## A Novel High-Speed Equalizer for QAM Signals\*

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Abstract: A high-speed equalizer based on a new algorithm; stop-and-go-DD-LMS CMA (SGLMS-CMA) for quadrature amplitude modulation (QAM) signals is presented. It integrates conventional constant modulus algorithm (CMA) and decision-direct least-mean-square (DD-LMS) under stop-and-go principle. Matlab simulations indicate that, compared with conventional CMA, the new algorithm performs five times faster in convergence speed,  $3\sim5dB$  improved in rudimental mean square error (MSE), 82% decreased in operation complexity and can correct a final phase ambiguity. As to the equalizer block in the system, synthesis results show that the SGLMS-CMA+DD-LMS equalizer's hardware consumption is only 5% greater than the CMA+DD-LMS equalizer's. Finally by using SMIC 0.  $18\mu m$  library to synthesis, the new equalizer is embedded into QAM demodulation chip, and test results show that the new equalizer acts better.

Key words: CMA; DD-LMS; equalizer; convergence speed; phase ambiguity

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### 1 Introduction

In order to remove inter-symbol interference (ISI) caused by multi-paths in band-limited time dispersive channels, an adaptive equalizer is widely used in communication systems. The algorithms used by equalizers can be sorted by working mode into two kinds: blind and non-blind. CMA is most popular in blind equalizers. It has no need for training sequence, no sensitivity to frequency ambiguity and phase ambiguity, and is easy to implement. But it is slow to be convergent, only has effect on symbols' amplitude and holds a rudimental phase offset. When CMA is convergent, the rudimental MSE is comparatively great. DD-LMS algorithm belongs to non-blind algorithms. It has effect on symbols' amplitude and phase, small rudimental MSE, and it is sure to be convergent with the steady channel. As its MSE is dependent tightly on decision values, DD-LMS should be used on condition that the decision values are basically right. Stop-and-go algorithm was proposed by Picchi and Prat in 1987<sup>[1]</sup>. Its basic principle is to stop updating adaptive taps while the reliability of decision output is not high enough. This algorithm is simply to implement, fast to be convergent with small rudimental MSE, but an uncertain parameter  $\beta$  exits in its cost function, which is fixed by experiment, and difficult to obtain optimal value. This limits stop-and-go algorithm's application.

According to the characteristics of the algorithms above, the equalizer use CMA and DD-LMS with decision feedback equalization (DFE) structure in high-capable QAM receiver, as shown in Fig. 1. It is called CMA + DD-LMS equalizer, which can receive data with low error-rate combining carrier-phase recovery (CPR) loop.

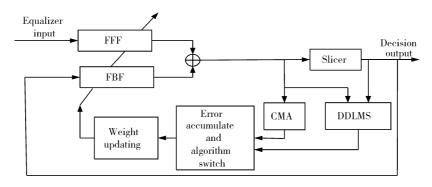


Fig. 1 Structure of the DFE equalizer

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DFE structure is infinite impulse response (IIR) filter in nature, including feedback forward filter (FFF) and feedback backward filter (FBF). FFF can counteract forward interferes, and FBF is used to wipe off the ISI of the former detected symbols [2]. At the beginning, equalizer uses a large step-size to converge the symbol amplitude in CMA stage. And when eye-pattern is almost open and CPR loop removes frequency ambiguity and phase ambiguity, it switches to DD-LMS stage. DD-LMS firstly uses an initial step (usually large) to continue iterative operation. When the MSE reaches to the critical value that is set, equalizer is thought to be convergent and gradually decreases the step to the final step until it converges completely. The description of DFE structure is in Ref. [3].

For the slow convergence speed of conventional CMA, especially in DFE structure, CMA's convergence almost costs 80% of the time of the whole equalization process, and the disadvantage of leaving a phase offset increases the complexity of CPR's design and generates the need of derotator<sup>[4]</sup>, recently many academicians dedicate to Modified CMA (MCMA). Reference [5] proposed to use CMA in the real part and imagine part of QAM signals, respectively, which can correct phase ambiguity and increase little hardware resource while convergence speed is not improved much; Reference [6] followed the separate method in Ref. [5] and proposed a new cost function fit for the square constellation, it can also correct phase ambiguity and improve rudimental MSE obviously; Reference [7] detruded the conclusion that only one part of real and imagine needs to be in equalization with independently and identically distributed (i. i. d) channels, which reduces 50% of operation complexity, but only fits to square constellation; CMA + AMA (Alphabet-Matched Algorithm) was proposed in Ref. [8], that is, in blind stage using CMA to initialize AMA, and this improves convergence speed apparently, but it is too complex to implement and cannot correct phase ambiguity. Those several algorithms above improve convergence speed no faster than 40% of the conventional CMA. In this paper, we propose a new MCMA named SGLMS-CMA using DFE structure, which modifies the cost function to include phase information, so it can correct phase ambiguity while convergence speed is five times faster and rudimental MSE 3~5dB lower than the conventional CMA.

