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The Driftless Electromigration Theory*
(Diffusion-Generation-Recombination-Trapping Theory)

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Abstract: Electromigration is the transport of atoms in metal conductors at high electronic current-densities which creates voids in the conductors and increases the conductors' electrical resistance. It was delineated in 1961 by Huntington; then modeled by the empirical electrical resistance formula derived by Black in 1969 to fit the dependences of the experimental electrical resistance and failure data on the electrical current density and temperature. Tan in 2007 reviewed 40-years' applications of the empirical Black formula to conductor lines interconnecting transistors and other devices in silicon integrated circuits. Since the first Landauer theory in 1957, theorists have attempted for 50 years to justify the drift force or electron momentum transfer assumed by Black as some electron-wind force to impart on the metal atoms and ions to move them. Landauer concluded in 1989 that the electron wind force is untenable even considering the most fundamental and complete many-body quantum transport theory. A driftless or electron-windless atomic void model for metal conductor lines is reviewed in this article. It was developed in the mid-1980 and described in 1996 by Sah in a homework solution. This model accounts for all the current and temperature dependences of experimental resistance data fitted to the empirical Black formula. Exact analytical solutions were obtained for the metal conductor line resistance or current, $R(t)/R(0) = J(t)/J(0) = [1-2(t/\tau_\alpha)^{1/\alpha}]^{-1/2}$, in the bond-breaking limit with $\alpha = 1$ to 2 and diffusion limit with $\alpha = 2$ to 4, from low to high current densities, where τ_α is the characteristic time constant of the mechanism, containing bond breaking and diffusion rates and activation energies of the metal.

Key words: electromigration; driftless void model; empirical Black formula; diffusion-generation-recombination-trapping

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1 Introduction

Thin and narrow aluminum and other composite metal conductor lines are employed to electrically connect the transistors and other devices (diodes, capacitors, inductors and resistors) in silicon semiconductor integrated circuits. The atoms of the metal conductor lines migrate at high electrical current densities leaving behind voids in the metal conductor lines. These voids increase the electrical resistance of the metal conductor lines and the current densities. Heat generation, from the higher electrical current density and higher resistance, increases the temperature which increases the migration rate of the atoms of the metal conductors. This regenerative feedback eventually 'burns' open the metal lines, disconnecting the transistors and devices, and causing the intergrated circuit to fail.

Electromigration (EM) is the term employed to describe the transport or migration of atoms in metals and metal lines at high electrical current-density (> $100kA/cm^2$) that creates or leaves behind voids in the metal and metal lines. It was delineated in 1961 by Huntington[1] in gold wires. In 1969, Black[2] gave an empirical equation to fit the experimental data of the electrical resistance of metal interconnect lines in semiconductor integrated circuits, as a power law of the electron-current density, J^α, and sample temperature, T, with a thermal activation energy, E_a. The time to failure, TTF, defined as 10% rise of the electrical resistance of the line, was defined by the Black formula: $TTF = AJ^{-\alpha}\exp(+E_a/k_BT)$ where A, α and E_a are adjustable parameters to fit the experimental data. Tan and Roy recently (2007) reviewed the 40-year applications of the Black formula[3]. Since the first Landauer theoretical analysis in 1957[4], theorists have attempted for 50 years to justify Black's drift force of the electrons, known as the electron wind

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force, to move an atom or ion, which was used to derive the Black formula. Landauer concluded in 1989[5] that the electron wind force is untenable even considering the most fundamental and complete many-body quantum transport theory.

In 1984, Sah obtained a resistance formula without any drift current while starting a PhD student on a thesis research project[6] to experimentally verify the formula. The details of the derivation algebra of this electrical resistance formula of a metal conductor line was given in 1996 as a solution for a homework problem in the Solution Manual of his undergraduate textbook for the device core-course[6]. This resistance formula accounts for both the current density and the temperature dependences of the experimentally observed electrical resistance rise of a conductor line that has been empirically fitted to the Black formula. The 1996-Sah derivation details used a generic void model, i. e. a one-dimensional scaling model that is a crude shape-independent approximation to the realistic random geometry of the voids. The transport theory of metal atoms excludes, deliberately at the onset of the derivation, any atomic transport due to the drift of the atomic ion in an electric field. This implies that the ion core of the metal atom is completely shielded by its valence electron cloud to provide a neutral atom which cannot be polarized much by the applied electric field. This is consistent with the inability to establish an electric field in metals due to the very high concentration of valence or conduction-band electrons in metals, which is the key in Landauer rejection [5] of the electron-wind force or drift force on the atomic ion. Thus, the only atomic migration or transport in the 1996-Sah model is via the diffusion of the neutral atom, or its ionic core and its spherical symmetrical valence electron cloud together. But the unique feature in this 1984-1996-Sah model is the inclusion of the "generation-recombination-trapping (GRT)" of conduction-band electrons and valence-band electrons or valence-bond electrons, both in the bulk of the conductor line near a void and also at the internal surface of the void. In this paper, we shall review this 1996-Sah windless-driftless model and we shall also trace through the physics-based arguments that are employed in the algebraic derivation of the resistance formula and the time to failure (TTF) formula defined as a 10% rise of the electrical resistance of the conductor.

