m_2 - 2m_{31}}{\left(\sqrt{2m_2 - m_{21}}\right)^{3/2}}\end{aligned} \tag{7}$$

The key idea behind delay metrics based on the PDF interpretation is to match the mean, variance, and skewness of the impulse response to those of the probability distributions which approximately accord with the real curve of the impulse response of the RLC circuit.

3 BSD: Birnbaum-Saunders delay

In this paper, a novel delay model based on the Birnbaum-Saunders distribution (BSD) is presented. The BSD model is more efficient than other PDF interpretation models with comparable accuracy because it omits table-lookup operations. The simplicity of the BSD model makes it suitable for 90nm and below technology. To choose the appropriate distribution to match the impulse response, there are two criterions: the first is that the PDF is unimodal or has a bell shape; the second is that the CDF monotonically ascends or has a nonnegative skewness. Figure 1 shows the Birnbaum-Saunders distribution, which agrees with the criterions and only has two parameters, γ the shape parameter and μ the scale parameter, which can be directly interpreted by the first two moments of the RLC circuit.

The PDF of Birnbaum-Saunders is:

$$f(t) = \frac{1}{2\sqrt{\pi}\gamma^2\mu t^2}\times\frac{t^2-u^2}{\sqrt{\dfrac{t}{u}}-\sqrt{\dfrac{u}{t}}}\mathrm{e}^{-\frac{1}{\gamma^2}\left(\frac{t}{u}+\frac{u}{t}-2\right)} \tag{8}$$

The CDF of Birnbaum-Saunders is:

$$F(t) = \Phi\left[\frac{1}{\gamma}\left(\sqrt{\frac{t}{u}}-\sqrt{\frac{u}{t}}\right)\right] \tag{9}$$

where Φ is the standard normal CDF function.

The mean and the variance of Birnbaum-Saunders PDF are respectively given by:

$$E(t) = \mu\left(1+\frac{\gamma^2}{2}\right) \text{ and } V(t) = \mu^2\gamma^2\left(1+\frac{5\gamma^2}{4}\right) \tag{10}$$

To match the μ and γ by the circuit moment m_k, from Eq. (8), we have:

$$\mu\left(1+\frac{\gamma^2}{2}\right) = -m_1,\quad \mu^2\gamma^2\left(1+\frac{5\gamma^2}{4}\right) = 2m_2 - m_1^2 \tag{11}$$

So,
$$\frac{\left(1+\dfrac{\gamma^2}{2}\right)^2}{\gamma^2\left(1+\dfrac{5\gamma^2}{4}\right)} = \frac{m_1^2}{2m_2-m_1^2} \tag{12}$$

