The Theory of Field-Effect Transistors XII. The Bipolar Drift and Diffusion Currents (1-2-MOS-Gates on Thin-Thick Pure-Impure Base)*

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Abstract: The previous report (XI) gave the electrochemical-potential theory of the Bipolar Field-Effect Transistors. This report (XII) gives the drift-diffusion theory. Both treat 1-gate and 2-gate, pure-base and impure-base, and thin and thick base. Both utilize the surface and bulk potentials as the parametric variables to couple the voltage and current equations. In the present drift-diffusion theory, the very many current terms are identified by their mobility multiplier for the components of drift current, and the diffusivity multiplier for the components of the diffusion current. Complete analytical drift-diffusion equations are presented to give the DC current-voltage characteristics of four common MOS transistor structures. The drift current consists of four terms: 1-D (One-Dimensional) bulk charge drift term, 1-D carrier space-charge drift term, 1-D E_X^2 (transverse electric field) drift term, 2-D drift term. The diffusion current consists of three terms: 1-D bulk charge diffusion term, 1-D carrier space-charge diffusion term, and 2-D diffusion term. The 1-D E_X^2 drift term was missed by all the existing transistor theories, and contributes significantly, as much as 25% of the total current when the base layer is nearly pure. The 2-D terms come from longitudinal gradient of the longitudinal electric field, which scales as the square of the Debye to Channel length ratio, at 25nm channel length with nearly pure base, $(L_D/L)^2 = 10^6$, but with impurity concentration of 10^{18}cm^{-3}, $(L_D/L)^2 = 10^{-2}$.

Key words: bipolar field-effect transistor theory; surface potential; drift and diffusion theory; single-gate impure-base; double-gate impure-base
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1　Introduction

In the previous report of this series on the theory of the bipolar field effect transistors (BiFET)[1] we have presented the general theory in the electron and hole electrochemical potentials representation (EC) to give the electron and hole currents, using the surface and bulk potentials as the parametric variables to relate the terminal DC currents and admittances to the terminal DC voltages applied to each of the terminal node (or point). In EC theory, the electron and hole currents are respectively proportional to the gradient of their electrochemical potentials times their concentrations. As stated in [1] and proven in [2], EC is mathematically as exact as and is completely equivalent to the more familiar drift and diffusion representation (DD) favored by transistor engineers. The proof and the interchangeable usage of EC and DD representations to facilitate device physics understanding and approximation were first given by Shockley[12]. The terms from the electrical drift current vector are labeled by the drift mobility multiplier, μ_n for electrons and μ_p for holes, therefore, the electron and hole drift current areal densities are $+q\mu_n n\boldsymbol{E}$ and $+q\mu_p p\boldsymbol{E}$. Similarly, the terms from the electrical diffusion current vector are labeled by the diffusivity multiplier, D_n for electrons and D_p for holes, giving $+qD_n\nabla n$ and $-qD_p\nabla p$. Our usage of identifiers μ_n and D_n for electrons and μ_p and D_p for holes, are designed to serve two purposes. The first purpose, just briefly stated, was to help keep track of the physics-origin of the many electric current terms when the transistor theory is represented by the two-component drift-diffusion model in this paper, in contrast to the one-component electrochemical potential theory in the previous report[1] where all terms are from the electrochemical potential gradient. The second purpose, as indicated in [2], was to allow for a different representation and approximation to the mobility for the drift current which dominates in the

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strong inversion range and to the diffusivity for the diffusion current which dominates in the weak inversion or subthreshold range, as we know the surface channel thickness and the surface potential well are very different in these two ranges, which would provide a better approximation model if this difference is accounted for. Another part of the second purpose is to provide a simple theory that can be readily extended to conditions of significant deviation from thermal equilibrium[3], namely the warm and hot carrier effects, in order to treat the effects of high electric fields and high current or particle densities at thermal non-equilibrium on the electron and hole currents. The two hot carrier effects were first formulated mathematically by Shockley in 1951 using the Maxwellian Distribution[4] to analyze mobility and drift velocity of the warm to hot electrons and holes under acoustical and optical phonon scattering, and in 1961 using the spike distribution or the lucky electron model[5] to analyze the interband impact generation of electron-hole pairs by an energetic primary electron or hole. Thus, we do not use the macroscopic Einstein relationship $D_n/\mu_n = k_B T/q$ and $D_p/\mu_p = k_B T/q$ which is valid only at thermal equilibrium[2,3] with the quantity temperature $T = T(\boldsymbol{r}, t) = T_L(\boldsymbol{r}, t)$ equal to the macroscopic lattice temperature defined by the average of the kinetic energy of the vibrating ionic cores of the crystal atoms or phonons, at thermal equilibrium with negligible phonon transport as heat and sound or the near thermal equilibrium heat transport temperature (in contrast to ultrasound). The generalized non-equilibrium Einstein relationships would be $D_n/\mu_n = k_B T_N/q$ and $D_p/\mu_p = k_B T_P/q$ where T_N and T_P are the warm and hot electron and hole temperatures, defined by their average kinetic energies dependent on the local macroscopic electric field. However, the equilibrium Einstein relationship facilitates the first order extension to the analysis of the nonequilibrium hot carrier effects, especially in compacting the transistor model, just like the exponential or electrochemical potential representation of the nonequilibrium electron and hole concentrations, by simply replacing $T(\boldsymbol{r}, t)$ by $T_N(\boldsymbol{r}, t)$ for hot electrons and $T_P(\boldsymbol{r}, t)$ for hot holes, separately or not necessarily at the same time (same transistor operating condition, such as the same applied terminal voltages). Just like the quasi-Fermi potentials for electrons and holes, these generalized Einstein relationships give the correct asymptotic limits as the electrons approach thermal equilibrium with the lattice vibration or phonons, $T_N(\boldsymbol{r}, t) \rightarrow T_L(\boldsymbol{r}, t)$ and holes, $T_P(\boldsymbol{r}, t) \rightarrow T_L(\boldsymbol{r}, t)$, although not necessarily simultaneously. The exponential representation or transform, first used by Shockley in 1951[4] for diode-transistor analyses, comes from a Boltzmann-Maxwellian-like distribution of electrons and holes in their kinetic energies. It was firmly established by Wolff in 1954[6] using the solution of the near-equilibrium ionized gas molecule model to give the Maxwellian distribution of the kinetic energies of the hot electrons, $\exp(-E/k_B T_N)$, and holes, $\exp(-E/k_B T_P)$ where the hot electron and hole temperatures are functions of the macroscopic electric field[4,7]. This Maxwellian distribution was proven by Baraff in 1962[8] with numerical computations of analytical formulas and by Kane in 1967[9] using the Monte Carlo method with simplified one-electron energy band structure, and later by Fischetti and Laux in 1988[10] using the complete one-electron energy band. For recent and future generations of nanometer transistors operating at low voltages, the acceleration potential can be barely above the kinetic energy threshold of electron-hole pair generation, making the orientation dependence of the threshold kinetic energy or acceleration potential, as indicated by Lu and Sah in 1995[11], very important on the impact generation rate of electron-hole pairs.

The exact equivalence of these two representations was first given, proven and applied by Shockley in his 1949 Bipolar Junction Transistor (BJT) invention article[12]. The main difference between the two representations is EC's simplicity and ease of making and justifying the approximations necessary to give analytical solutions, approximations, and numerical solutions, as benchmarks for compacting the transistor models, and DD's complexities with very many more terms and numerically difficult if not impossible computation of some of the drift current terms. Nevertheless, we shall complete the general formulation of the bipolar field-effect transistor theory, started in [1], by presenting in this report the more frequently used drift-diffusion theory. A new key result was the discovery of the substantial contributions to the total current by the 1-Dimensional E_X^2 term or transverse electric field term, which was neglected in all the previous drift-diffusion-theory-based compact models and their applications. This E_X^2 term can contribute as much as 25% or more of the total channel current in nanometer size, thin and pure or impure (low-impurity-concentration) silicon base with two MOS gates such as the FinFET, SOI and TFT device geometries.

2 The General Drift and Diffusion Bipolar FET Theory

The bipolar electrochemical potential theory of Field-Effect Transistors has been given completely in the previous report[1] in this series of exposition. As in

[1], we start from the Shockley equations to derive drift and diffusion current formulas of BiFET and to derive some analytical approximations.

A detailed algebraic derivation of the three equations, (1), (2) and (3) listed below, are given respectively in equations (1A) to (1H), (2A) to (2E) and (3A) to (3E), also listed below. They are then used to obtain the surface-potential-coupled Current and Voltage equations of the BiFET. The Cartesian coordinate unit vectors are defined by $\boldsymbol{r} = \boldsymbol{i}_X x + \boldsymbol{i}_Y y + \boldsymbol{i}_Z z$ and the volume element is given by $d\mho = dx dy dz$. In order to simplify the notation and also to bring out the device and material physics in scaling, we normalize the x-axis to the Debye Screening Length of pure silicon, $L_D = (\varepsilon_S k_B T/2q^2 n_i)^{1/2}$, $X = x/L_D$, and the y-axis to the channel length L, $Y = y/L$. The Debye screening Length of impure silicon can be used for x-normalization of transistors with impure base layer. The impure Debye screening length is obtained from replacing $2n_i$ by the sum of the equilibrium electron and hole concentrations $N_E + P_E$. A nonequilibrium local Debye screening Length can also be defined by using $N(x,y,z) + P(x,y,z)$ in place of $2n_i$, provided the elemental volume $dxdydz$ is sufficiently large to contain many particles or electrons and holes to retain statistical significance.

Shockley Equations (Three of Six at DC for No-Generation-Recombination-Trapping[1].)

