System-Level Modeling and Simulation of Force-Balance MEMS Accelerometers*

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Abstract: This paper presents two approaches for system-level simulation of force-balance accelerometers. The derivation of the system-level model is elaborated and simulation results are obtained from the implementation of those strategies on the fabricated silicon force-balance MEMS accelerometer. The mathematical model presented is implemented in VHDL-AMS and SIMULINK TM, respectively. The simulation results from the two approaches are compared and show a slight difference. Using VHDL-AMS is flexible, reusable, and more accurate. But there is not a mature solver developed for the language and this approach takes more time, while the simulation model can be easily built and quickly evaluated using SIMULINK.

Key words: MEMS; force-balance accelerometers; system-level simulation; mathematical model
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1 Introduction

With the development of MEMS (micro-electro-mechanical-systems), micromachined accelerometers have become more important over the past decades. Multiple approaches to acceleration sensing have been proposed. But, most accelerometers use the capacitive-based mechanism because of its structural simplicity, high accuracy, low temperature sensitivity, low noise performance, good DC response, and compatibility with CMOS readout electronics. Moreover, force-balance MEMS accelerometers based on capacitive mechanism can improve the system stability and widen the measurement range using a closed loop structure.

Using the system-level simulation can simulate the running status of the system, optimize the structure of accelerometers, and eventually reduce costs and design time. Different methods to model a microsystem have been discussed[1]. Today, commercial software like COVENTOR and INTELLISUITE are widely used in the simulation of MEMS devices. INTELLISUITE has the most advantages in the process simulation of MEMS devices, such as anisotropic etching, bonding, and so on. COVENTOR is well-known for the system level simulation of many MEMS devices, such as the micromirror and microgyroscopic. Moreover, the elementary models like beam and anchor are custom, and cannot be modified according to the needs of users. Especially for the system-level simulation of a force-balance MEMS accelerometer, the feedback voltage is continuously applied to the proof mass of accelerometer proportional to the output signal and cannot be accomplished by use of COVENTOR.

Currently, there are two widely used approaches to accomplish the system-level simulation of the force-balance MEMS accelerometer. First, establish a lumped electrical network that is equivalent to the system, and then use commercially available simulators such as SPICE™ or MATLAB™ for system-level simulations[2]. Second, use a set of coupled ordinary differential equations (ODEs) that represent the dynamical behavior of the MEMS system and use the hardware description language VHDL-AMS to describe those ODEs. The former approach creates a lumped-parameter macromodel that describes general characteristics of MEMS systems. So, if dimensions of structures or locations of elements in the system change, the model has to be rebuilt. In addition, it is difficult to establish equivalent circuits for complex MEMS configurations. However, using VHDL-AMS can overcome these disadvantages. Because of the relatively small number of state variables in those ODEs, the macromodel can be quickly evaluated and simulated. Meanwhile, using VHDL-AMS, such models can be readily inserted into circuit simulators for behavioral simulation at the system level. The available language facilities of VHDL-AMS and the requirements of the applied method are discussed[3~5].

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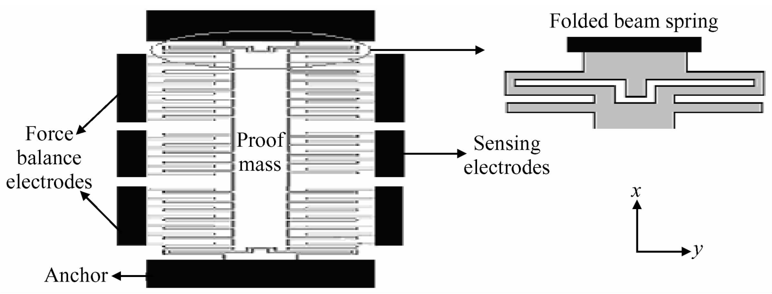

Fig. 1 Sensing element of a force-balance MEMS accelerometer

This paper concentrates on system-level simulation force-balance accelerometers using two approaches. The derivation of the system-level model is elaborated and simulation results are obtained from the implementation of those strategies on a fabricated silicon force-balance MEMS accelerometer. Comparing the two design methods, the better approach can be confirmed.