### 2 CMA and DD-LMS

CMA is proposed by Godard in 1980, and its cost function is:

$$J_n = E[(|Y_n|^2 - R_{CMA}^2)^2]$$
 (1)

where  $R_{\text{CMA}}^2 = E[|I_k|^4]/E[|I_k|^2]$ ,  $I_k$  represents the ideal constellation points, and E[.] denotes the expected value<sup>[2]</sup>.

The weight updating function is given by

$$C_{n+1} = C_n - \mu e_{n\_CMA} X_n^*$$
  
=  $C_n - \mu Y_n (|Y_n|^2 - R_{CMA}^2) X_n^*$  (2)

where  $C_n$  is the weight of equalizer taps at time n,  $\mu$  is called the step-size of the equalizer,  $e_{n\_{\rm CMA}}$  is the error function,  $X_n$  is the input of equalizer at time n, and  $X^*$  is the complex conjugate form of the received data vector at time n. Assumed the taps' length is L, the equalizer output is:

$$Y_n = \sum_{i=0}^{L-1} C_{n,i} X_{n-i}$$
 (3)

 $R_{\rm CMA}^2$  is different for each QAM mode and signal power, and when the minimum distance of constellation points is 0. 125 for 64QAM,  $R_{\rm CMA}^2 = 0.22656$ .

DD-LMS is used widely for its facility, and its cost function is given by

$$J_n = E(|I_{n-\Delta} - Y_n|^2) \tag{4}$$

where  $I_{n-\Delta}$  is the transmitted symbol corresponding to the current output of equalizer, and  $\Delta$  is the sum delay of the transmitter delay, channel delay, other modules before equalizer and equalizer self delay. Its cost function is given by

$$C_{n+1,f} = C_{n,f} + \mu e_{n\_LMS} X_n^*$$

$$= C_{n,f} + \mu (D_n - Y_n) X_n^*$$

$$C_{n+1,b} = C_{n,b} + \mu e_{n\_LMS} D_n^*$$

$$= C_{n,b} + \mu (D_n - Y_n) D_n^*$$
(6)

where  $D_n$  is the output of equalizer slicer at time n,  $L_{\rm f}$  and  $C_{\rm n,f}$  are the forward taps' length and weight at time n, respectively, while  $L_{\rm b}$  and  $C_{\rm n,b}$  are the backward taps' length and weight at time n, respectively. The output is:

$$Y_{n} = \sum_{i=0}^{L_{f}-1} C_{n,f,i} X_{n-i} + \sum_{i=-L_{b}}^{1} C_{n,b,i} D_{n-i}$$
 (7)

### 3 SGLMS-CMA

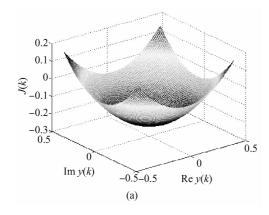
The cost function of the conventional CMA only includes amplitude information, which makes it have no effect on phase. We can improve it as follows:

$$J_n = E[(|Y_n|^2 - R_{CMA}^2)^2 + |I_{n-\Delta} - Y_n|^2]$$
 (8)

The above function transforms from one-stage convergence to two-stage convergence, including double-amplitude equalization and single-phase equalization. Assumed that in CMA stage the values output by slicer are basically right, Eq. (8) alters into Eq.(9).

$$J_{n} = E[(|Y_{n}|^{2} - R_{CMA}^{2})^{2} + |D_{n} - Y_{n}|^{2}]$$
 (9)

The mesh plots of Eqs. (9) and (1) are shown in



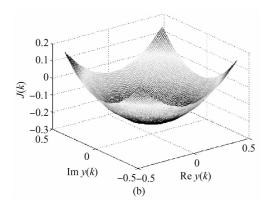


Fig. 2 Mesh plots of cost function (a) Conventional CMA; (b) SGLMS-CMA

Fig. 2, from which we can see the meshes of the two algorithms' cost functions are very close, confirming that CMA reacts as a dominant part in SGLMS-CMA. According to the Orthogonality principle, when the taps' length tends to infinitude and  $E[e_nX^*]=0$ , the equalization is convergent to the optimal point globally. At that point MSE is the least, accordingly  $\nabla J_n = 0^{\lfloor 2 \rfloor}$ .