Let
$$\frac{m_1^2}{2m_2-m_1^2} = k \tag{13}$$

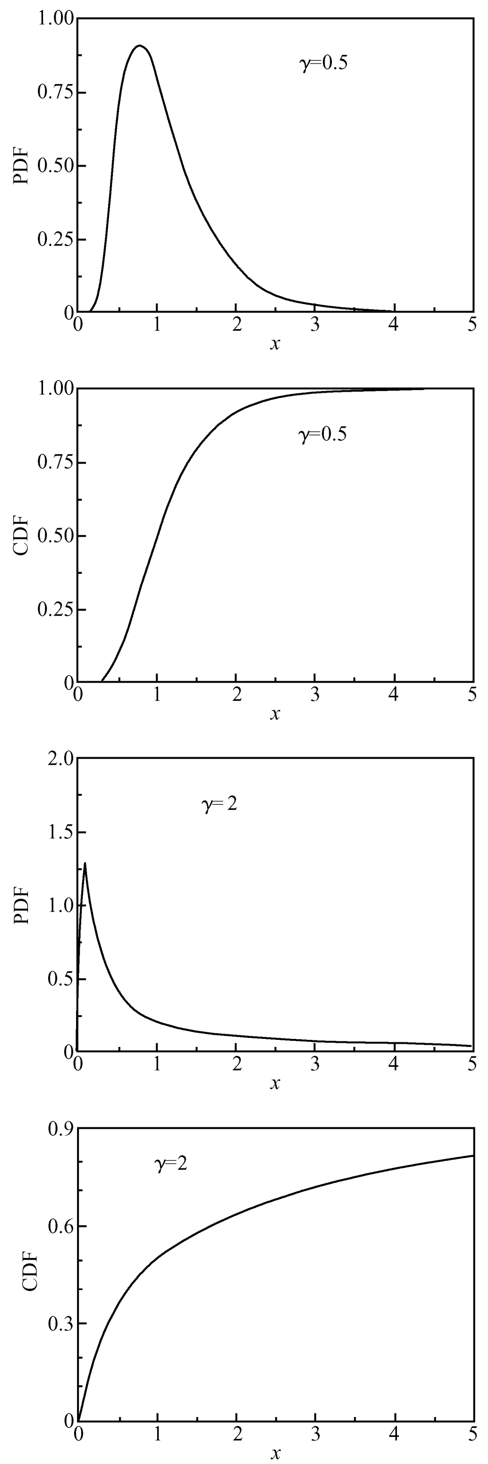

Fig. 1　PDF and CDF of Bimbaum-Saunders

Then,

$$(5k-1)\gamma^4+4(k-1)\gamma^2-4=0 \quad (14)$$

Solving Eqs. (11) and (14), we have:

$$\gamma^2=2\times\frac{1-k+\sqrt{5k^2+3k}}{5k-1},\mu=\frac{1-5k}{4k+\sqrt{k^2+3k}}\times m_1 \quad (15)$$

Because the CDF of the Birnbaum-Saunders distribution is a normal standard distribution, we can directly solve the delay of the RC circuit $D(0)=0.5$. From Eq. (9), the 50% delay $t_{0.5}=\mu$. There is no perfect

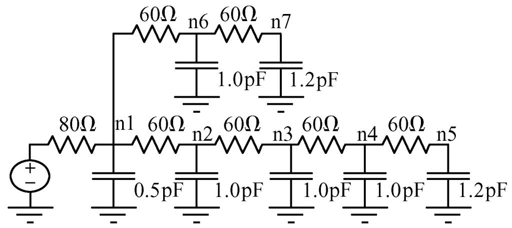

Fig. 2　A simple RC circuit to compare the accuracy of WED, D2M, PRIMO, and H-gamma

distribution for moment matching. All the methods based on PDF interpretation have one or more variations to reality, such as PRIMO, H-gamma, Weibull, and LnD[13]. The BSD is not an exception. Particularly in 90nm and below technology, increasing capacitances due to higher wiring densities extend the mismatching. Therefore, we propose a revised algorithm that can effectively improve the accuracy of BSD. From experimental results, we found that the different became smaller as k decreased. The revised algorithm is:

$$t_{\text{delay}}=\mu(R-k)^o \quad (16)$$

where R is a constant about 2.1 and o is a coefficient that varies from 0 to 1.

4　Experiment

To verify the BSD for the interconnect metric and to compare it with other methods, we first apply the BSD to a representative industrial example[7,8] as shown in Fig. 2. The tests have been performed on several nets from an industrial ASIC part in 0.20μm technology. The nets were identified as those with a large Elmore error in order to provide a realistic, but challenging set to the delay metrics. We use SPICE for the golden result. Results are presented in Table 1. As seen in Table 1, BSD produced more accurate delays for near-end n1 and n3, than D2M, and gave more accurate results compared with other table look-up methods at nodes: n2, n4, n5, n6, and n7.