2 Theoretical principles of force-balance MEMS accelerometers

Capacitive sensing with a closed loop structure based on electrostatic forces is the most important detection method because of its numerous advantages, such as good linearity, low temperature sensitivity, and, especially, compatibility with CMOS readout electronics.

2.1 Operation of force-balance MEMS accelerometers

The force-balance accelerometer system consists of two main parts: the sensing element and the interface electronics. As shown in Fig. 1, the sensing element comprises a proof mass suspended by two folded beam springs on either end, differential sensing capacitances, and the force balance part. Movable fingers are mounted on the mass, and stator fingers are attached to anchors that are also used as electrodes. The movable fingers and the stators establish differential sensing capacitances that are evaluated by a signal pick-off circuit. In the initial state, there is no inertial acceleration and the proof mass rests in the null position. The differential capacitances are equal, and the output voltage is zero. If an inertial acceleration is applied on the proof mass in the x direction, the balance breaks and position variation of the proof mass is sensed, causing relative changes of differential capacitances. As shown in Fig. 2, the capacitances changes are converted into a voltage signal in phase with the carrier signal at the input terminal of the buffer amplifier. The signal is amplified and then sent to the high pass filter. The output of the filter is demodulated, amplified, and then filtered by a low-pass filter. The output of the filter is the output signal of the circuit. Proportional to the output signal, the feedback voltage is continuously applied to the proof mass, and then the deflected proof mass is pulled back to the null position.

2.2 Mathematical model

Mechanically, the sensing element can be approximated by a second order damping spring-mass system. The ordinary differential equation for this system where an inertial force F is applied on the mass M is

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \qquad (1)$$

where B is the damping coefficient, K is the spring constant, and x represents the movement of the mass.

The interface electronics consists of a capacitive sensing circuit (or a readout circuit), which converts mechanical displacements into electrical signal, and an electrostatic feedback part. The capacitive sensing is based on the principle of the capacitive divider. Figure 3 shows the sketch map of the sensing mechanism, including two differential capacitive dividers.

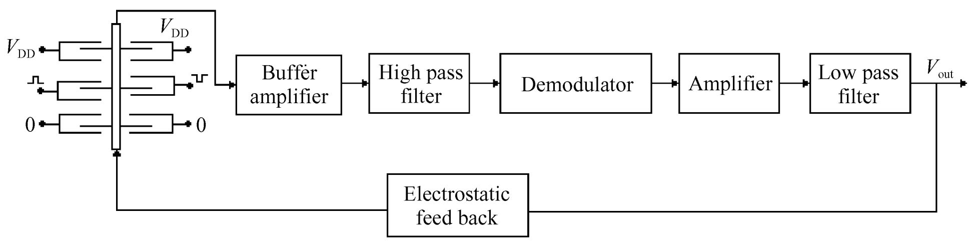

Fig. 2 Interface electronics of a force-balance MEMS accelerometer

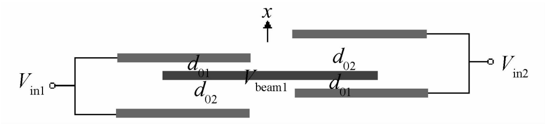

Fig. 3　Sketch map of the sensing mechanism

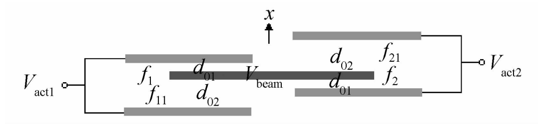

Fig. 4　Sketch map of the force-balance mechanism

The carrier signals (V_{in1} and V_{in2}) that have the same amplitude and reverse phases are applied on the test electrodes. When the proof mass moves in the x direction, the capacitance of one sensing capacitor increases and the capacitance of the other capacitor decreases. Thus, a modulated sense signal (V_{beam1}) proportional to the displacement is generated on the proof mass or movable fingers.