2 Derivation of 1996-Sah Formulas of Electrical Resistance and Time-To-Failure

In this derivation, the interconnection metal conductor line in silicon semiconductor integrated circuit application was employed as the scheme. A cross sectional-solid view of the silicon metal-oxide-semiconductor field-effect-transistor is given in Fig. 1 [6] which shows the two metal lines connecting the gate and drain of the transistor to the preceding and following or other transistors. The atomic picture used to derive our driftless formula of the electrical resistance of the metal interconnect line is given in Fig. 2[6]. The critical dimensions are drawn to scale in Fig. 2 and are described in the figure caption of Fig. 2. The vertical axis is the Y-axis which is the width direction of the metal line. The horizontal axis is the X-axis which is the length direction of the metal line and the direction of the electrical current. The crystalline metal is employed for the metal line. The face center cubic cell of the aluminum crystal is shown in Fig. 3.

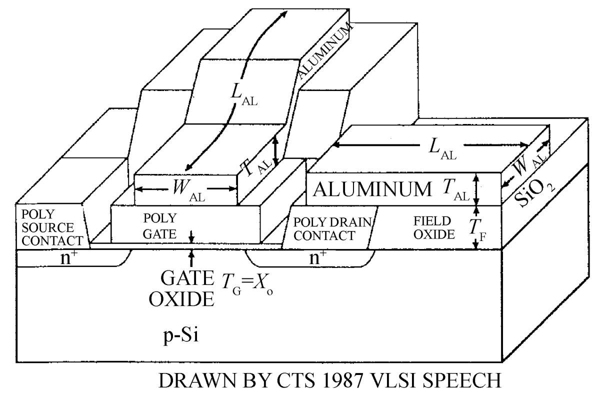

Fig. 1 The cross sectional-solid view of an example n-inversion channel silicon MOS Field-Effect Transistor showing the two aluminum metal conductor lines labeled L_{AL}, which interconnect the gate and the drain of this MOSFET to the other transistors in a silicon integrated circuit. The aluminum interconnect line to the source is not shown. From [6].

There are two rate limiting analytical solutions of the transport of the metal atoms in the metal line: (a) metal-bond-breaking rate limited and (b) diffusion limited by diffusion of the activated metal atoms in the metal line, either via the bulk vacancy mechanism (self-diffusion) or along the grain boundaries (surface diffusion). Bulk vacancy pathway is modeled.

The transport or rate equations are listed below. In the flux equation, (1), **we drop the atomic and atomic-ion-core drift current terms at the onset,** $\mu_{mx}E_X m^x + \mu_{m+} E_X m^+$, where $m = m^x + m^+$, m^x is the density of the neutral atoms and m^+ is the density of the ionized atoms. These are the fudge-factor terms assumed by all previous momentum-transfer/electron-wind-force models, using current-proportional fudge factors. There are two basic considerations that diminish the ion drift term, $\mu_{m+} E_X m^+$, due to the electric field E_X. (i) The electric field is very small, vanish-

