Quantum Measurement of Single Electron State by a Mesoscopic Detector *

Hu Xuening†, Luo Junyan, and Li Xinqi

(State Key Laboratory for Superlattices and Microstructures, Institute of Semiconductors, Chinese Academy of Sciences, Beijing 100083, China)

Abstract: A realistic measurement setup for a system such as a charged two-state (qubit) or multi-state quantum system measured by a mesoscopic detector, is theoretically studied. To properly describe the measurement-induced back-action, a detailed-balance preserved quantum master equation treatment is developed. The established framework is applicable for arbitrary voltages and temperatures.

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1 Introduction

The measurement of a single electron state in solid-state systems has attracted widespread attention in recent years[1~4]. This renewed interest largely stems from the new field of quantum computation, since a quantum-measurement procedure is needed, for instance, at the end of a computation to read the final results, or even in the course of computation for the purpose of error correction. A possible implementation of quantum measurement in solid states is a charge qubit measured by a mesoscopic detector, which could be, for example, the quantum point contact (QPC)[1~4]. Very recently, an elegant experiment was performed by employing a QPC to measure the quantum dot occupation by an extra electron, which is further associated with a single electron spin state[5]. This experiment clearly demonstrated the extremely high sensitivity of the QPC detector, implying its possible wide application in the future. It is therefore important to develop a reliable theoretical description for this important quantum measurement device.

This measurement problem was first studied theoretically by Gurvitz[1], followed by many other groups[2~4]. Here we mention three typical approaches employed in the literature: (1) the so-called Bloch equation approach developed in Ref. [1] and a number of other papers by Gurvitz et al.; (2) the quantum trajectory technique from quantum optics by Goan et al.[6,7]; and (3) the Bayesian approach by Korotkov et al.[8~10]. In spite of their different forms in appearance, these three approaches are equivalent in essence. In particular, all of them are based on the same (unconditional) Lindblad master equation. However, as clearly manifested in Ref. [1], the associated Lindblad master equation would result in the universal equal occupation probability on the qubit states in a stationary state. Obviously, under finite voltages this result breaks down the detailed balance condition, which is thus valid only at a high voltage limit[11,12]. In this work, we extend the study to arbitrary voltages. Furthermore, beyond the two-state qubit, we will also consider a multi-state system.

2 Model description and formalism

As schematically shown in Fig. 1, for general-

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† Corresponding author. Email: xnHu@red.semi.ac.cn

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ity (rather than the two-state qubit), let us consider an electron in a one-dimensional array of coupled quantum wells, which is measured by a mesoscopic QPC. The entire system Hamiltonian reads

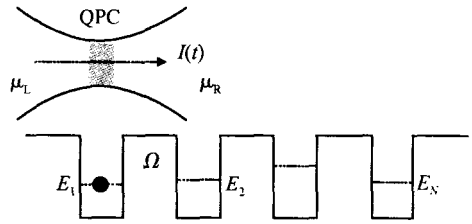

Fig. 1　Schematic illustration of using the mesoscopic quantum-point-contact to measure an electron in multiple coupled quantum wells

$$H = H_0 + H' \tag{1a}$$

$$H_0 = H_S + \sum_k (\varepsilon_k^L c_k^\dagger c_k + \varepsilon_k^R d_k^\dagger d_k) \tag{1b}$$

$$H_S = \sum_{j=1}^{N} \varepsilon_j c_j^\dagger c_j + \sum_{j=1}^{N-1} (\Omega_j c_{j+1}^\dagger c_j + \text{H.c.}) \tag{1c}$$

$$H' = \sum_{k,q} (\Omega_{qk} + \sum_j \chi_{qk}^j c_j^\dagger c_j)\, c_k^\dagger d_q + \text{H.c.} \tag{1d}$$