Poisson Equation

$$\nabla\cdot(\varepsilon_S \boldsymbol{E}) = \rho(x,y,z) = q(P - N - P_{IM}) \quad \text{Poisson Equation 3-Dimensional Impure (1A)}$$

$$= \varepsilon_S[+(\partial E_X/\partial x) + (\partial E_Y/\partial y)] = q(P - N - P_R + N_R) \quad \text{2-Dimensional Impure (1B)}$$

$$= \varepsilon_S(k_B T/q)[-(\partial^2 U/\partial x^2) - (\partial^2 U/\partial y^2)] = q(P - N - P_F + N_F) \quad \text{2-Dimensional Impure (1C)}$$

$$= qn_i[\exp(U_P - U) - \exp(U - U_N) - \exp(+U_F) + \exp(-U_F)] \quad \text{2-Dimensional Impure (1D)}$$

$$[-(\partial^2 U/\partial x^2) - (\partial^2 U/\partial y^2)] = (q^2/\varepsilon_S k_B T)(P - N - P_F + N_F) \quad \text{2-Dimensional Impure (1E)}$$

$$= (1/2L_D^2)[\exp(U_P - U) - \exp(U - U_N) - \exp(+U_F) + \exp(-U_F)] \quad \text{2-Dimensional Impure (1F)}$$

$$[-(\partial^2 U/\partial X^2) - (L_D/L)^2(\partial^2 U/\partial Y^2)] = (P - N - P_F + N_F)/2n_i \quad \text{2-Dimensional Impure (1G)}$$

$$= [\exp(U_P - U) - \exp(U - U_N) - \exp(+U_F) + \exp(-U_F)]/2 \quad \text{2-Dimensional Impure (1H)}$$

$$[-(\partial^2 U/\partial X^2) - (L_D/L)^2(\partial^2 U/\partial y^2)] = [\exp(U_P - U) - \exp(U - U_N)]/2 \quad \text{2-Dimensional Pure (1)}$$

Electron Current Continuity Equation

$$\nabla\cdot \boldsymbol{j}_N + q(g_N - r_N) = \partial n/\partial t \quad \text{Time-Dependent Electron 3-D Continuity Equation (2A)}$$

$$\nabla\cdot \boldsymbol{J}_N + q(G_N - R_N) = 0 \quad \text{DC Steady State Electron 3-D Continuity Equation (2B)}$$

$$\nabla\cdot \boldsymbol{J}_N = 0 \quad \text{GRT-free DC Steady State Electron 3-D Continuity Equation (2C)}$$

$$\oiiint \nabla\cdot \boldsymbol{J}_N\, d\mho = \oiint \boldsymbol{J}_N\cdot d\boldsymbol{S} = 0 \quad \text{Divergence Theorem, Kirchoff Law 3-D Continuity Equation (2D)}$$

$$\oiiint \nabla\cdot \boldsymbol{J}_N\, d\mho = \oiint \boldsymbol{J}_N\cdot d\boldsymbol{S} = \oint J_{NX}\partial yZ + \oint J_{NY}\partial xZ = 0 \quad \text{Electron 2-D Continuity Equation (2E)}$$

$$\therefore \int J_{NX}\partial yZ + \int J_{NY}\partial xZ = \textbf{constant} = -I_{DN} \quad \text{Kirchoff Law 2-D Electron Current Continuity (2)}$$

Hole Current Continuity Equation

$$\nabla\cdot \boldsymbol{j}_P + q(g_P - r_P) = \partial p/\partial t \quad \text{Time-Dependent Hole 3-D Continuity Equation (3A)}$$

$$\nabla\cdot \boldsymbol{J}_P + q(G_P - R_P) = 0 \quad \text{DC Steady State Hole 3-D Continuity Equation (3B)}$$

$$\nabla\cdot \boldsymbol{J}_P = 0 \quad \text{GRT-free DC Steady State Hole 3-D Continuity Equation (3C)}$$

$$\oiiint \nabla\cdot \boldsymbol{J}_P\, d\mho = \oiint \boldsymbol{J}_P\cdot d\boldsymbol{S} = 0 \quad \text{Divergence Theorem, Kirchoff Law 3-D Continuity Equation (3D)}$$

$$\oiiint \nabla\cdot \boldsymbol{J}_P\, d\mho = \oiint \boldsymbol{J}_P\cdot d\boldsymbol{S} = \oint J_{PX}\partial yZ + \oint J_{PY}\partial xZ = 0 \quad \text{Hole 2-D Continuity Equation (3E)}$$

$$\therefore \int J_{PX}\partial yZ + \int J_{PY}\partial xZ = \textbf{constant} = -I_{DP} \quad \text{Kirchoff Law 2-D Hole Current Continuity (3)}$$

Voltage Equation, *X*-Equation. (1966-Sah-Pao Model[2,13,14])

Integrating the Poisson Equation (1F) and (1H) by quadrature along the X-axis from X_1 to X_2 and U_1 to U_2 with the assumption of spatially constant impurity concentration, $P_{IM}(x,y) = P_{IM}$, we get the general solution as follows[1]. Let $X_2 = X$ be a variable, then

$$(\partial U/\partial X)_2^2 - (\partial U/\partial X)_1^2 = F^2(U_2, U_P, U_N, U_1, P_{IM}, E_Y) \quad \text{2-Dimensional Impure (4A)}$$

$$(\partial U/\partial X)^2 = F^2(U, U_P, U_N, U_1, P_{IM}, E_Y) + (\partial U/\partial X)_1^2$$

$$= F^2(U, U_P, U_N, U_1, P_{IM}, E_Y, E_{X1}) \quad \text{2-Dimensional Impure (4B)}$$

$$= +\exp(U - U_N) - \exp(U_1 - U_N) + \exp(U_P - U) - \exp(U_P - U_1)$$
$$+ (P_{IM}/n_i)\times(U - U_1) + 2(L_D/L)^2 \int (\partial^2 U/\partial Y^2)\partial_X U + (\partial U/\partial X)_1^2$$

$$\text{2-Dimensional Impure (4C)}$$

$$= + \exp(U - U_N) - \exp(U_1 - U_N) + \exp(U_P - U) - \exp(U_P - U_1) \quad \text{1-Dimensional Pure} \quad (4)$$

Integrating the Poisson Equation (1F) and (1H) twice along X axis, from X_1 to X_2, and let $X_2 = X$ as a variable, we get the general Voltage Equation or X-equation as follows[1]:

$$\iint \rho \partial x \partial x = \int \varepsilon E_X \partial x - \int \varepsilon E_{X1} \partial x + \varepsilon \iint (\partial E_Y / \partial y) \partial x \partial x = -\varepsilon(V - V_1) - (x - x_1)\varepsilon E_{X1} + \varepsilon \iint (\partial E_Y / \partial y) \partial x \partial x$$

$$U(X, Y) - U_1(X_1, Y) = (C_\varepsilon / C_{X-X1}) \times (\partial U / \partial X_1)_1$$
$$- (x_\varepsilon / L)^2 \iint (\partial^2 U / \partial Y^2) \partial X \partial X - (1/2)(x_\varepsilon / L_{D\varepsilon})^2 \iint (\rho / q n_i) \partial X \partial X \quad \text{2-Dimensional terms} \quad (5)$$

$$(\partial U / \partial X_1)_1 = \text{sign}(U - U_1) \times \{\exp(U - U_N) - \exp(U_1 - U_N) + \exp(U_P - U) - \exp(U_P - U_1)$$
$$+ (P_{IM} / n_i) \times (U - U_1) + 2(L_D / L)^2 \int (\partial^2 U / \partial Y^2) \partial_X U + (\partial U / \partial X)_1{}^2\}^{1/2} \quad (5A)$$

Current Equation, *Y*-Equation. (1996-Sah Model[15,16,17])

Electron Current

$$\boldsymbol{J}_N = + q\mu_n N\boldsymbol{E} + qD_n \nabla N = - qD_n N \nabla U_N \quad \text{3-Dimensional} \quad (6A)$$
$$= J_{NX} \boldsymbol{i}_X + J_{NY} \boldsymbol{i}_Y = - qD_n N[(\partial U_N / \partial x)\boldsymbol{i}_X + (\partial U_N / \partial y)\boldsymbol{i}_Y] \quad \text{2-Dimensional} \quad (6B)$$
$$J_{NX} \boldsymbol{i}_X = - qD_n N(\partial U_N / \partial x)\boldsymbol{i}_X = 0 \quad \textbf{Gradual-Channel} \quad \therefore U_N(x, y) = U_N(y) \quad (6C)$$
$$J_{NY} \boldsymbol{i}_Y = (q\mu_n N E_Y) + (qD_n \nabla N) \boldsymbol{i}_Y \neq 0 \quad (6D)$$
$$\oiiint \boldsymbol{J}_N \cdot d\boldsymbol{S} = \oint J_{NX} \partial y Z + \oint J_{NY} \partial x Z = \oint J_{NY} \partial x Z = 0 \quad \therefore \int J_{NY} \partial x Z = \textbf{constant} = - I_{DN} \quad (6)$$