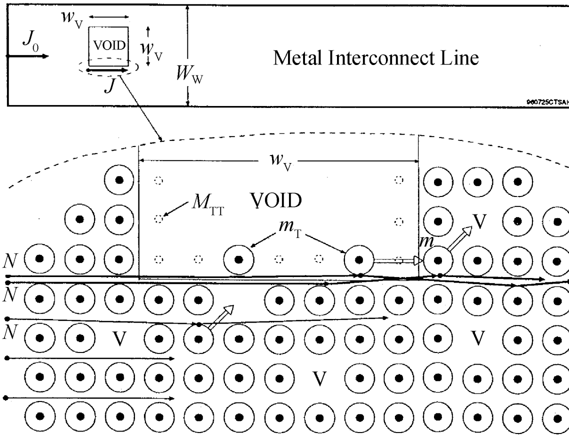

Fig. 2　The cross section view of an ideal void model for electromigration in a metal interconnect line. The void in the metal interconnect line is enlarged in the lower figure, which is approximately to scale. From 1996-Sah[6] with additional details on the characteristic numbers. Small dots = electrons. Solid arrows = electron trajectories. Small broker circles = Aluminum atom trapping sites on the interior surface of the void. Circled dots = Aluminum Ion Core = Al^{+3} with radius of r_{AL+3} = 0.26Å (radii from Table 162.2 of [7]). m^{+3} is the density or number of the Al^{+3} core. Large circles = valence electron orbits in isolated Aluminum in free space = 1.044Å to 1.312Å (radii from Table 162.2 of [7]). $a_{AL-FCCube}$ = 4.05Å = size of the edge of the Face Centered Cube of the aluminum crystal[8]. Interatomic core distance in FCCube d_{Al-Al} = 2.86Å[8]. Cubic average aluminum volume density and aluminum atom separation $= N_{AL} = 4/a^3 =$ 6.02×10^{22} Al/cm^3 and $a_{AL-AL} = N_{AL}^{-3}$ = 2.55Å. N = Average aluminum Valence Electron Concentration and electron-electron separation $= 3N_{AL} = 1.81\times10^{23}$ cm^{-3} and $a_{electron-electron} = N^{-3}$ = 1.77Å. V = Aluminum vacancy = missing Aluminum atom = missing Aluminum Ion Core with its three valence bond electrons or three valence electrons in the Debye sphere of Debye length or radius of $\lambda_{Debye-screening-length}$ = 0.576Å. $\lambda_{electron-mean-free-path}$ = 150Å. Open-arrow = Track of Aluminum atom = inverse track of Aluminum vacancy. J_0 is the average electrical current density per unit cross-sectional area of the conductor line, far from the location of the void, V. w_V is the physical width of the void, which increases during current stress towards the two width edges of the conductor line with linewidth of W_W. When w_V reaches W_W the conductor line is opened and its electrical resistance is infinite. TTF is the defined as the time when the electrical resistance increases by 10% or $w_V(t = \text{TTF}) = W_W/10$. M_{TT} is the total number metal atom site on the interior surface of the Void, which is the sum of the occupied and the unoccupied interior surface sites, each by a metal atom. m_T is the number of interior surface site of the Void which is occupied by a metal atom, or the number of trapped metal atoms (adatom) on the interior surface of the Void. m is the total number of mobile or activated metal atoms, each is 'jumping' into its adjacent atomic vacancy site V by the vacancy diffusion mechanism with an atomic diffusivity of D_m and thermal activation energy of E_A which is the potential barrier height between the vacancy and its adjacent aluminum atoms. V is the volume density of the vacancy sites. N is the volume concentration of the conduction electrons in the metal line.

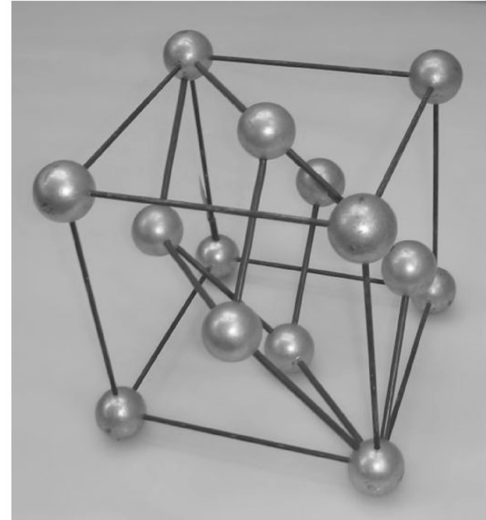

Fig. 3　The crystalline metal model for the metal conductor line. Picture shows the Face Centered Cubic Unit Cell of the Aluminum metal with a cell edge of a = 4.05Å and interatomic distance of d_{AL-AL} = 2.86Å. The aluminum atomic ion cores, Al^{+3}, represented by the silver balls, has a radius of r_{AL+3} = 0.26Å based on the core electron charge distribution[7] and the picture is 0.42Å, about twice larger, hence not to scale. The Debye length or radius of the conduction band or valence shell electrons is λ_{Debye} = 0.576Å, which gives the tightly bound electron distribution or spherical Debye electron cloud around the aluminum core when the aluminum atom moves to the adjacent vacancy site or the vacancy mechanism of diffusion.