Using stochastic gradient algorithm, the weight updating function is given by

$$C_{n+1} = C_n - \frac{1}{2}\mu \nabla J_n$$

$$= C_n - \frac{1}{2}\mu \frac{2\partial J_n}{\partial C^*}$$

$$= C_n - \frac{1}{2}\mu \frac{2\partial J_n}{\partial Y_n} X_n^*$$

$$= C_n - \mu \times 2[(|Y_n|^2 - R_{CMA}^2) Y_n - (D_n - Y_n)] X_n^*$$
(10)

We can see that the error function includes  $e_{n_{\text{CMA}}}$  of Eq. (2), and some part like  $e_{n_{\text{LMS}}}$  of Eq. (6). Getting rid of the coefficient 2 by including its effect on the step-size, Eq. (10) alters into Eq. (11).

$$C_{n+1} = C_n - \mu(|Y_n|^2 - R_{CMA}^2) Y_n X_n^* + \mu(D_n - Y_n) X_n^*$$

$$= C_n - \mu_{CMA} e_{n\_CMA} X_n^* + \mu_{LMS} e_{n\_LMS} X_n^*$$
 (11)

Because of the dissimilar implement targets of CMA and DD-LMS, Equation (11) uses different

steps separately to enhance the new algorithm's adaptive ability. In order to ensure the values output by slicer are basically right in CMA stage, CMA's power constant  $R_{\rm CMA}^2$  is used. According to the stop-and-go principle, when the decision is dependable, DD-LMS participates into the updating of weights, otherwise it does not. So the final weight updating function of SGLMS-CMA is given by

$$C_{n+1,f} = C_{n,f} - \mu_{\text{CMA}} e_{n_{\text{CMA}}} X_{n}^{*} + f_{n} \mu_{\text{LMS}} e_{n_{\text{LMS}}} X_{n}^{*}$$

$$C_{n+1,b} = C_{n,b} - \mu_{\text{CMA}} e_{n_{\text{CMA}}} X_{n}^{*} + f_{n} \mu_{\text{LMS}} e_{n_{\text{LMS}}} D_{n}^{*}$$
(12)

 $where^{\tiny \lceil 10 \rceil}$ 

$$f_{n} = \begin{cases} 1, & \text{if } \operatorname{sgn}(|Y_{n}|^{2} - |D_{n}|^{2}) = \\ & \operatorname{sgn}(|Y_{n}|^{2} - R_{\text{CMA}}^{2}) \\ 0, & \text{if } \operatorname{sgn}(|Y_{n}|^{2} - |D_{n}|^{2}) \neq \\ & \operatorname{sgn}(|Y_{n}|^{2} - R_{\text{CMA}}^{2}) \end{cases}$$
(14)

SGLMS-CMA combines conventional CMA and DD-LMS under stop-and-go principle, and its weight updating function includes CMA part and DD-LMS part. The phase information is added into the updating function on condition that the direction of DD-LMS convergence is basically right, which avoids error spreading and result converging to the local optimal point effectively, and double convergent effect would enhance the convergent speed markedly.

# 4 Simulation results and VLSI implement

In order to verify our proposed algorithm SGLMS-CMA, we built a transmitting and receiving system of QAM signal on the platform of Simulink in Matlab, and simulated in every QAM mode. By way of observing the performance variety, we compare the results of SGLMS-CMA and conventional CMA. This paper only discusses 64QAM.