Hole Current

$$\boldsymbol{J}_P = + q\mu_p P\boldsymbol{E} - qD_p \nabla P = - qD_p P \nabla U_P \quad \text{3-Dimensional} \quad (7A)$$
$$= J_{PX} \boldsymbol{i}_X + J_{PY} \boldsymbol{i}_Y = - qD_p P[(\partial U_N / \partial x)\boldsymbol{i}_X + (\partial U_N / \partial y)\boldsymbol{i}_Y] \quad \text{2-Dimensional} \quad (7B)$$
$$J_{PX} \boldsymbol{i}_X = - qD_p P(\partial U_P / \partial x)\boldsymbol{i}_X = 0 \quad \textbf{Gradual-Channel} \quad \therefore U_P(x, y) = U_P(y) \quad (7C)$$
$$J_{PY} \boldsymbol{i}_Y = (q\mu_p P E_Y) - qD_p \nabla P \boldsymbol{i}_Y \neq 0 \quad (7D)$$
$$\oiiint \boldsymbol{J}_P \cdot d\boldsymbol{S} = \oint J_{PX} \partial y Z + \oint J_{PY} \partial x Z = \oint J_{PY} \partial x Z = 0 \quad \therefore \int J_{PY} \partial x Z = \textbf{constant} = - I_{DP} \quad (7)$$

Total Terminal Current

(Flowing into the Drain and Source Nodes from Outside)

$$I_D = I_{DN} + I_{DP} = - I_S \quad (\text{integrate } x = 0 \text{ to } x_B;\ y \text{ can be any value between 0 to } L.) \quad (8)$$

By following the derivation steps in the 1996-Sah transistor theory [15,16], the general drift-diffusion current equation can be obtained. The two dimensional Poisson equation (1B) can be rearranged as follows:

$$P - N - P_{IM} = (\varepsilon_S / q)(\partial E_X / \partial x + \partial E_Y / \partial y) \quad (9)$$

Using the above equation, the drift term in the drain electron current (6D) and (6) can be rearranged as follows:

$$\int N E_Y \partial x = \int (P - P_{IM}) E_Y \partial x - (\varepsilon_S / q) \int (\partial E_X / \partial x + \partial E_Y / \partial y) E_Y \partial x$$
$$= \int (P - P_{IM}) E_Y \partial x - (\varepsilon_S / q) \int (\partial E_X / \partial x) E_Y \partial x - (\varepsilon_S / q) \int (\partial E_Y / \partial y) E_Y \partial x \quad (10)$$

where x is integrated from the front interface $x = 0$ to the back interface $x = x_B$.

$$\int (\partial E_X / \partial x) E_Y \partial x = [E_X E_Y(x = x_B) - E_X E_Y(x = 0)] - \int E_X (\partial E_Y / \partial x) \partial x$$
$$= [(\partial V / \partial x)(\partial V / \partial y)(x = x_B) - (\partial V / \partial x)(\partial V / \partial y)(x = 0)] - \int E_X (\partial E_X / \partial y) \partial x$$
$$= [(\partial V / \partial x)(\partial V / \partial y)(x = x_B) - (\partial V / \partial x)(\partial V / \partial y)(x = 0)] - \frac{1}{2} (\partial / \partial y) \int E_X{}^2 \partial x$$
$$(11)$$

$$\int (\partial E_X / \partial x) E_Y \partial x = (L_D L)^{-1} (kT/q)^2 [(\partial U / \partial X)(\partial U / \partial Y)(X = X_B) - (\partial U / \partial X)(\partial U / \partial Y)(X = 0)]$$
$$- (L_D L)^{-1} (kT/q)^2 \frac{1}{2} (\partial / \partial Y) \int (\partial U / \partial X)^2 \partial X$$
$$= (L_D L)^{-1} (kT/q)^2 [(\partial U / \partial X)_{S2} (\partial U_{S2} / \partial Y) - (\partial U / \partial X)_{S1} (\partial U_{S1} / \partial Y)]$$
$$- (L_D L)^{-1} (kT/q)^2 \frac{1}{2} (\partial / \partial Y) \int (\partial U / \partial X)^2 \partial X$$
$$= (2qn_i / \varepsilon_S)(L_D / L)(kT/q) \{(\partial U / \partial X)_{S2} (\partial U_{S2} / \partial Y) - (\partial U / \partial X)_{S1} (\partial U_{S1} / \partial Y)$$
$$- \frac{1}{2} (\partial / \partial Y) \int (\partial U / \partial X)^2 \partial X\} \quad (12)$$

where $(\partial U/\partial X)_{S1}$ and $(\partial U/\partial X)_{S2}$ are normalized surface electric fields and U_{S1} and U_{S2} are the normalized surface potentials, respectively at the front and back interfaces $x=0$ and x_B. The drain total, electron drift and diffusion currents are then given by the following four terms with integration perpendicular to the channel from $x=0$ to x_B and $X=x/L_D=0$ to $X=X_B$. Three are due to drift with the electron mobility multiplier, μ_n, and one is due to diffusion with the electron diffusivity multiplier, D_n.

$$
\begin{aligned}
I_{DN} &= -\iint J_{NY}\partial x\partial z = -qW[\mu_n\int NE_Y\partial x + D_n(\partial/\partial y)\int N\partial x] \\
&= -qW\{+\mu_n\int(P-P_{IM})E_Y\partial x - \mu_n(\varepsilon_S/q)\int(\partial E_X/\partial x)E_Y\partial x - \mu_n(\varepsilon_S/q)\int(\partial E_Y/\partial y)E_y\partial x + D_n(\partial/\partial y)\int N\partial x\} \\
&= -qW\{+(L_D/L)(kT/q)\mu_n\int(P_{IM}-P)(\partial U/\partial Y)\partial X \\
&\quad -(L_D/L)(kT/q)\mu_n 2n_i[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y)-(\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)-\frac{1}{2}(\partial/\partial Y)\int(\partial U/\partial X)^2\partial X] \\
&\quad -(L_D/L)^3(kT/q)\mu_n n_i(\partial/\partial Y)\int(\partial U/\partial Y)^2\partial X \\
&\quad +(L_D/L)D_n(\partial/\partial Y)\int N\partial X\}
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
I_{DN} = & -kT\mu_n n_i L_D(W/L)\times \\
& \{+\int(P_{IM}-P)/n_i(\partial U/\partial Y)\partial X && \text{ionized impurity charge or bulk charge drift term} \\
& -2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y)-(\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)] && \text{carrier space-charge drift term} \\
& +\partial/\partial Y\int(\partial U/\partial X)^2\partial X && \text{transverse electric-field drift term} \\
& -(L_D/L)^2\,\partial/\partial Y\int(\partial U/\partial Y)^2\partial X\} && \text{2-dimensional drift term} \\
& -qD_n n_i L_D(W/L)\times\partial/\partial Y\int N/n_i\,\partial X && \text{carrier-injection diffusion term}
\end{aligned}
\tag{14}
$$

Shown in (14) above, the drain electron current by the drift mechanism has been broken into four terms. By following the 1996-Sah transistor theory[15], they are called: (i) 1-dimensional fixed ionized impurity charge or bulk charge drift term indicated by the factor $(P_{IM}-P)$, (ii) 1-dimensional mobile carrier space-charge drift term, (iii) 1-dimensional transverse electric-field drift term, and (iv) 2-dimensional drift term indicated by the pre-factor $(L_D/L)^2$ which is very important in lowly doped short channels since $L_D\sim 25\mu m$ and current and future nanometer transistors have channel length of $L\sim$ 25nm to 100nm making the ratio $(L_D/L)^2=(25\times10^{-3}/25\times10^{-6})^2=10^6$. The drain electron current by the diffusion mechanism is still given by one term in Eq. (14), called carrier-injection diffusion term since it depends on the barrier height of the source/channel junction. By using Eq. (9) again, this diffusion term can be broken into three terms.

$$
\begin{aligned}
I_{DN} = & -kT\mu_n n_i L_D(W/L)\times \\
& \{+\int(P_{IM}-P)/n_i(\partial U/\partial Y)\partial X && \text{ionized impurity charge or bulk charge drift term} \\
& -2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y)-(\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)] && \text{carrier space-charge drift term} \\
& +\partial/\partial Y\int(\partial U/\partial X)^2\partial X && \text{transverse electric-field drift term} \\
& -(L_D/L)^2(\partial/\partial Y)\int(\partial U/\partial Y)^2\partial X\} && \text{2-dimensional drift term} \\
& -qD_n n_i L_D(W/L)\times \\
& \{-(\partial/\partial Y)\int(P_{IM}-P)/n_i\,\partial X && \text{ionized impurity charge or bulk charge diffusion term} \\
& +2(\partial/\partial Y)[(\partial U/\partial X)_{S2}-(\partial U/\partial X)_{S1}] && \text{carrier space-charge diffusion term} \\
& +2(L_D/L)^2(\partial/\partial Y)\int(\partial^2U/\partial Y^2)\partial X\} && \text{2-dimensional diffusion term}
\end{aligned}
\tag{15}
$$

These three diffusion current terms are called: (v) 1-dimensional fixed ionized impurity charge or bulk charge diffusion term indicated by the factor $(P_{IM}-P)$, (vi) 1-dimensional carrier space-charge diffusion term, and (vii) 2-dimensional diffusion term indicated by the pre-factor $(L_D/L)^2$.

The drain hole current can be obtained using similar algebraic steps:

$$
\begin{aligned}
I_{DP} = & -kT\mu_p\, n_i L_D (W/L) \times \\
& \{ -\int (P_{IM} + N)/n_i (\partial U/\partial Y)\partial X \quad \text{ionized impurity charge or bulk charge drift term} \\
& + 2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y) - (\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)] \quad \text{carrier space-charge drift term} \\
& - \partial/\partial Y \int (\partial U/\partial X)^2 \partial X \quad \text{transverse electric-field drift term} \\
& + (L_D/L)^2\, \partial/\partial Y \int (\partial U/\partial Y)^2 \partial X \} \quad \text{2-dimensional drift term} \\
& - qD_p\, n_i L_D (W/L) \times \\
& \{ -(\partial/\partial Y) \int (P_{IM} + N)/n_i\, \partial X \quad \text{ionized impurity charge or bulk charge diffusion term} \\
& - 2(\partial/\partial Y)[(\partial U/\partial X)_{S2} - (\partial U/\partial X)_{S1}] \quad \text{carrier space-charge diffusion term} \\
& - 2(L_D/L)^2 (\partial/\partial Y) \int (\partial^2 U/\partial Y^2)\partial X \} \quad \text{2-dimensional diffusion term} \quad (16)
\end{aligned}
$$

In the following sections, the general drift and diffusion theory are applied to the four volume-application and volume-produced MOSFET structures.