ingly small, in a metal with its very high concentration of conduction electrons, such as $N = 1.81\times10^{23}$ cm^{-3} in Aluminum[7]. (See the captions of Figs. 2 and 3 for the numbers.) (ii) The neutral aluminum atom, with ion core radius of a_{AL+3} = 0.26Å, and a conduction-electron Debye screening length or Debye Cloud radius of λ_{Debye} = 0.576Å～0.58Å, is not likely to be ionized by the small applied electric field. The ionization probability can also be estimated by the cubic separation of the electron and the Al^{+3} with a $d_{AL\text{-}electron} = a_{AL\text{-}AL} - a_{electron\text{-}electron}$ = 2.55Å − 1.77Å = 0.68Å, which is of the same order as the Debye screening length, λ_{Debye} = 0.58Å, based on a spherical electron distribution. The electric field in metal is rather small to even significantly polarized the tightly bound electrons to the aluminum ion core, $Al^{+3} + 3e^-$, to make the first term from the drift of the neutral aluminum, $\mu_{mx}E_X m^x$, significant. These are consistent with Landauer's rejection of the electron-wind model based on his many-body (many-electron) considerations[5] that are akin to the Debye electron screening of the atomic ion core Al^{+3} by the high density of conduction electrons in aluminum.

It is interesting and useful to point out that a similar consideration as above for metals can be made also for semiconductors with low or intrinsic conductiv-

ities, and with high conductivity from high concentrations of impurity or small energy gaps. The Debye screening length or radius would be much larger due to lower electron and hole concentrations, enable the polarization and ionization of the host and impurity atoms by the applied electric field. Furthermore, semiconductor devices contain fabricated composition changes, such as a p/n junction, which gives high build-in internal electric field. Thus, the drift under the electric field of both the ionized and also the neutral but polarized host and impurity atoms are possible, and in fact are causes of the long term, irreversible degradations of the reliability of semiconductor devices.

The atomic transport equations and the current and resistance equations are now listed below in equations (1) to (8). Here the sum of the polarized but neutral atom and the ionized atom gives the total atom concentration, $m = m^x + m^+$. We drop the ions, m^+ and for simplicity of notation, we use m for the neutral atoms. Thus, $m = m^x + m^+ \simeq m^x$. This notation simplification is reflected by Equation (1) given below in which the two drift and one ion diffusion terms are dropped.

$$F_M = -D_{mx}\partial m^x/\partial x + \mu_{mx}E_X m^x - D_{m+}\partial m^+/\partial x + \mu_{m+}E_X m^+$$

$$= -D_{mx}\partial m^x/\partial x + 0 - D_{m+}\partial m^+/\partial x + 0 \quad \text{(electron - windless = driftless)}$$

$$\equiv -D_m \partial m/\partial x \quad \text{(neutral atom diffusion only)} \tag{1}$$

$$\partial m/\partial t = -\partial F_M/\partial x + e_m m_T - c_m(M_{TT} - m_T)m \quad \text{(neutral atom continuity)} \tag{2}$$

$$= +D_m \partial^2 m/\partial x^2 + (e_m + c_m m)m_T - c_m M_{TT} \quad \text{(diffusion + trapping)} \tag{3}$$

$$\partial m_T/\partial t = -(e_m + c_m m)m_T + c_m M_{TT} \quad \text{(trapping)} \tag{4}$$

where

$$e_m = e_{m1}J \text{ or}$$

$$e_m = e_{m2}J^2 \tag{5}$$

$$J = J_0/[1-(w_V/W_W)] \equiv J_0/(1-\omega) \quad \text{where } \omega \equiv (w_V/W_W) \tag{6}$$

$$R/R_0 = 1/[1-(w_V/W_W)] \equiv 1/(1-\omega) \tag{7}$$

$$(R/R_0) - 1 = \Delta R/R_0 = \omega/(1-\omega) = 0.1 \text{ defines the Time to Failure or TTF} \tag{8}$$

Here the D's and μ's are the respective diffusivities and mobilities. e_m is the emission rate of the atoms trapped at the void surface trapping sites. c_m is rate of capture of mobile atoms by the unoccupied surface traps $(M_{TT} - m_T)$ at the interior surface of the void.

The two alternative current dependent forms of Equation (5) give the release or emission rate of the metal atoms trapped at the void's surface, m_T, due to the interaction or collision of the conduction electron with the valence 'bond' electron (or the electron in the Debye electron cloud) to break the metallic bond, as shown by the solid arrows in Fig. 2. This activates or releases the trapped metal atom, m_T, from its surface trapping site, M_{TT}. Thus, this emission rate is proportional to electron density, N, or the electron current density, $e_m = e_{m1}J$, since the electron number is proportional to the electron current density, $J = qv_d N$ where v_d is the drift velocity. At higher current densities, we have a J^2 dependence, $e_m = e_{m2}J^2$ because the drifting electrons also enhance vacancy migration to provide more vacancy sites for the released "mobile atoms" to move into.