The signal V_{beam1} can be derived from the conversation equation of charge (Eq. 2),

$$\left(\frac{\varepsilon\varepsilon_0 A}{d_{01}-x}+\frac{\varepsilon\varepsilon_0 A}{d_{02}+x}\right)(V_{in1}-V_{beam1})+\left(\frac{\varepsilon\varepsilon_0 A}{d_{01}+x}+\frac{\varepsilon\varepsilon_0 A}{d_{02}-x}\right)(V_{in2}-V_{beam1})=0 \quad (2)$$

where ε is the relative dielectric constant, ε_0 is the dielectric constant, and d_{01} and d_{02} represent the spaces between movable fingers and stator fingers.

The same capacitor structures are used as electrostatic feedback. Figure 4 shows the sketch map of the force-balance mechanism. The actuation voltages (V_{act1} and V_{act2}) are applied on the test electrodes, and the feedback signal is directly applied on the movable electrodes (or the proof mass). Then, attractive forces are generated between the two electrodes. The electrostatic force components (f_1, f_2, f_{11}, f_{21}) are:

$$f_1=\frac{1}{2}(V_{act1}-V_{beam})^2\frac{\varepsilon\varepsilon_0 A}{(d_{01}-x)^2} \quad (3)$$

$$f_2=\frac{1}{2}(V_{act2}-V_{beam})^2\frac{\varepsilon\varepsilon_0 A}{(d_{01}+x)^2} \quad (4)$$

$$f_{11}=\frac{1}{2}(V_{act1}-V_{beam})^2\frac{\varepsilon\varepsilon_0 A}{(d_{02}+x)^2} \quad (5)$$

$$f_{21}=\frac{1}{2}(V_{act2}-V_{beam})^2\frac{\varepsilon\varepsilon_0 A}{(d_{02}-x)^2} \quad (6)$$

where V_{beam} represents the final potential of the proof mass considering the test signal (V_{beam1}) and the feedback voltage.

3　VHDL-AMS simulation

As the extension of VHDL, the VHDL-AMS language possesses capabilities for describing and simulating analog and mixed-signal systems at different levels of abstraction in electrical and non-electrical energy domains. New concepts of VHDL-AMS include 'quantities', which represent continuous-time unknowns in ODEs of the system, and 'simultaneous statements', which describe and implement these ODEs. Any micro-electro-mechanical-system that can be described with ODEs can be easily simulated in VHDL-AMS, together with the electronic circuit that controls the system[6,7].

3.1　Model organization

Based on the mathematical model presented above, the force-balance MEMS accelerometer system can be modeled in VHDL-AMS. There are four different blocks forming the system, as shown in Fig. 5. Each of the blocks is represented in VHDL_AMS code as an entity. The four blocks are connected by five 'quantities' (force_a: the resultant force applied on the proof mass; x: the displacement of the proof mass; V_{out}: the analog output voltage; force_fb: the electrostatic feedback force).

3.2　Simulation results

The entire force-balance MEMS accelerometer system is simulated using VHDL-AMS in the simulation environment SIMPLORER6.0™ from ANSOFT™[8]. Major parameters of the accelerometer