3 Applications to MOS Transistors

For ease of presentation, we collect the previous equations that are used to completely define the solution of a specific transistor structure. They are listed and renumbered below, as the Voltage or X equations and the Current or Y equations. They are to be solved simultaneously, analytically as much as possible, but exactly in order to serve as the bench mark, for a given transistor structure to give the drain terminal electron and hole currents, I_{DN} and I_{DP}, as a function of the terminal electric potentials U_{G1S}, U_{G2S} of gate1, gate2 and U_{DS} of drain, all relative to the source which may be connected to the absolute reference, the remote ground, previously designated by the subscript B (for Base) or F (for Fermi at thermal equilibrium). In order to give analytical solutions, we use what we called the thin transistor model in which the transistor base layer is thin so the electron and hole currents, confined in the thin base layer, are nearly parallel to the length direction of the thin base layer, which gives a physical and pictorial basis of our model, and the approximations we shall make to give analytical formulas. This was what Shockley called the "gradual channel" approximation or just "gradual approximation" in his 1952 FET invention paper[18], which was based on the assumption that the conduction channel width does not change very fast in the channel length direction. It can also be called and is actually the long channel approximation. However, none can be completely justified in any of the transistor geometries or the practical combination of the transistor channel length and base impurity concentrations. Nevertheless, it gives us the start, by dropping the 2-D term which is proportional to $(L_D/L)^2$ and may not be small enough (1% contribution) to be dropped, because the 1-D solution does provide and has provided simple, useful and easy to understand analytically results, widely used for more than 40 years, since their first use by Sah during the mid-1960's[13,14]. So the best and new physical-picture-based condition to give the iterative-able analytical solutions is to assume that both the electron and hole currents flow along the channel direction (y = axis), which were used in (6C) for the electron current namely, $J_{NX} = -qD_n N(\partial/\partial x) U_N(x, y) = 0$ or $(\partial/\partial x) U_N(x, y) = 0$, then, since $N(x, y) \neq 0$, so $U_N(x, y) = U_N(y)$, similarly in (7C) for the hole current, $J_{PX} = -qD_p P \times (\partial/\partial x) U_P(x, y) = 0$ or $(\partial/\partial x) U_P(x, y) = 0$, then, since $P(x, y) \neq 0$, so $U_P(x, y) = U_P(y)$. The general voltage or X equations from (4B) and (4C) and (5) are

$$
\begin{aligned}
(\partial U/\partial X)^2 &= F^2(U, U_P, U_N, U_1, P_{IM}, E_Y) + (\partial U/\partial X)_1^{\,2} \\
&= F^2(U, U_P, U_N, U_1, P_{IM}, E_Y, E_{X1}) \qquad \text{2-D Impure (4B)} \qquad (17) \\
&= +\exp(U - U_N) - \exp(U_1 - U_N) + \exp(U_P - U) - \exp(U_P - U_1) \\
&\quad + (P_{IM}/n_i)(U - U_1) + 2(L_D/L)^2 \int (\partial^2 U/\partial Y^2)\partial_X U + (\partial U/\partial X)_1^{\,2} \\
&\qquad \text{2-D Impure (4C)} \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
U(X, Y) - U_1(X_1, Y) = & \ (C_\varepsilon / C_{X-X1}) \times (\partial U/\partial X_1)_1 \\
& - (x_\varepsilon/L)^2 \iint (\partial^2 U/\partial Y^2)\partial X \partial X - (1/2)(x_\varepsilon/L_D)^2 \iint (\rho/qn_i)\partial X \partial X \qquad \text{2-D terms (5)} \qquad (19)
\end{aligned}
$$

The general current or Y equations from (15) to (16) are

$$
I_{DN} = -kT\mu_n\, n_i L_D (W/L) \times \{ + \int (P_{IM} - P)/n_i (\partial U/\partial Y)\partial X
$$

$$- 2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y) - (\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)] + \partial/\partial Y\int(\partial U/\partial X)^2\partial X$$
$$- (L_D/L)^2\, \partial/\partial Y\int(\partial U/\partial Y)^2\partial X\}$$
$$- qD_n\, n_i L_D(W/L) \times \{- \partial/\partial Y\int(P_{IM} - P)/n_i\, \partial X$$
$$+ 2\partial/\partial Y[(\partial U/\partial X)_{S2} - (\partial U/\partial X)_{S1}] + 2(L_D/L)^2\, \partial/\partial Y\int(\partial^2 U/\partial Y^2)\partial X\} \quad (20)$$

$$I_{DP} = - kT\mu_p\, n_i L_D(W/L) \times \{- \int(P_{IM} + N)/n_i(\partial U/\partial Y)\partial X$$
$$+ 2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y) - (\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)] - \partial/\partial Y\int(\partial U/\partial X)^2\partial X$$
$$+ (L_D/L)^2\, \partial/\partial Y\int(\partial U/\partial Y)^2\partial X\}$$
$$- qD_p\, n_i L_D(W/L) \times \{- \partial/\partial Y\int(P_{IM} + N)/n_i\, \partial X$$
$$- 2\partial/\partial Y[(\partial U/\partial X)_{S2} - (\partial U/\partial X)_{S1}] - 2(L_D/L)^2\, \partial/\partial Y\int(\partial^2 U/\partial Y^2)\partial X\} \quad (21)$$

3.1 1-Gate MOSFETs

We will consider two base-layer thicknesses. The finite base-thickness MOS transistor consists of two kinds of base material fabrication processes, hence base materials properties. The evaporated Silicon on Insulator thin-film field-effect transistors (SOI TFT) are widely used in the large array but relatively slower circuits such as the pixel driver in the large area flat panel LCD (Liquid Crystal Display) for TV sets. The separation by implanted oxide (SIMOX) Silicon on Insulator transistors (SOI SIMOX) with thin silicon surface layer insulated by a layer of oxygen implanted thermally converted SiO_2 are widely used in very fast circuits for game stations. The semi-infinite base-thickness 1-Gate MOS transistor with the remote contact on the back surface that has little effect on the transistor characteristics are the traditional bulk transistor which is used in all MOS signal processing circuits to this day except the two SOI transistors just described. It is evident that the finite base-thickness solutions would degenerate into the semi-infinite base-thickness solution. However, we will see that there are additional features from the remote boundary conditions that can be applied to the remote back surface of the semi-infinite base-thickness bulk transistor, which is too specific for the finite-thickness SOI transistors.

Finite-Thickness Impure-Base (SOI SIMOX and SOI TFT)

The general 1-gate MOS transistor has a finite silicon base thickness of x_B from $x = 0$ to $x = x_B$ with a dielectric constant of $\varepsilon_S = \kappa_{silicon}\varepsilon_0$ where $\varepsilon_0 = 8.854 \times 10^{-14}$ F/m is the permittivity or electric constant of free space and $\kappa_{Si} = 11.7$ is the relative dielectric constant of silicon while for the silicon dioxide, $\kappa_{SiO2} = 3.90$ giving a ratio of $\kappa_{Si}/\kappa_{SiO2} = 11.7/3.90 = 3.00$. The 1-gate or front gate is composed of a gate conductor over a gate insulator or oxide of thickness x_O with dielectric constant of $\varepsilon_O = 3.9\varepsilon_0$, which is located from $x = - x_O$ to $x = 0$ and the oxide/silicon interface plane is located at $x = 0$. Oxide and interface trapped charges can be readily included in the general voltage equation and it is understood as part of the flat-band gate voltage which is absorbed into the applied gate voltage. The distance between the drain and source contact is L, which is the active channel length and it is determined by the contact material properties and contact geometry. The back surface, at plane $x = x_B$ at all y from $y = 0$ to $y = L$, is taken as a free surface or a surface covered by an infinite thick oxide with no interface charge. Thus, $U(x = x_B, y) = U_0(y)$ and $E_X = (kT/q)[(\partial/\partial x)U(x = x_B, y)] = 0$. One cannot and must not assign a condition for the second derivative, $\partial^2 U(x, y)/\partial x^2$ at the back interface, $x = x_B$, or any (x, y) point since it not only forces electrical neutrality at all y at the back interface which cannot true in general and which must be determined by the applied potential to the terminals which determines the spatial variation of the electron and hole concentrations, which is determined by the Poisson Equation, but such an assumption on $\partial^2 U(x, y)/\partial x^2$ also over-specifies the problem which usually one cannot find a physically realistic situation to give such a specification. Furthermore, the insulating back boundary at $x = x_B$ is indeed not inconsistent with the general assumption that the electron and hole currents both flow in the y direction, i.e. at $x = x_B$, both $\partial U_N/\partial x = 0$ and $\partial U_P/\partial x = 0$. Finally, the most important, is the location of the source and drain contacts for the applied potentials to the drain and source. They are at $(x_B, y = L)$ for the drain and $(x_B, y = 0)$ for the source. We do not specify how these are connected to the external lead, which is covered by the contact theory. Therefore the current-voltage solutions are those limited by the 'intrinsic' transistor with perfect drain and source contacts or are those with very long 'intrinsic' transistor which is current limiting to dominate the electrical characteristics while the source and drain contacts appear as nearly short circuits or the electric potential type of boundaries[19]. In real transistors, the contacts must be taken into account in the analysis, which has been taken into account in the transistor theory only as the electrochemical potential type of boundaries[19]. So, based on the above, with the source as the reference for the potentials applied to the terminals, $U_{GS} = qV_{GS}/k_BT$ and $U_{DS} = qV_{DS}/kT$, the X-equations for this 1-Gate thin-base of finite-thickness-base but impure, MOS transistor (with $\rho_{OX} = 0$ and $\rho_{INTERFACE} = 0$), valid for all y values between the source ($x = x_B$, $y = 0$) and the drain ($x = x_B$, $y = L$), are