Analytical solutions of (1) to (8) can be obtained for the two asymptotic cases of void-surface-bond-breaking rate limited and bulk-diffusion-limited. These are listed in the following subsections.

2.1 Void-Surface Bond-Breaking Rate Limited

In this case, we have in equation (3) the following inequality,

$$D_m \partial^2 m/\partial x^2 \ll (e_m + c_m m)m_T - c_m M_{TT} \tag{9}$$

therefore (1) to (8) reduce to the following equations and their analytical solution. Here k is a proportionality constant relating the width of the void, w_V, and the number of atoms, m, moved from the interior surface of the void which increases the size of the void.

$$\partial w_V/\partial t = k\partial m/\partial t = k(e_m + c_m m)m_T - kc_m m M_{TT} \tag{10}$$

$$= k(e_m)m_T \tag{11}$$

$$= ke_{m1}J \times M_{TT}\exp(-E_A/k_B T) \tag{12}$$

$$\partial(w_V/W_W)/\partial t = k(e_{m1}J_0/W_W)M_{TT}\exp(-E_A/k_B T)/[1-(w_V/W_W)] \tag{13}$$

$$= \partial\omega/\partial t = (1/\tau_1)/(1-\omega) \tag{14}$$

$$\int dt/\tau_B = t/\tau_1 = \int[1-(w_V/W_W)]d(w_V/W_W) = \int(1-\omega)d\omega \tag{15}$$

$$= \{1-[1-(w_V/W_W)]^2\}/2 = \omega(1-\omega/2) \tag{16}$$

$$1/\tau_1 = k(e_{m1}M_{TT}/W_W)J_0\exp(-E_A/k_B T) \tag{17}$$

$$(R/R_0)-1=\omega/(1-\omega) \quad (18)$$
$$t_\infty/\tau_1=1/2 \quad \text{Breakdown Time at } R=\infty \quad (19)$$
$$(R/R_0)-1=\Delta R/R_0=t/\tau_1\propto J_0 \quad \text{Short Time and Low } J_0 \quad (20)$$
$$\text{TTF}=t(\Delta R/R_0=0.1)=\tau_1/10 \quad (21)$$
$$=(W_W/10ke_{m1}M_{TT})(J_0)^{-1}\exp(+E_A/k_BT) \quad (22)$$

At high current densities, when $e_m=e_{m2}J^2$ then (24)

$$\text{TTF}=(W_W/10ke_{m2}M_{TT})(J_0)^{-2}\exp(+E_A/k_BT) \quad (25)$$

The general solution of the resistance as a function of time, from combining (15) to (16), is

$$R/R_0=[1-2(t/\tau_1)]^{-1/2} \quad (26)$$

which has initial linear rise with stress time and which breaks up at $t=\tau_1/2$ as indicated by (19).

2.2 Bulk Diffusion Rate Limited

In this case, detrapping of m_T or generation of m is much faster than diffusion of m. Thus, m and m_T on the void's surface, $x=0$, has reached the steady-state as soon as the current is turned on. The boundary condition at $x=0$ and the kinetics in the region $x>0$ are given below.

At $x=0$

$$e_m m_T-c_m m(M_{TT}-m_T)=0 \text{ with } m_T(x=0,t)=M_{T0} \text{ and } m(x=0,t)=M_0 \quad (27)$$
$$M_0=e_mM_{T0}/[c_m(M_{TT}-M_{T0})]=(e_m/c_m)(M_{T0}/M_{TT}) \quad (28)$$
$$=(e_{m1}J_0/c_m)[1/(1-\omega)]\exp(-E_A/k_BT) \quad (29)$$

In $x>0$

$$\partial m/\partial t=D_m\partial^2m/\partial x^2 \quad (30)$$
$$m(x,t)=M_0\text{erfc}[x/2(D_mt)^{1/2}] \quad (31)$$
$$F_M(x=0,t)=-D_m\partial m/\partial x|_{x=0}=D_mM_0/(\pi D_mt)^{1/2} \quad (32)$$
$$Q_M(x=0,t)=\int F_M(x=0,t)\mathrm{d}t=2M_0(D_mt/\pi)^{1/2} \quad (33)$$

Let $w_V(t)=kQ_M(x=0,t)$ where k is proportionality constant (34)

Let $D_m=D_{m0}\exp(-E_{dif}/k_BT)$ Vacancy or Grain Boundary (35)