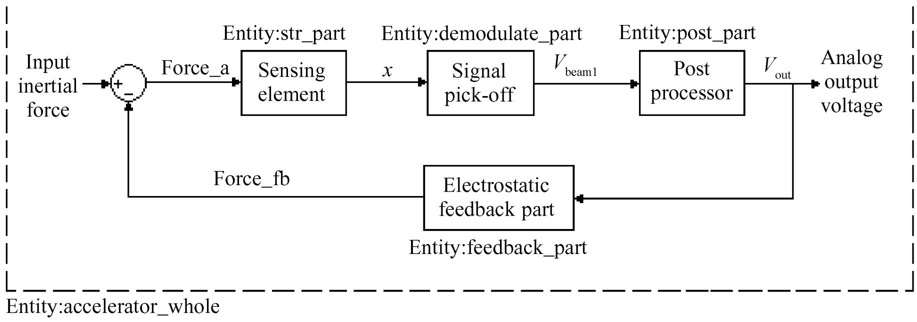

Fig. 5　VHDL_AMS scheme of the entire accelerometer system

system under investigation are listed in Table 1. The value of parameters M, K and C_s are derived from the static analysis of the sensing element in ANSYS™, and the details of the calculation are not presented in this paper. To characterize the system, a step acceleration signal, which is assumed to be zero from 0s to 0.001s and then ascends to the value of 50g, is applied on the proof mass. Figure 6 shows the step responses that include a step change wave of the input inertial force caused by the step acceleration, the displacement of the proof mass, the modulated sense signal, the demodulated voltage signal before low-pass filtering, the feedback electrostatic force, and the final output voltage. The feedback electrostatic force remains close to the input inertial force, so the proof mass vibrates around its null position, which considerably reduces the effect of the nonlinear viscous damping. In addition, the transient characteristic for the sinusoidal input is shown in Fig. 7. The displacement and the output voltage curves are also sinusoidal with the same frequency and different phase delays due to the sensing element and the interface circuit.

Table 1 Major parameters of the accelerometer system

Power supply	18V	d_{01}	3μm
Frequency of carrier signals	1MHz	d_{02}	17μm
Magnifying times of the readout circuit	100	M	0.112735mg
Number of force balance electrodes pairs	24	K	147.11N/m
Number of sensing electrodes pairs	18	V_{act1}	18V
Total sense capacitance C_s	1.08pF	V_{act2}	0V