$$U_{GS} - U_S = (C_D/C_O) \times (\partial U/\partial X)_S - (x_O/L)^2 \iint (\partial^2 U/\partial Y^2)\partial X \partial X \text{ from (19)} \quad (22)$$

$$(\partial U/\partial X)^2 = F^2(U, U_P, U_N, U_0, P_{IM}, E_Y) = +\exp(U - U_N) - \exp(U_0 - U_N) + \exp(U_P - U) - \exp(U_P - U_0) + (P_{IM}/n_i) \times (U - U_0) + 2(L_D/L)^2 \int (\partial^2 U/\partial Y^2)\partial_X U \text{ from (17)} \quad (23)$$

$$(\partial U/\partial X)_S = \mathrm{sign}(U_S - U_0) \times \{F^2(U_S, U_P, U_N, U_0, P_{IM}, E_Y)\}^{1/2} \quad (24)$$

$$X_B = \int \mathrm{sign}(U - U_0)\partial_X U/F \text{ (Integrated from } U = U_0 \text{ to } U = U_S\text{) (The Thickness Equation)} \quad (25)$$

From the current equations (20) to (21), with $(\partial U/\partial X)_{S2} = 0$, $(\partial U/\partial X)_{S1} = (\partial U/\partial X)_S$, $U_{S2} = U_0$ and $U_{S2} = U_S$, we have

$$\begin{aligned} I_{DN} = &- kT\mu_n n_i L_D (W/L) \times \{+\int (P_{IM} - P)/n_i (\partial U/\partial Y)\partial X \\ &+ 2(\partial U/\partial X)_S(\partial U_S/\partial Y) + \partial/\partial Y \int (\partial U/\partial X)^2 \partial X - (L_D/L)^2(\partial/\partial Y)\int(\partial U/\partial Y)^2 \partial X\} \\ &- qD_n n_i L_D(W/L) \times \{-(\partial/\partial Y)\int (P_{IM} - P)/n_i \partial X \\ &- 2(\partial/\partial Y)[(\partial U/\partial X)_S] + 2(L_D/L)^2(\partial/\partial Y)\int(\partial^2 U/\partial Y^2)\partial X\} \end{aligned} \quad (26)$$

$$\begin{aligned} I_{DP} = &- kT\mu_p n_i L_D(W/L) \times \{-\int [(P_{IM} + N)/n_i](\partial U/\partial Y)\partial X \\ &- 2(\partial U/\partial X)_S(\partial U_S/\partial Y) - (\partial/\partial Y)\int(\partial U/\partial X)^2 \partial X + (L_D/L)^2 \partial/\partial Y \int (\partial U/\partial Y)^2 \partial X\} \\ &- qD_p n_i L_D(W/L) \times \{-(\partial/\partial Y)\int (P_{IM} + N)/n_i \partial X \\ &+ 2\partial/\partial Y[(\partial U/\partial X)_S] - 2(L_D/L)^2(\partial/\partial Y)\int(\partial^2 U/\partial Y^2)\partial X\} \end{aligned} \quad (27)$$

By dropping the 2-dimensional term in (22), we obtain

$$(\partial U/\partial X)_S = (C_O/C_D) \times (U_{GS} - U_S) \quad (28)$$

Following the x-independent longitudinal electric field approximation used in the 2006-Jie-Sah analytical model[17], $(\partial U/\partial Y)(X, Y) \simeq \partial U_S/\partial Y$, substituting (28) into both (26) and (27), and dropping the 2-dimensional terms in (26) and (27), we get

$I_{DN} = -kT\mu_n n_i L_D(W/L) \times$

$\{+(\partial U_S/\partial Y)\int[(P_{IM} - P)/n_i]\partial X$ — ionized impurity charge or bulk charge drift term

$+(\partial/\partial Y)[(C_O/C_D) \times (2U_{GS} \times U_S - U_S^2)]$ — carrier space-charge drift term

$+(\partial/\partial Y)\int(\partial U/\partial X)^2\partial X\}$ — transverse electric-field drift term

$-qD_n n_i L_D(W/L) \times$

$\{-\partial/\partial Y\int(P_{IM} - P)/n_i \partial X$ — ionized impurity charge or bulk charge diffusion term

$+(\partial/\partial Y)[(C_O/C_D) \times 2U_S]\}$ — carrier space-charge diffusion term (29)

$I_{DP} = -kT\mu_p n_i L_D(W/L) \times$

$\{-(\partial U_S/\partial Y)\int(P_{IM} + N)/n_i \partial X$ — ionized impurity charge or bulk charge drift term

$-\partial/\partial Y[(C_O/C_D) \times (2U_{GS} \times U_S - U_S^2)]$ — carrier space-charge drift term

$-\partial/\partial Y\int(\partial U/\partial X)^2\partial X\}$ — transverse electric-field drift term

$-qD_p n_i L_D(W/L) \times$

$$\{-\partial/\partial Y\int(P_{IM}+N)/n_i\,\partial X \qquad \text{ionized impurity charge or bulk charge diffusion term}$$
$$-\partial/\partial Y\,[(C_O/C_D)\times 2U_S]\} \qquad \text{carrier space-charge diffusion term (30)}$$

If the silicon base of a single gate nMOSFET is very highly doped, then we have $P_{IM}/n_i\times(U-U_0)\gg(P-P_0)>(N-N_0)$, which is the depletion assumption in the silicon base. Using (23) and dropping the 2-dimensional term, the bulk charge drift and diffusion terms and transverse electric-field drift term in the drain electron current can be evaluated analytically as follows.

$$\begin{aligned}
\int(P_{IM}-P)/n_i\,\partial X &= \int[-\exp(U_P-U)+(P_{IM}/n_i)]/(\partial U/\partial X)\times\partial U \text{ from } U=U_S \text{ to } U_0\\
&= \int \text{sign}(U-U_0)\times\\
&\quad [\exp(U-U_N)-\exp(U_0-U_N)+\exp(U_P-U)-\exp(U_P-U_0)+(P_{IM}/n_i)\times(U-U_0)]^{-1/2}\\
&\quad \times\partial[\exp(U_P-U)-\exp(U_P-U_0)+(P_{IM}/n_i)\times(U-U_0)]\\
&\approx \int[0 \quad -0 \quad +\exp(U_P-U)-\exp(U_P-U_0)+(P_{IM}/n_i)\times(U-U_0)]^{-1/2}\\
&\quad \times\partial[\exp(U_P-U)-\exp(U_P-U_0)+(P_{IM}/n_i)\times(U-U_0)]\times\text{sign}(U_S-U_0)\\
&= -2[\exp(U_P-U_S)-\exp(U_P-U_0)+(P_{IM}/n_i)\times(U_S-U_0)]^{1/2}\times\text{sign}(U_S-U_0)\\
&\approx -2[0 \quad -0 \quad +(P_{IM}/n_i)\times(U_S-U_0)]^{1/2}\times\text{sign}(U_S-U_0)\\
&= -2(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{1/2}\times\text{sign}(U_S-U_0)
\end{aligned}$$

$$\int(P_{IM}-P)/n_i\,\partial X \approx -2\,(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{1/2}\times\text{sign}(U_S-U_0) \tag{31}$$

$$\begin{aligned}
\int(\partial U/\partial X)^2\partial X &= \int(\partial U/\partial X)\,\partial U \text{ from } U=U_S \text{ to } U_0\\
&= \int \text{sign}(U-U_0)\times[\exp(U-U_N)-\exp(U_0-U_N)\\
&\quad +\exp(U_P-U)-\exp(U_P-U_0)+(P_{IM}/n_i)\times(U-U_0)]^{1/2}\partial U\\
&\approx \text{sign}(U_S-U_0)\times\int[0 \quad -0\\
&\quad +0 \quad -0 \quad +(P_{IM}/n_i)\times(U-U_0)]^{1/2}\partial U\\
&= -(2/3)(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{3/2}\times\text{sign}(U_S-U_0)
\end{aligned}$$

$$\int(\partial U/\partial X)^2\partial X \approx -(2/3)(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{3/2}\times\text{sign}(U_S-U_0) \tag{32}$$

The analytical formula for the drain electron current for the single-gate nMOSFETs with highly doped silicon base-well can be obtained:

$$\begin{aligned}
I_{DN} = &-kT\mu_n n_i L_D(W/L)\times\text{sign}(U_S-U_0)\times\\
&(\partial/\partial Y)\{-(4/3)\times(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{3/2} && \text{ionized impurity charge or bulk charge drift term}\\
&+\text{sign}(U_S-U_0)\times(C_O/C_D)\times(2U_{GS}-U_S)\ U_S && \text{carrier space-charge drift term}\\
&-(2/3)\times(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{3/2}\} && \text{transverse electric-field drift term}\\
&-qD_n n_i L_D(W/L)\times\text{sign}(U_S-U_0)\times\\
&(\partial/\partial Y)\{+2\times(P_{IM}/n_i)^{1/2}\times(U_S-U_0)^{1/2} && \text{ionized impurity charge or bulk charge diffusion term}\\
&+\text{sign}(U_S-U_0)\times(C_O/C_D)\times 2U_S\} && \text{carrier space-charge diffusion term}
\end{aligned} \tag{33}$$

$$\begin{aligned}
I_{DN} = &-qD_n n_i L_D(W/L)\times(\partial/\partial Y)\{(C_O/C_D)\times(2U_{GS}+2-U_S)U_S\\
&+\text{sign}(U_S-U_0)\times(P_{IM}/n_i)^{1/2}\times[(-2)\times(U_S-U_0)^{3/2}+2\times(U_S-U_0)^{1/2}]\}
\end{aligned} \tag{34}$$