Let $M_{T0}=M_{TT}\exp(-E_A/k_BT)$ Thermally activated void surface atoms (36)

$$\mathrm{d}(w_V/W_W)/\mathrm{d}t=(\mathrm{d}/\mathrm{d}t)(kQ_M/W_W)\equiv \mathrm{d}\omega/\mathrm{d}t=(k/W_W)F_M(x=0,t) \quad (37)$$
$$\int(1-\omega)\mathrm{d}\omega=\omega(1-\omega/2)\equiv(t/\tau_2)^{1/2} \quad (38)$$
$$=2\text{k}(\text{e}_{m1}\text{J}_0/\text{c}_m\text{W}_M)\,exp[-(\text{E}_A+\text{E}_{Dif}/2)/\text{k}_B\text{T}](\text{D}_{m0}/\pi)^{1/2}\int \mathrm{d}t/t^{1/2} \quad (39)$$
$$=2k(e_{m1}J_0/c_mW_M)\exp[-(E_A+E_{Dif}/2)/k_BT](D_{m0}/\pi)^{1/2}\,2t^{1/2} \quad (40)$$
$$\tau_2\equiv(\pi/D_{m0})(c_mW_M/4ke_{M1})^2J_0^{-2}\exp[+(2E_A+E_{Dif})/k_BT] \quad (41)$$
$$t=[\omega(1-\omega/2)]^2\tau_2 \quad (42)$$
$$\Delta R/R_0=(R-R_0)/R_0=\omega/(1-\omega) \quad (43)$$
$$t_\infty=t(\omega=1)=\tau_2/4 \quad \text{Breakdown Time at } R=\infty \quad (44)$$
$$\Delta R/R_0=\omega=(t/\tau_2)^{1/2}\propto J_0 \quad \text{Short Time and Low } J_0 \quad (45)$$
$$\text{TTF}=t(\Delta R/R_0=0.1)=\tau_2/10 \quad (46)$$
$$=\text{TTF}_{20}\times(J_0)^{-2}\times\exp[+(2E_A+E_{Dif})/k_BT] \quad (47)$$

At high current densities, enhanced vacancy generation by the high electron current gives a higher bond breaking or emission rate of trapped atoms, $e_m=e_{m2}J^2$ (48)

then $\text{TTF}=\text{TTF}_{2\infty}\times(J_0)^{-4}\times\exp[+(2E_A+E_{Dif})/k_BT]$ (49)

The exact solution valid for all times can be obtained from (42) and (43) and is given by

$$R/R_0=[1-2(t/\tau_2)^{1/2}]^{-1/2} \quad (50)$$

3 The Compact Model

The amazingly simple asymptotic metal line resistance formulas, limited by the bond-breaking rate and diffusion rate, given respectively by (26) and (50), immediately suggest the following analytical compact model formula to cover the entire range of both bond breaking and diffusion limited mechanisms. These are summarized and compared below.

$$J(t)/J_0=R(t)/R_0=1/[1-(w_V/W_W)]$$

$$= [1-2(t/\tau_\alpha)^{1/\alpha}]^{-1/2} \quad \text{(For } \textbf{The Compact Model}, \ \alpha = \text{adjustable)} \tag{51}$$

$$t_\infty = (1/2)^\alpha \tau_\alpha \quad \text{(Breakdown time Infinite Resistance)} \tag{52}$$

$$\mathrm{TTF}(10\%) = [1-(1/1.10)^2]^\alpha \tau_\alpha \quad \text{(Time To Failure 10\% rise of resistance)} \tag{53}$$

$$\mathrm{TTF}(20\%) = [1-(1/1.20)^2]^\alpha \tau_\alpha \quad \text{(Time To Failure 20\% rise of resistance)} \tag{54}$$

$$\mathrm{TTF}(50\%) = [1-(1/1.50)^2]^\alpha \tau_\alpha \quad \text{(Time To Failure 50\% rise of resistance)} \tag{55}$$

$$\mathrm{TTF}(100\%) = [1-(1/2.00)^2]^\alpha \tau_\alpha = (3/4)^\alpha \tau_\alpha \quad \text{(Time To Failure 100\% rise of resistance)} \tag{56}$$