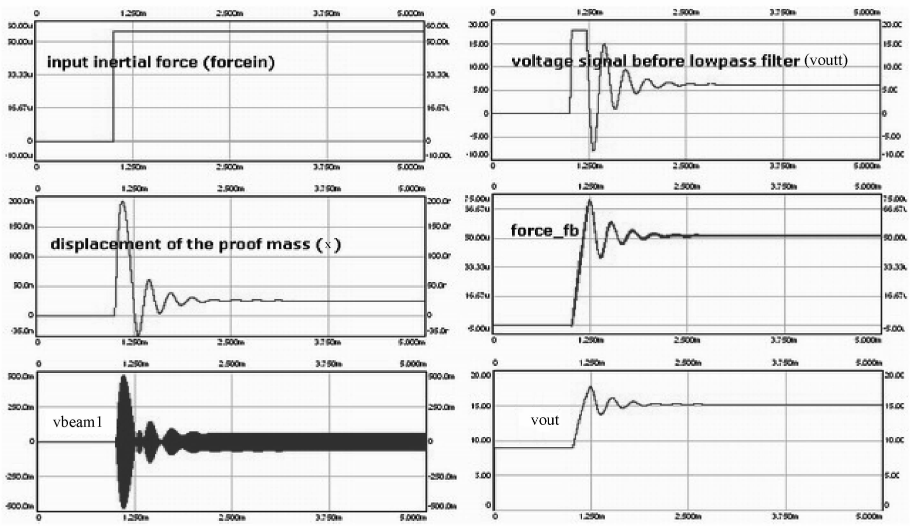

Fig.6 Step responses of the accelerometer system

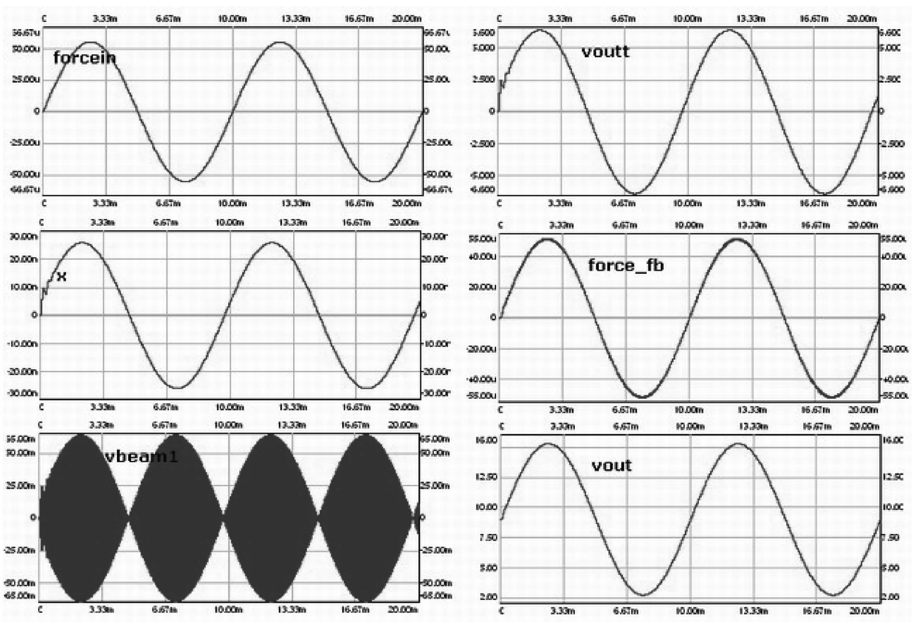

Fig.7 Transient characteristics of the accelerometer system for the sinusoidal input

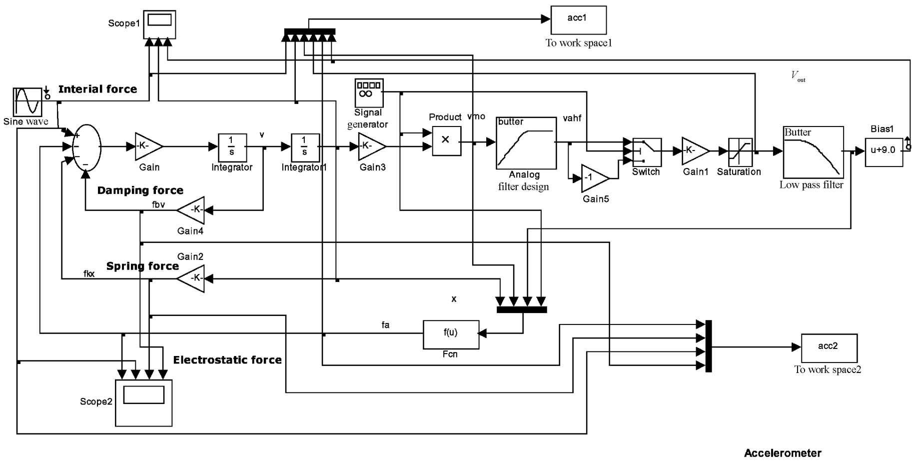

Fig. 8　SIMULINK simulation model of the entire closed-loop accelerometer system

4　SIMULINK simulation

As a comparison, the mathematical model described above is also implemented in SIMULINK™. The simulation model established in SIMULINK is shown in Fig. 8. Using SIMULINK, Figure 9 shows the transient characteristic of the accelerometer system (which has the same parameters listed in Table 1) for the sinusoidal input.

5　Comparison of VHDL-AMS and SIMULINK simulation

Table 2 compares the simulation results of the two modeling approaches. Figure 7, Figure 9, and Table 2 demonstrate that the results are similar. The tiny difference may come from the necessary simplification and linearization of mathematical equations in the process of establishing the SIMULINK simulation model. Compared with the equivalent lumped electrical network established in SIMULINK, using the hardware description language VHDL-AMS to describe and solve the governing ODEs provides higher accuracy and more simulation flexibility. In VHDL-AMS simulation, if the parameters of the system change, the model can be modified easily. However, in SIMULINK simulation, some model parameters have to be recalculated. Meanwhile, VHDL-AMS modeling can account for nonlinearity that cannot be introduced in SIMULINK. Moreover, the blocks, which are built in VHDL-AMS and form the whole system, can be invoked as components in the simulation of other MEMS devices or systems. But, there is not a mature