Semi-Infinite-Thickness Impure-Base (Bulk MOSFET)

This is the traditional or classic MOSFET with a single front MOS gate and a semi-infinite thick impure-base layer so that the remote boundary condition at the back boundary does not affect the transistor characteristics, except near and at flat-band, which has caused numerical divergence in compact modeling using the surface potential as the parameter, first reported and fixed by McAndrews, Gildenblat and collaborators[20,21]. Its fundamental cause was later traced[2] to the remote charge neutrality condition at the far boundary on the back. The remote boundary condition, such as open and short circuited, $U_N(x=\infty,y)-U_P(x=\infty,y)=U_{NP}(y)$ and $U_N(x=\infty,y)-U_P(x=\infty,y)=0$, showed little effect on the surface channel current except at flatband which is usually ignored because it is far below the parasitic leakage current of a real transistor, except when numerical solutions are need to connect the transistor properties, such as capacitance, on the two sides of the flatband. The preceding results for the 1-Gate impure thin-base are readily applied to the bulk MOSFET, such as the very thick (~500μm) impure-base ($P_{IM}=10^{17\text{-}18}\text{cm}^{-3}$)

on a 12-inch diameter silicon wafer, which has been the basic building block (BBB) device of all integrated circuits. Our previous analysis showed that the characteristic length of the FET is the Debye screening length of the semiconductor, which for the pure silicon base is about 25μm at room temperature, and which decreases as the impurity concentration increases, by the ratio of $(2n_i/P_{IM})^{1/2}$. So, for the traditional bulk MOSFET with ion-implanted impure base layer of $P_{IM}=10^{17\text{-}18}\text{cm}^{-3}$ and a room temperature intrinsic carrier concentration of $n_i=10^{10}\text{cm}^{-3}$, the impure Debye length drops to $25\mu m\times(2\times10^{10}/2\times10^{18})^{1/2}=2.5nm$. Thus, the current nanometer technology of 10 to 50nm dimensions would still qualify as long and thick transistors. For this case, $E_X(x=x_B\to\infty, y)=0$ still applies, and $U_0(x=x_B\to\infty, y)=U_0(y)$ can be assumed to take some boundary values determined by the contacts, such as $U_0(y=0)=U_{SB}$ and $U_0(y=L)=U_{DB}$ which replaces the meaningless or no longer viable thickness equation (25) so there is still enough conditions for the number of unknowns. In the past, it has also been assumed that there is no hole current flowing out through the terminals, so $\partial U_P(y)/\partial y=0$ or $U_P(y)=U_P=$ constant such as U_F. Alternative boundary conditions could be $U_0(x=x_B\to\infty, y=0)=0$ and $U_S(x=0, y=L)=U_{DB}$. Another pair was $U_0(x=x_B\to\infty, y)=0$ and $\partial^2U(x=x_B\to\infty, y)/\partial X^2=0$, the so-called remote charge neutrality condition to removed the imaginary electric field near flatband[2]. Thus, the voltage equations from finite-base-thickness (22) to (24) reduces to the following for the infinite-base-thickness Bulk transistor, with $U_0=0$ and the thickness equation removed. Or, more generally, for a body or source bias of $U_0(y)=U_{SB}=$ constant.

$$U_{GS}-U_S=(C_D/C_O)\times(\partial U/\partial X)_S-(x_O/L)^2\iint_{oxide}(\partial^2U/\partial Y^2)\partial X\partial X \quad \text{from (22)} \tag{35}$$

$$(\partial U/\partial X)^2=F^2(U,U_P,U_N,U_0,P_{IM},E_Y)=+\exp(U-U_N)-\exp(U_0-U_N)+\exp(U_P-U)-\exp(U_P-U_0)$$
$$+(P_{IM}/n_i)\times(U-U_0)+2(L_D/L)^2\int(\partial^2U/\partial Y^2)\partial_XU \quad \text{from (23)} \tag{36}$$

$$n_i\times[\exp(U_P-U_0)-\exp(U_0-U_N)]-P_{IM}=0\ \text{(remote charge neutrality)} \tag{37}$$

The general current or Y equations from (26) to (27) are

$$I_{DN}=-kT\mu_n n_i L_D(W/L)\times\{+\int(P_{IM}-P)/n_i(\partial U/\partial Y)\partial X$$
$$+2(\partial U/\partial X)_S(\partial U_S/\partial Y)+(\partial/\partial Y)\int(\partial U/\partial X)^2\partial X-(L_D/L)^2(\partial/\partial Y)\int(\partial U/\partial Y)^2\partial X\}$$
$$-qD_n n_i L_D(W/L)\times\{-(\partial/\partial Y)\int(P_{IM}-P)/n_i\partial X$$
$$-2(\partial/\partial Y)[(\partial U/\partial X)_S]+2(L_D/L)^2(\partial/\partial Y)\int(\partial^2U/\partial Y^2)\partial X\} \tag{38}$$

$$I_{DP}=-kT\mu_p n_i L_D(W/L)\times\{-\int(P_{IM}+N)/n_i(\partial U/\partial Y)\partial X$$
$$-2(\partial U/\partial X)_S(\partial U_S/\partial Y)-\partial/\partial Y\int(\partial U/\partial X)^2\partial X+(L_D/L)^2\,\partial/\partial Y\int(\partial U/\partial Y)^2\partial X\}$$
$$-qD_p n_i L_D(W/L)\times\{-\partial/\partial Y\int(P_{IM}+N)/n_i\partial X$$
$$+2\partial/\partial Y[(\partial U/\partial X)_S]-2(L_D/L)^2\,\partial/\partial Y\int(\partial^2U/\partial Y^2)\partial X\} \tag{39}$$

The further simplification of the above current equations when $P_{IM}=10^{17\text{-}18}\text{cm}^{-3}$ has been discussed by us[17]. By using the x-independent longitudinal electric field approximation[17] and the depletion assumptions[17] in the 2006-Jie-Sah analytical models, the analytical expressions for these drift and diffusion terms were derived.

$$+\int(P_{IM}-P)/n_i(\partial U/\partial Y)\partial X\approx+(\partial U_S/\partial Y)\int(P_{IM}-P)/n_i\partial X \tag{40}$$

$$\int(P_{IM}-P)/n_i\partial X=\int[-\exp(U_P-U)+(P_{IM}/n_i)]/(\partial U/\partial X)\times\partial U \text{ from } U=U_S \text{ to } 0$$
$$=\int\text{sign}(U)\times$$
$$[\exp(U-U_N)-\exp(-U_N)+\exp(U_P-U)-\exp(U_P)+(P_{IM}/n_i)\times U]^{-1/2}$$
$$\times\partial[\exp(U_P-U)-\exp(U_P)+(P_{IM}/n_i)\times U]$$
$$\approx\int[0\quad -0\quad +\exp(U_P-U)-\exp(U_P)+\exp(U_P)\times U]^{-1/2}$$
$$\times\partial[\exp(U_P-U)-\exp(U_P)+\exp(U_P)\times U]\times\text{sign}(U_S)$$

$$\int (P_{IM} - P)/n_i \,\partial X = -2[\exp(U_P - U_S) - \exp(U_P) + \exp(U_P) \times U_S]^{1/2} \times \mathrm{sign}(U_S) \quad (41)$$
$$\approx -2[0 \quad - \exp(U_P) + \exp(U_P) \times U_S]^{1/2} \times \mathrm{sign}(U_S)$$

$$\int (P_{IM} - P)/n_i \,\partial X = -2[\exp(U_P) \times U_S - \exp(U_P)]^{1/2} \times \mathrm{sign}(U_S) \quad (42)$$
$$\approx -2[\exp(U_P) \times U_S \quad - 0 \quad]^{1/2} \times \mathrm{sign}(U_S)$$

$$\int (P_{IM} - P)/n_i \,\partial X = -2[\exp(U_P) \times U_S]^{1/2} \times \mathrm{sign}(U_S) \quad (43)$$

Substituting (41−43) into (40), respectively

$$\int (P_{IM} - P)/n_i (\partial U/\partial Y) \partial X \approx (\partial/\partial Y)\{\exp(U_P/2)[\exp(-U_S) - 1 + U_S]^{3/2}\} \times (-4/3)\,\mathrm{sign}(U_S) \quad (44)$$

$$\int (P_{IM} - P)/n_i (\partial U/\partial Y) \partial X \approx (\partial/\partial Y)\{\exp(U_P/2)[U_S - 1]^{3/2}\} \times (-4/3)\,\mathrm{sign}(U_S) \quad (45)$$

$$\int (P_{IM} - \mathrm{P})/n_i (\partial U/\partial Y) \partial X \approx (\partial/\partial Y)\{\exp(U_P/2)[U_S]^{3/2}\} \times (-4/3)\,\mathrm{sign}(U_S) \quad (46)$$

$$\int (\partial U/\partial X)^2 \partial X = \int (\partial U/\partial X)\, \partial U \text{ from } U = U_S \text{ to } 0$$
$$= \int \mathrm{sign}(U) \times [\exp(U - U_N) - \exp(-U_N) + \exp(U_P - U) - \exp(U_P) + (P_{IM}/n_i) \times \mathrm{U}]^{1/2} \partial U$$