Empirical Fit Parameters:

$$\tau_\alpha = A_\alpha J_0^{-\alpha} \exp(+E_\alpha / k_B T) \tag{57}$$

$$\alpha = 1 \text{ to } 4 \tag{58}$$

Bond Breaking Limited $\alpha = 1$ to 2

$$\tau_\alpha = \tau_1 = (W_W / k M_{TT})(1/e_m) \exp(+E_A / k_B T) \tag{59}$$

$$= \tau_{1L} = (W_W / k M_{TT})(1/e_{m1}) J_0^{-1} \exp(+E_A / k_B T) \quad \text{Low } J_0 \tag{60}$$

$$= \tau_{1H} = (W_W / k M_{TT})(1/e_{m2}) J_0^{-2} \exp(+E_A / k_B T) \quad \text{High } J_0 \tag{61}$$

$$R(t)/R_0 = [1 - 2(t/\tau_1)]^{-1/2} \quad \text{All times} \tag{62}$$

$$= t/\tau_1 \propto J_0 \quad \text{Short time } t \ll \tau_1 \tag{63}$$

$$R(t = t_\infty)/R_0 = \infty \text{ at } t = t_{1\infty} = \tau_1/2 \quad \text{Breakdown time} \tag{64}$$

Diffusion Limited $\alpha = 2$ to 4

$$\tau_\alpha = \tau_2 = (\pi / D_{m0})(c_m W_W / 4 k e_m)^2 \exp[+(2E_A + E_{Dif}) / k_B T] \tag{65}$$

$$= \tau_{2L} = (\pi / D_{m0})(c_m W_W / 4 k e_{m1})^2 J_0^{-2} \exp[+(2E_A + E_{Dif}) / k_B T] \quad \text{Low } J_0 \tag{66}$$

$$= \tau_{2H} = (\pi / D_{m0})(c_m W_W / 4 k e_{m2})^2 J_0^{-4} \exp[+(2E_A + E_{Dif}) / k_B T] \quad \text{High } J_0 \tag{67}$$

$$R(t)/R_0 = [1 - 2(t/\tau_2)^{1/2}]^{-1/2} \quad \text{All times} \tag{68}$$

$$= (t/\tau_2)^{1/2} \propto J_0 \quad \text{Short time } t \ll \tau_2 \tag{69}$$

$$R(t = t_\infty)/R_0 = \infty \text{ at } t = t_{2\infty} = \tau_2/4 \quad \text{Breakdown time} \tag{70}$$

The initial rise of the resistance is proportional to the current density, J_0, independent of the rate limiting mechanism, as shown by (63) and (69). The mechanism determines only the initial time dependence, linear in time, t, as shown by (63) if bond breaking is rate limiting, and square-root in time, $t^{1/2}$, as shown by (69) if diffusion is rate limiting. As anticipated, the time to failure (10% increase in resistance in order to be in the linear range) increases linearly with the line width, W_w, if bond-breaking is rate limiting, and parabolically with the line width, W_w^2, if diffusion is rate limiting. The material properties in the time constants should provide the quantitative clues for better materials, processing steps, and geometries, to attain longer lasting interconnect lines, complementing if not replacing the empirical rules accummulated for four decades by semiconductor integrated circuit engineers

4 Conclusions

The resistance and time-to-failure (defined as 10% rise of resistance) formulas of a metal conductor line are derived based on diffusion transport of the neutral metal atoms via the vacancy pathway and the trapping or generation-recombination-trapping of the metal atoms at the atom trapping sites on the interior surface of the voids. These 1986-1996-Sah formulas are identical in form to the empirical 1969-Black formula. $\mathrm{TTF} = AJ^{-\alpha} \times \exp(+E_\alpha / k_B T)$ where A, α and E_α were empirical parameters introduced by Black to be adjusted to fit the experimental data, which have varied over a wide range, $\alpha = 1$ to 4 or more and $E_\alpha = 0.5$eV or less to 1.5eV or more. These parameters in the 1986-1996-Sah formulas are fundamental materials properties of the metal conductor, therefore, they provide the quantitative clues to search for metals and compound conductors such as silicides that would give higher endurance against electromigration failure. In addition, they provide fundamental inputs to design physical structures for higher endurance. This materials and device physics based resistance formula advances the technology of electromigration into an engineering science from 40 years of empirical recipe that was started by the 1969-Black formula. Refinements of the underlying physics from the simple constant rate coefficients are expected, such as Debye screening, and valence-conduction electron concentration or local current density dependent bond-breaking and bond-interaction or electron-hole interaction rates, all of which could give further insights on the device design and processing to enhance the endurance of metal interconnect lines in semiconductor integrated circuits.

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References

[1] R. B. Huntington and A. R. Grone, "Current-Induced Marker Motion in Gold Wires," J. Phys. Chem. Solids, 20(1/2), 76－87, January 1961. See also the references cited by these authors.

[2] James R. Black, "Electromigration-A Brief Survey and Some Recent Results," IEEE Trans. ED-16(4), 338－347, April 1969. See also the references cited by Black.