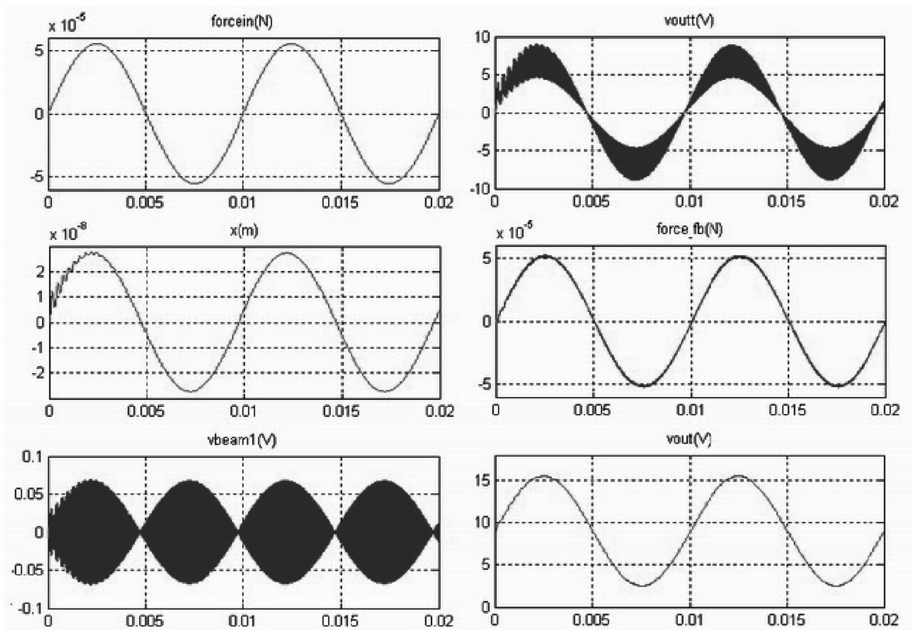

Fig. 9　Transient characteristics of the accelerometer for the sinusoidal input using SIMULINK

Table 2 Comparison of simulation results using different modeling approaches

Input acceleration	V_{out}/V	
	Using VHDL-AMS	Using SIMULINK
1g	9.1249	9.1326
10g	10.2492	10.2948
20g	11.5001	11.5938
30g	12.7543	12.8955
40g	14.0137	14.2021
50g	15.2799	15.5204
Nonlinearity	0.2051%	0.2708%

solver for the VHDL-AMS language accepted by most researchers as it the case for SIMULINK. Moreover, using SIMULINK to model the system is easier and faster. It takes almost 1min to carry out a 0.01-second simulation, while for the same case, the VHDL-AMS simulation requires about 7min.

6 Conclusion

To aid system designers, the system-level modeling and simulation of force-balance MEMS accelerometers is studied. A mathematical model, which involves both the mechanical component and interface electronics, is developed and governing equations are derived. The models presented are implemented in VHDL-AMS and SIMULINK™. Step responses of the accelerometer system and the transient characteristic for the sinusoidal input are illustrated using VHDL-AMS in the simulation environment SIMPLORER6.0™ from ANSOFTTM. The simulation results for SIMULINK are also shown in this paper. The simulation results obtained by the two approaches are compared and show a tiny difference. Using VHDL-AMS is flexible, reusable, and more accurate. But there is not a mature solver developed for the language and this approach requires more time. With SIMULINK, the simulation model can be easily built and quickly evaluated. System designers should choose the proper approach depending on the system requirements.

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力平衡 MEMS 加速度计的系统级建模与仿真*

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摘要： 利用两种不同方法完成力平衡加速度计的系统级仿真.系统级模型参数在已完成的硅基力平衡 MEMS 加速度计结构上提取，数学模型和仿真分别采用 VHDL-AMS 和 SIMULINK TM 来实现.对仿真结果进行比较，两者之间有细微的差别.利用 VHDL-AMS 进行仿真更灵活、准确，模型可重复再用，但是过程中要消耗更多时间.而使用 SIMULINK 可以方便、快捷地建立模型.

关键词： 微机电系统；力平衡加速度计；系统级模型；数学模型

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