Using the different depletion assumptions in (41−43), respectively

$$\int (\partial U/\partial X)^2 \partial X \approx \int \mathrm{sign}(U) \times [0 \quad - 0 \quad + \exp(U_P - U) - \exp(U_P) + \exp(U_P) \times U]^{1/2} \partial U$$
$$\approx \{\exp(U_P/2)[\exp(-U_S) - 1 + U_S]^{3/2}\} \times (-2/3)\,\mathrm{sign}(U_S) \quad (47)$$

$$\int (\partial U/\partial X)^2 \partial X \approx \int \mathrm{sign}(U) \times [0 \quad - 0 \quad + 0 \quad - \exp(U_P) + \exp(U_P) \times U]^{1/2} \partial U$$
$$= \{\exp(U_P/2)[U_S - 1]^{3/2}\} \times (-2/3)\,\mathrm{sign}(U_S) \quad (48)$$

$$\int (\partial U/\partial X)^2 \partial X \approx \int \mathrm{sign}(U) \times [0 \quad - 0 \quad + 0 \quad - 0 \quad + \exp(U_P) \times U]^{1/2} \partial U$$
$$= \{\exp(U_P/2)[U_S]^{3/2}\} \times (-2/3)\,\mathrm{sign}(U_S) \quad (49)$$

By substituting (35) without 2-dimensional term, (41−43), (44−46), and (47−49) into (38) without 2-dimensional terms, respectively, three 1-dimensional analytical drain electron current expressions can be obtained.

$I_{DN} = -kT\mu_n n_i L_D (W/L) \times \mathrm{sign}(U_S) \times$
$(\partial/\partial Y)\{-(4/3) \times \exp(U_P/2) \times [U_S - 1 + \exp(-U_S)]^{3/2}$ — ionized impurity charge or bulk charge drift term
$+ \mathrm{sign}(U_S) \times (C_O/C_D) \times (2U_{GS} - U_S) U_S$ — carrier space-charge drift term
$-(2/3) \times \exp(U_P/2) \times [U_S - 1 + \exp(-U_S)]^{3/2}\}$ — transverse electric-field drift term
$- qD_n n_i L_D (W/L) \times \mathrm{sign}(U_S) \times$
$(\partial/\partial Y)\{+2 \times \exp(U_P/2) \times [U_S - 1 + \exp(-U_S)]^{1/2}$ — ionized impurity charge or bulk charge diffusion term
$+ \mathrm{sign}(U_S) \times (C_O/C_D) \times 2U_S\}$ — carrier space-charge diffusion term (50)

$I_{DN} = -kT\mu_n n_i L_D (W/L) \times \mathrm{sign}(U_S) \times$
$(\partial/\partial Y)\{-(4/3) \times \exp(U_P/2) \times (U_S - 1)^{3/2}$ — ionized impurity charge or bulk charge drift term
$+ \mathrm{sign}(U_S) \times (C_O/C_D) \times (2U_{GS} - U_S) U_S$ — carrier space-charge drift term
$-(2/3) \times \exp(U_P/2) \times (U_S - 1)^{3/2}\}$ — transverse electric-field drift term
$- qD_n n_i L_D (W/L) \times \mathrm{sign}(U_S) \times$
$(\partial/\partial Y)\{+2 \times \exp(U_P/2) \times (U_S - 1)^{1/2}$ — ionized impurity charge or bulk charge diffusion term
$+ \mathrm{sign}(U_S) \times (C_O/C_D) \times 2U_S\}$ — carrier space-charge diffusion term (51)

$I_{DN} = -kT\mu_n n_i L_D (W/L) \times \mathrm{sign}(U_S) \times$
$(\partial/\partial \mathrm{Y})\{-(4/3) \times \exp(U_P/2) \times (U_S)^{3/2}$ — ionized impurity charge or bulk charge drift term
$+ \mathrm{sign}(U_S) \times (C_O/C_D) \times (2U_{GS} - U_S)\, U_S$ — carrier space-charge drift term
$-(2/3) \times \exp(U_P/2) \times (U_S)^{3/2}\}$ — transverse electric-field drift term
$- qD_n n_i L_D (W/L) \times \mathrm{sign}(U_S) \times$
$(\partial/\partial Y)\{+2 \times \exp(U_P/2) \times (U_S)^{1/2}$ — ionized impurity charge or bulk charge diffusion term
$+ \mathrm{sign}(U_S) \times (C_O/C_D) \times 2U_S\}$ — carrier space-charge diffusion term (52)

3.2 2-Gate MOSFETs

The results just obtained for thick and thin 1-Gate can be easily extended to 2-Gate transistors. We shall first give the equations for the general 2-asymmetrical-Gate finite-thickness impure base, then simplify to the supposedly ultimate 2-symmetrical-Gate finite-thickness pure-base. These are known as the FinFETs[22] while the (2-)wrap-around-gate(s) finite-thickness-width impure-base is known as the nanowire FET[23].

2-Asymmetrical-Gate Finite-Thickness Impure-Base

We apply the general results, listed in (17) to (21), with the two gates labeled by 1 in the front at $x=0$ and 2 in the back at $x=x_B$, and we apply the same kind of boundary condition to the second gate as we have applied to the first gate. Thus, by inspection of (17) to (21), the results for the asymmetrical 2-Gate Impure-Base transistor are easily obtained. Here the asymmetry is from the two different gates, from two different applied gate voltages, different oxide thicknesses, different oxide charges, and/or different gate-material-work-functions. The gate flat-band voltages are included in the applied gate voltages, U_{G1} and U_{G2}. The terminal voltages, applied to or measured at each terminal contact point, are all measured with respect to a common reference point, such as $U(\boldsymbol{r}=\infty)=0$ at infinity by virtual of the Coulomb Law, or at some reference point, $\boldsymbol{r}_R \equiv (x_R, y_R, z_R)$ where $R=B$ for a real or virtual base contact with a applied constant potential U_B relative to that at infinity which is the Coulomb zero potential reference. We can also find a reference point $R=F$ that is always at equilibrium (or nearly so under all transient and steady-state application conditions of the transistor) so that the electron and hole electrochemical potentials or quasi-Fermi potentials at this reference point are equal and given by $U_P(\boldsymbol{r}_R=\boldsymbol{r}_F)=U_F$ and $U_N(\boldsymbol{r}_R=\boldsymbol{r}_F)=U_F$ and U_F is given by $P_F=n_i\exp(+U_F)$, $N_F=n_i\exp(-U_F)$, and that is electrically neutral, $\rho(\boldsymbol{r}_R=\boldsymbol{r}_F)=q(P_F-N_F-P_{IM})=0$. In fact, any point can be used as a reference as long as its electric and electron and hole electrochemical potentials, and their first and second derivatives are given at a particular excitation or applied voltages to the terminals and applied optical and particle exposure to the transistor. To wit, this boundary value problem is precisely defined mathematically, not the perceived uncertainty disguised as the floating base. The choice of the reference potential point is one to make the solution the simplest, for example the source contact point in the 2-Gate MOSFET.

$$(\partial U/\partial X)^2 = F^2(U,U_P,U_N,U_{S2},P_{IM},E_Y) + (\partial U/\partial X)_{S2}{}^2 = F^2(U,U_P,U_N,U_{S2},P_{IM},E_Y,E_{X2}) \text{ from (17)} \quad (53)$$

$$= +\exp(U-U_N)-\exp(U_{S2}-U_N)+\exp(U_P-U)-\exp(U_P-U_{S2}) + (P_{IM}/n_i)\times(U-U_{S2}) + 2(L_D/L)^2\int(\partial^2 U/\partial Y^2)\partial_X U + (\partial U/\partial X)_{S2}{}^2 \quad \text{from (18)} \quad (54)$$

$$\partial U/\partial X = \text{sign}(U-U_{S2})\times\{F^2(U,U_P,U_N,U_{S2},P_{IM},E_Y,E_{X2})\}^{1/2} \quad (55)$$

$$U_{G1}-U_{S1}(Y) = (C_D/C_O)\times(\partial U/\partial X)_{S1} - (x_{O1}/L)^2\iint_{\text{oxide1}}(\partial^2 U/\partial Y^2)\partial X\partial X - (1/2)(x_{O1}/L_D)^2\iint(\rho_{\text{oxide1}}/qn_i)\partial X\partial X \quad \text{from (19)} \quad (56)$$

$$U_{G2}-U_{S2}(Y) = (C_D/C_O)\times(\partial U/\partial X)_{S2} - (x_{O2}/L)^2\iint_{\text{oxide2}}(\partial^2 U/\partial Y^2)\partial X\partial X - (1/2)(x_{O2}/L_D)^2\iint(\rho_{\text{oxide2}}/qn_i)\partial X\partial X \quad \text{from (19)} \quad (57)$$

$$X_B = \int \text{sign}(U-U_{S2})\partial_X U/F \text{ (Integrated from } U=U_{S2} \text{ to } U=U_{S1}\text{) (The Thickness Equation)} \quad (58)$$

Two distinct boundary conditions at $0 < X = X_0 < X_B$ in the finite-thickness base can be specified from the polarity of the voltage applied to the two gates: the boundary condition $(\partial U/\partial X)_0=0$ when the two gate-to-source voltages have the same polarity, and the boundary condition $U_0=U_{SB}$ when the two gate-to-source voltages have the opposite polarity. These two conditions at $X=X_0$ result in different approaches to be used to numerically compute the above voltage and thickness equations.