To further verify the effectiveness of BSD, an experiment has been demonstrated for a single interconnect delay metric in a 90nm technology with dual damascene structure by BSD, as shown in Fig. 3. The parameters of the interconnect were extracted from

Table 1　Comparison between D2M, PRIMO, H-gamma WED

Node	Elmore	D2M	PRIMO	H-γ	WED	RLD	BSD	SPICE/ps
n1	552	299	241	194	246	242	128	196
n2	804	514	498	486	485	474	469	476
n3	996	696	699	701	698	689	716	700
n4	1128	830	836	840	855	853	862	844
n5	936	905	909	912	943	949	896	919
n6	684	420	376	355	386	377	342	374
n7	756	492	450	431	470	460	470	453

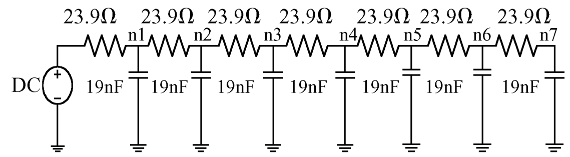

Fig.3 An interconnect metric in 90nm technology to demonstrate the accuracy of BSD

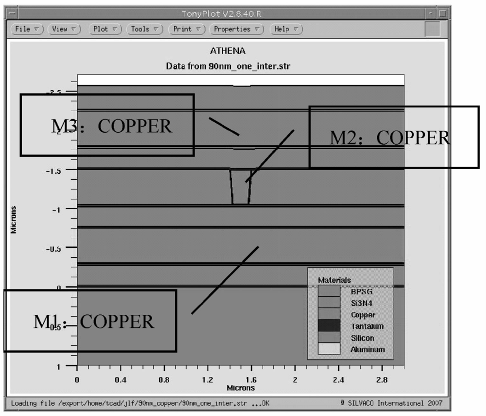

Fig.4 TCAD simulation for interconnect in 90nm technology

Silvaco TCAD by rigorous verified technology simulation[14], as shown in Fig. 4.

Results are shown in Table 2. The D2M metric became more accurate in this experiment than above, but the WED metric became more inaccurate. Nevertheless, there is the same level of difference between the two experiments' BSD errors. The experiments show BSD can get a satisfactory accuracy delay in different situations by adjusting the revisable parameters.

5 Comparison

From the above two experiments and other reference experiments, we found that BSD always underestimates delay at near-end nodes and overestimates at far-end nodes. This variation can be reduced by rectifying revisable parameters in practical industrial applications. Compared with PRIMO, WED, H-gamma, and RLD, BSD avoids complicated table lookup operations with comparable accuracy. Table 1 shows that BSD can approach a more accurate metric at near-end nodes and far-end nodes than D2M. Because of the characteristics of the Birnbaum-Saunders distribution, the 10/90 slew can be achieved directly by BSD. The above two experiments show that the accuracy of all other methods mentioned fluctuates with different process technologies because of the imperfection matching of the PDF and the response curve. However, the parameters of these models are always invariable. BSD supplies a revisable step to solve the problem, which can rectify the metric for different process technology.

Table 2 Delay metric for interconnect of 90nm technology compared with D2M, WED

Node	D2M	WED	BSD	SPICE/ps
n1	130	808	3	50
n2	323	270	145	202
n3	516	484	452	438
n4	687	682	716	663
n5	823	843	870	821
n6	918	953	934	918
n7	966	1010	950	965

6 Conclusion

In this paper, we proposed a novel interconnect delay metric called BSD. Based on the Birnbaum-Saunders distribution, it can quickly and accurately accomplish interconnect delay estimation by the first two moment matching without a table lookup operation. This method also integrates a revisable method to rectify the metric with different process technologies. The accuracy and efficiency of BSD make it suitable for 90nm and below technology. Because of its simplicity to implement, BSD can be a good choice for DFY and DFM tools development.

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一种基于 Birnbaum-Saunders 分布的新型互连时延统计模型

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摘要： 基于统计概率分布的互连时延模型具有效率高、准确性好的特点，但此类方法往往包含一些查表运算. 本文提出了一种基于 Birnbaum-Saunders 分布的互连线时延模型，避免了查表运算，且仅需要采用前两个瞬态，计算简单，准确性较好，并提出了一种精度修正算法，使该方法具有更好的适应性.

关键词： 时延模型；互连线；瞬态；统计概率分布

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