[3] Cher Ming Tan and Arijit Roy, "Electromigration in ULSI Interconnects," Materials Science and Engineering: R: Reports, 58(1－2), 1－76, October 2007. There are 327 references cited and analyzed in this comprehensive review. See also Electromigration in ULSI Interconnects, World Scientific Publishing Company, to be published in winter, 2009.

[4] Rolf Landauer, "Spatial Variation of Currents and Fields Due to Localized Scatters in Metallic Conduction," IBM Journal of Research and Development, 1, 223－231, July 1957. See also references cited by Professor Landauer.

[5] Rolf Landauer, "Comment on Lodder's 'Exact' Electromigration Theory," Solid State Communication, 72(9), 867－868, 2 October 1989. See also references cited in this article and in earlier Landauer papers. There were also many post-1989 theory articles which Professor Landauer no longer responded, and they all failed to overcome the Landauer rejections of the presence of any electron-wind or atomic-ion drift force.

[6] Chih-Tang Sah, Fundamentals of Solid-State Electronics, Solution Manual, World Scientific Publishing Company, Singapore, 1996. For the cross-sectional-solid view of the example-MOSFET, see Figure 940.1 on page 148, in Section 940 ELECTROMIGRATION-Introduction. For the empirical Black formula, see equation (941.1) on page 149 in Section 941 Empirical Characteristics of Electromigration. For the original atomic picture and analytical solution of the void model and a compact list of step-by-step algebraic derivations of the resistance and TTF formulas, see Problem P.941.1 and Figure P941.1 on pages 174 to 176 in Section 999, PROBLEMS AND SOLUTIONS. This solution was obtained by Sah in June 1984, when he started Ann Chen's Ph. D. (1984-1988) research project in Electrical and Computer Engineering at the University of Illinois in Urbana-Champaign. The contract and grant proposals to undertake an investigation of fundamental electromigration physics failed to win support from SRC and US-NSF due to lack of fund.

[7] Chih-Tang Sah, Fundamentals of Solid-State Electronics, World Scientific Publishing Company, Singapore, 1991. For the atomic ion core radii see Section 162 Many-Proton and Many-Electron Atoms on pages 87 to 92 and Table 162.2 Radii of One-Electron Radial Charge Distribution on page 91. For valence or conduction electron concentration in metal and semiconductor crystals, see Table 413.1 on page 356.

[8] Chih-Tang Sah, Fundamentals of Semiconductor Materials Physics, Lecture Notes of graduate course Physics-EE 435, Conduction of Electricity in Solids, taught by Sah during 1961-1988 at the Department of Physics and Department of Electrical and Computer Engineering, University of Illinois in Urbana-Champaign, and Lecture Notes of the advanced graduate course EEL-7936, Fundamentals of Semiconductor Materials Physics, taught by Sah at the University of Florida, 1988-2008. To be published by the World Scientific Publishing Company.

无飘移电迁移理论(扩散产生复合俘获理论)*,**

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摘要： 电迁移是在高电流密度下金属导体中原子输运，它在导体中产生空位，增加导体的电阻. Huntington 于 1961 年描述电迁移；Black 于 1969 年推导经验电阻公式用以模拟电迁移，给出电阻和失效实验数据对电流密度和温度的依赖关系. Tan 于 2007 年总结 Black 经验公式在连接晶体管和其他硅集成电路器件的导线方面四十年运用情况. 自 1957 年第一个 Landauer 理论以来，理论工作者用 50 年试图论证飘移力，即 Black 假设的电子动量转移，又称电子风力，作用到金属原子和离子上，可使它们移动. Landauer 于 1989 年断定，即使运用最基本和完备的多体量子输运理论，电子风力也站不住脚. 本文回顾用于金属导线的无飘移或无电子风力原子空位模型. 萨在八十年中提出这模型，在 1996 年以课外作业题解的形式描述这模型. 该模型解释了用 Black 经验公式拟合的电阻实验数据的电流和温度依赖关系. 得到精确解析方程，描述两种极限情形下金属导线电阻或电流，$R(t)/R(0)=J(t)/J(0)=[1-2(t/\tau_\alpha)^{1/\alpha}]^{-1/2}$；从低到高电流密度，价键断裂情形 $\alpha=1\sim2$ 和原子扩散情形 $\alpha=2\sim4$. 其中 τ_α 是电迁移机理的时间特征常数，含有价键断裂率，原子扩散系数和激发能.

关键词： 电迁移；无飘移空位模型；BLACK 经验公式；扩散产生复合俘获
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