The general current or Y equations from (20) to (21) or (15) to (16) are

$$\begin{aligned} I_{DN} = & -kT\mu_n n_i L_D(W/L)\times\{+\int(P_{IM}-P)/n_i(\partial U/\partial Y)\partial X \\ & -2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y)-(\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)] \\ & +\partial/\partial Y\int(\partial U/\partial X)^2\partial X-(L_D/L)^2\,\partial/\partial Y\int(\partial U/\partial Y)^2\partial X\} \\ & -qD_n n_i L_D(W/L)\times\{-\partial/\partial Y\int(P_{IM}-P)/n_i\,\partial X \\ & +2\partial/\partial Y[(\partial U/\partial X)_{S2}-(\partial U/\partial X)_{S1}]+2(L_D/L)^2\,\partial/\partial Y\int(\partial^2 U/\partial Y^2)\partial X\} \end{aligned} \quad (59)$$

$$I_{DP} = -kT\mu_p n_i L_D(W/L) \times \{-\int (P_{IM} + N)/n_i(\partial U/\partial Y)\partial X$$
$$+2[(\partial U/\partial X)_{S2}(\partial U_{S2}/\partial Y) - (\partial U/\partial X)_{S1}(\partial U_{S1}/\partial Y)]$$
$$-\partial/\partial Y\int(\partial U/\partial X)^2\partial X + (L_D/L)^2\,\partial/\partial Y\int(\partial U/\partial Y)^2\partial X\}$$
$$-qD_p n_i L_D(W/L) \times \{-\partial/\partial Y\int(P_{IM} + N)/n_i\,\partial X$$
$$-2\partial/\partial Y[(\partial U/\partial X)_{S2} - (\partial U/\partial X)_{S1}] - 2(L_D/L)^2\partial/\partial Y\int(\partial^2 U/\partial Y^2)\partial X\} \quad (60)$$

Symmetrical-Gate Thin Pure-Base (FinFET)

This is a special case with simpler solutions than the general asymmetrical case just described. It is the FinFET geometry that has been analyzed by many[24] due to its promising technological importance[22]. Due to the complete symmetry between the two gates, including applied gate voltages which contain the offset from workfunction differences of the contacts, by virtue of connecting the two gates, the solution is symmetrical with respect to the bisection plane, $x = x_B/2$ where $U(x = x_B/2, y) = U_0(y)$ is a maximum (when $U_{GS} > 0$) or minimum (when $U_{GS} < 0$) and at this plane, we have $\partial U(x = x_B/2, y)/\partial x = 0$. The symmetry of two gates also gives $U_{S1}(y) = U_{S2}(y) = U_S(y)$, while $(\partial U/\partial X)_{S1} = -(\partial U/\partial X)_{S2} = (\partial U/\partial X)_S$. The solutions are obtained by treating just half of the thickness of the channel, $x = 0$ to $x = x_B/2$, giving the following equations from the general results just obtained for the asymmetrical gates.

$$(\partial U/\partial X)^2 = F^2(U, U_P, U_N, U_0, P_{IM}, E_Y) = +\exp(U - U_N) - \exp(U_0 - U_N) + \exp(U_P - U) - \exp(U_P - U_0)$$
$$+(P_{IM}/n_i) \times (U - U_0) + 2(L_D/L)^2\int(\partial^2 U/\partial Y^2)\partial_X U \text{ from (54)} \quad (61)$$

$$(\partial U/\partial X)_S = \mathrm{sign}(U_S - U_0) \times \{F^2(U_S, U_P, U_N, U_0, P_{IM}, E_Y)\}^{1/2} \quad \text{from (55)} \quad (62)$$

$$U_{GS} - U_S = (C_D/C_O) \times (\partial U/\partial X)_S \quad \text{(1-D term)}$$
$$-(x_O/L)^2\iint_{\text{oxide}}(\partial^2 U/\partial Y^2)\partial X\partial X - (1/2)(x_O/L_D)^2\iint(\rho_{\text{oxide}}/qn_i)\partial X\partial X \text{ (2-D terms) from (56)} \quad (63)$$

$$X_B = 2\int \mathrm{sign}(U - U_0)\partial_X U/F \quad \text{(Integrated from } U = U_0 \text{ to } U = U_S\text{) (The Thickness Equation)} \quad (64)$$

The general current or Y equations are

$$I_{DN} = -kT\mu_n n_i L_D(W/L) \times \{+\int(P_{IM} - P)/n_i(\partial U/\partial Y)\partial X$$
$$+2(\partial U/\partial X)_S(\partial U_S/\partial Y) + \partial/\partial Y\int(\partial U/\partial X)^2\partial X - (L_D/L)^2\,\partial/\partial Y\int(\partial U/\partial Y)^2\partial X\}$$
$$-qD_n n_i L_D(W/L) \times \{-\partial/\partial Y\int(P_{IM} - P)/n_i\,\partial X$$
$$-2\partial/\partial Y[(\partial U/\partial X)_S] + 2(L_D/L)^2\,\partial/\partial Y\int(\partial^2 U/\partial Y^2)\partial X\} \quad (65)$$

$$I_{DP} = -kT\mu_p n_i L_D(W/L) \times \{-\int(P_{IM} + N)/n_i(\partial U/\partial Y)\partial X$$
$$-2[(\partial U/\partial X)_S(\partial U_S/\partial Y)] - \partial/\partial Y\int(\partial U/\partial X)^2\partial X + (L_D/L)^2\partial/\partial Y\int(\partial U/\partial Y)^2\partial X\}$$
$$-qD_p n_i L_D(W/L) \times \{-\partial/\partial Y\int(P_{IM} + N)/n_i\,\partial X$$
$$+2\partial/\partial Y[(\partial U/\partial X)_S] - 2(L_D/L)^2\,\partial/\partial Y\int(\partial^2 U/\partial Y^2)\partial X\} \quad (66)$$

By dropping all the 2-dimensional terms and P_{IM} terms, both the current equation and voltage equation can be simplified as follows:

$$\partial U/\partial X = \mathrm{sign}(U - U_0) \times F(U, U_P, U_N, U_0)$$
$$= \mathrm{sign}(U - U_0) \times [\exp(U - U_N) - \exp(U_0 - U_N) + \exp(U_P - U) - \exp(U_P - U_0)]^{1/2} \quad (67)$$

$$U_{GS} - U_S = (C_D/C_O) \times (\partial U/\partial X)_S \quad (68)$$

$$I_{DN} = -kT\mu_n n_i L_D(W/L) \times$$
$$\{+\partial/\partial Y[(C_O/C_D) \times (2U_{GS} \times U_S - U_S^2)] \quad \text{carrier space-charge drift term}$$
$$+\partial/\partial Y\int(\partial U/\partial X)^2\partial X\} \quad \text{transverse electric-field drift term}$$
$$-qD_n n_i L_D(W/L) \times \{+\partial/\partial Y[(C_O/C_D) \times 2U_S]\} \quad \text{carrier space-charge diffusion term} \quad (69)$$

$$
\begin{aligned}
I_{DP} = & -kT\mu_p n_i L_D(W/L) \times \\
& \{-\partial/\partial Y[(C_O/C_D)\times(2U_{GS}\times U_S - U_S^2)] && \text{carrier space-charge drift term} \\
& -\partial/\partial Y\int(\partial U/\partial X)^2\partial X\} && \text{transverse electric-field drift term} \\
& - qD_p n_i L_D(W/L)\times\{-\partial/\partial Y[(C_O/C_D)\times 2U_S]\} && \text{carrier space-charge diffusion term (70)}
\end{aligned}
$$

The further simplification of the above equations for the one-section and two-sections unipolar solution of the bipolar theory of the FinFET with pure base has been described in [25] and [26].

4 Summary

The general analytical solutions in the drift-diffusion representation of the bipolar DC current-voltage characteristics of the bipolar field effect transistor are derived. The solutions cover both the one-gate finite thickness SOI and TFT and semi-infinite thickness bulk MOSFET, and the general two-gate transistor and its simplest case of symmetrical two-gate and pure-base, the FinFET. The drift and diffusion current terms are easily identified by the multipliers of drift mobility and diffusivity. This distinction also serves to include high electric-field effects such as the mobility variation with electric field, and the interband impact generation of electron-hole pairs in the high-electric-field drain junction space-charge region when the drain-source voltage exceeds the drain-gate voltage. This is a principle mechanism for standby current and power dissipation.

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场引晶体管理论:XII. 双极飘移扩散电流（薄及厚、纯及不纯基体,单及双 MOS 栅极）*,**

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摘要： 上篇论文(XI)报告双极场引晶体管的电化电流理论，这篇论文(XII)报告飘移扩散理论．两篇都讨论了单栅双栅，纯基不纯基，薄基厚基的情形．两篇都用表面及内部电势作为参变量耦合电压方程和电流方程．在这飘移扩散理论中，有许多电流项，属飘移电流的用迁移率因子标识，属扩散电流的用扩散率因子标识．给出完备飘移扩散解析方程，用以计算四种常用 MOS 晶体管的直流电流电压特性．飘移电流有四项：一维体电荷漂移项，一维载子空间电荷漂移项，一维横向电场漂移项，二维漂移项．扩散电流有三项：一维体电荷扩散项，一维载子空间电荷扩散项，二维扩散项．现有的晶体管理论都没认识到一维横向电场漂移项．当基区几乎是纯基，这项贡献显著，约总电流的 25%．二维项来自纵向电场的纵向梯度，它随德拜长度对沟道长度比率的平方按比例变化．当沟道长度为 25nm，几乎纯基时，$(L_D/L)^2=10^6$，杂质浓度 $10^{18}cm^{-3}$ 时，$(L_D/L)^2=10^{-2}$．

关键词： 双极场引晶体管理论；表面势；飘移扩散理论；单栅不纯基；双栅不纯基
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