The lengths of FFF and FBF in this equalizer are 7 and 15, respectively. At first we set the weight of FFF's center tap 1 + 0i, while others 0 + 0i. The original data firstly is modulated by 64QAM, of which the minimum distance is 0. 125, then gets across the Additive White Gaussion Noise (AWGN) channel with Signal to Noise Ratio (SNR) 33dB, lastly passes the channel with ISI, and enters into equalizer. The channel used is given by h = [1, 0.5, 0.25, 0.125, 0.0625,0. 03125, 0. 03125, which is the approximate channel model of ETS300473 standard constituted by European Broadcast Union. The step-size of CMA is 2<sup>-8</sup>, and the initial step-size of DD-LMS is  $2^{-5}$ . When the MSE of 100 symbol cycles is accumulated, it is compared with the critical MSE that is set, and when the equalizer arrives to the final convergence in CMA stage,

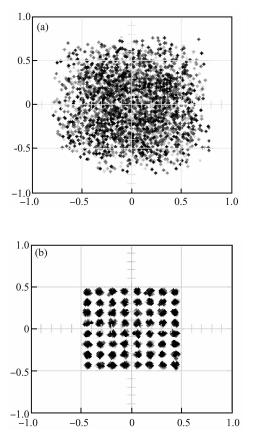


Fig. 3 Constellation results (a) Equalizer input constellation; (b) Convergent constellation in CMA stage

the accumulated MSE must be less than -36dB.

The constellation figures of the equalizer data are shown in Fig. 3 (without frequency ambiguity and phase ambiguity).

The performance standards for the equalization algorithm are rudimental MSE, convergence speed and operation complexity,  $etc^{[2]}$ .

The MSE curves of CMA and SGLMS-CMA are shown in Fig. 4. When the equalizer is convergent in CMA stage, the rudimental MSE of SGLMS-CMA is about  $3{\sim}5\text{dB}$  improved than conventional CMA.

The comparison result about convergence speed is shown in Fig. 5, and axis Y represents the iterations every time. They are initialized again and again after being convergent. In the same time, the conventional CMA converges for 64 times corresponding to about 50000 iterations in every convergence, while the SGLMS-CMA for 360 times corresponding to 8000 iterations. So the later one's convergence speed is five times faster and is much more steady-going than the former one.

For operation complexity, since the step-size's multiplication is usually implemented by shift, according to the above result, conventional CMA needs 880  $\times\,10^4$  times of multiplication and  $440\times10^4$  times of addition, while SGLMS-CMA only needs  $145.6\times10^4$  times of multiplication and  $92\times10^4$  times of addition

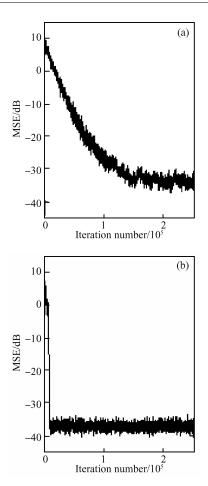


Fig. 4 Rudimental MSE (a) Conventional CMA; (b) SGLMS-CMA

from initiation to convergence. If we only consider the times of multiplication and addition, SGLMS-CMA reduces 82% operations of conventional CMA. In addition, SGLMS-CMA can correct phase ambiguity, as shown in Fig. 6. The result of correction is the same as analysis in Ref. [4] in detail.

The new equalizer proposed is improved from the conventional high-capable equalizer, so it uses DFE construct. It adopts SGLMS-CMA to replace CMA, while the control policy is the same as before. Firstly it uses SGLMS-CMA to converge the symbol amplitude and correct phase ambiguity, and when the MSE reduces to the critical value that is set, the equalizer would transform to DD-LMS to converge completely. We do not propose the equalizer that merely use SGLMS-CMA, for the rudimental MSE would reduce some in DD-LMS stage.

The new equalizer is described by verilog, and synthesized by SMIC  $0.18\mu m$  library, as shown in Fig. 7. The reports of Design Compiler shows that, the resource consumption of the new SGLMS-CMA + DD-LMS equalizer is 5% more than that of the conventional equalizer using CMA + DD-LMS. But as SGLMS-CMA can correct phase ambiguity, which

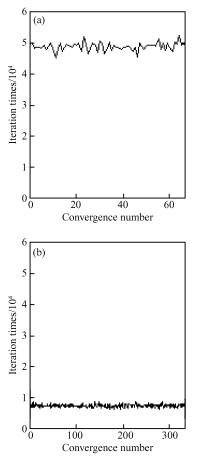


Fig. 5 Convergence times (a) Conventional CMA; (b) SGLMS-CMA

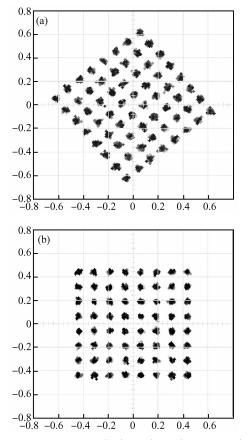


Fig. 6 Convergence constellation when phase excursion is 40° (a) Conventional CMA; (b) SGLMS-CMA

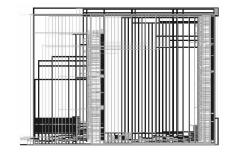


Fig. 7 Synthesis result of the new equalizer

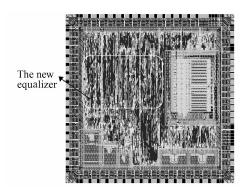


Fig. 8 Layout of the new DVB-C demodulation chip

makes the system save the derotator of CPR, the resource consumption of the new demodulation system is almost equal to that using conventional equalizer. Considering the high request of the application, we embed the new equalizer in the DVB-C (Digital Video Broadcasting for Cable systems) demodulation chip, as shown in Fig. 8. Tests show that it can receive the transport stream clearly, as shown in Fig. 9.

Table 1 gives a comparison between the DVB-C chip with new equalizer and that with conventional equalizer on condition of 64QAM mode and 355MHz carrier. Lock time of the chip with new equalizer is reduced to 15% of the origin one with conventional equalizer, while hardware resource is some decreased and other performance is little affected. The lock speed now is more than five times faster, since CPR, STL (symbol timing loop) and equalizer compose a

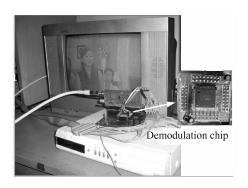


Fig. 9 Test platform of the chip

Table 1 Performance comparison

1		
DVB-C chip	With new equalizer	With conventional equalizer
Process/μm	0.18	0.18
Die size/mm <sup>2</sup>	$2.72 \times 2.72$	$2.73 \times 2.72$
Gate cost <sup>1</sup>	138,803	143,360
Lowest C/N ratio	21.2	21.6
Lowest signal level input/(dB • $\mu$ V)	32	31
Highest signal level input/(dB • μV)	105	102
Lock time/ms	2.2	14.8

Note 1: This number is not including SRAMs' cost that has no relation with equalizer.

Note 2:C/N ratio means Carry/Noise ratio.

whole loop and affect each other in the demodulation system. The improvement of equalizer would enhance the whole system's performance obviously.

### 5 Conclusion

In this paper, we propose a high-speed equalizer based on a new MCMA named SGLMS-CMA by combining conventional CMA and DD-LMS under stopand-go principle. Matlab simulations indicated that, compared with conventional CMA, the new algorithm performs five times faster and rudimental MSE 3 ~ 5dB lower, corrects a final phase ambiguity and decreases 82% of operation complexity. As to hardware consumption, synthesis results show that SGLMS-CMA + DD-LMS equalizer only increases 5% hardware consumption than CMA + DD-LMS equalizer, but SGLMS-CMA equalizer makes the whole system spare the derotator. That is, the new chip saves some

hardware resource. By using SMIC 0.18 $\mu$ m library to synthesis, the new equalizer is embedded in the DVB-C demodulation chip, and test results show that the chip's lock speed is more than fix time faster, and it can receive the transport stream clearly. Thus, the proposed equalizer is suitable for the high performance occasion with DFE structure.

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## 一种应用于 QAM 信号的新型高速均衡器\*

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摘要:提出了一种基于新型算法 SGLMS-CMA 的高速 QAM 均衡器. SGLMS-CMA 算法在 stop-and-go 原理指导下结合了经典 CMA 算法和 DD-LMS 算法. MATLAB 仿真表明,与经典 CMA 算法相比,新型算法具有收敛速度快 5 倍,剩余 MSE 改善 3~5dB,运算复杂度减少了 82%,可纠正残留相偏的优点.综合结果表明,采用 SGLMS-CMA + DD-LMS 的新型均衡器与 CMA + DD-LMS 均衡器相比,系统中均衡器模块的硬件开销只增加 5%.最后使用 SMIC  $0.18\mu m$  工艺库对新型均衡器进行综合,并且嵌入到 QAM 解调芯片中进行流片,最终测试结果表明新型均衡器性能优越.

关键词: CMA; DD-LMS; 均衡器; 收敛速度; 残留相偏

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