In this decomposition, the free part of the total Hamiltonian, H_0, contains Hamiltonians of the measured system (H_S) and the QPC reservoirs (the last two terms). The operator $c_j^\dagger$ (c_j) corresponds to the creation (annihilation) of an electron in the j_{th} well. For simplicity we assume that each well contains a single bound state ε_j and is coupled only to its nearest neighbors with couplings Ω_j and Ω_{j-1}. $c_k^\dagger$ (c_k) and $d_k^\dagger$ (d_k) are, respectively, the electron creation (annihilation) operators of the left and right reservoirs of the QPC. The Hamiltonian H' describes electron tunneling between the two reservoirs of the QPC detector with, for instance, tunneling amplitude $\Omega_{qk} + \sum_j \chi_{qk}^j c_j^\dagger c_j$, which generally depends on the measured electron's position that is characterized by the occupation operator $c_j^\dagger c_j$. This dependence properly describes the correlation between the detector and the measured system, which enables us to draw out measurement information from the output current, and simultaneously propagates the back-action of the detector onto the measured system, causing state dephasing and relaxation.

Statistically, the measurement back-action onto the measured system is described by a quantum master equation (QME) that is satisfied by the reduced density matrix. Regarding the tunneling Hamiltonian H' as a perturbation, the second-order cumulant expansion gives rise to a formal equation for the reduced density matrix[13]:

$$\dot{\rho}(t) = -\mathrm{i}L\rho(t) - \int_0^t \mathrm{d}\tau \langle L'(t)\, G(t,\tau)\, L'(\tau)\, G^\dagger(t,\tau)\rangle \rho(t) \tag{2}$$

Here the Liouvillian superoperators are defined as, $L(\cdots) \equiv [H_S, (\cdots)]$, $L'(\cdots) \equiv [H', (\cdots)]$, and $G(t,\tau)(\cdots) \equiv G(t,\tau)(\cdots) G^\dagger(t,\tau)$ with $G(t,\tau)$ the usual propagator (Green's function) associated with H_S. The reduced density matrix $\rho(t) = \mathrm{Tr}_D[\rho_T(t)]$, results from tracing out all the detector degrees of freedom from the entire density matrix. Following Ref. [11], the unified QME is obtained as

$$\dot{\rho} = -\mathrm{i}L\rho - \frac{1}{2}[Q, \tilde{Q}\rho - \rho Q^\dagger] \tag{3}$$

where $Q \equiv \Omega_0 + \sum_j \chi_j c_j^\dagger c_j$. For simplicity, we have assumed $\Omega_{qk} = \Omega_0$ and $\chi_{qk}^j = \chi_j$, i.e. the tunneling amplitudes are of reservoir-state independence. Other quantities and notations in Eq. (3) are explained as follows. $\tilde{Q} = \tilde{Q}^{(+)} + \tilde{Q}^{(-)}$, and $\tilde{Q}^{(\pm)} = \tilde{C}^{(\pm)}(L)\, Q$, with $\tilde{C}^{(\pm)}(L)$ the spectral function of the QPC reservoirs. Under wide-band approximation, $\tilde{C}^{(\pm)}(L)$ can be explicitly carried out as: $\tilde{C}^{(\pm)}(L) = \eta[x/(1 - e^{-x/T})]_{x=-L\mp V}$, where $\eta = 2\pi g_L g_R$, with g_L (g_R) the density of states (DOS) of the left (right) reservoir, and T is the temperature. In this work we use the unit system of $\hbar = e = k_B = 1$. The meaning of the superoperator function $\tilde{C}^{(\pm)}(L)$ will become more clear by explicitly carrying out the matrix elements of $\tilde{Q}^{(\pm)}$. In the eigen-state basis $\{|E_m\rangle\}$, we easily obtain $\tilde{Q}_{mn}^{(\pm)} = \tilde{C}^{(\pm)}(\pm\omega_{mn})\, Q_{mn}$, where $\omega_{mn} = E_m - E_n$ and $Q_{mn} = \langle E_m | Q | E_n \rangle$. In this derivation, the simple algebra $\langle E_m | LQ | E_n \rangle = \langle E_m | (H_S Q - Q H_S) | E_n \rangle = (E_m - E_n)\, Q_{mn}$ has been used. Here we see clearly that the Liouvillian operator "L" in $\tilde{C}^{(\pm)}(L)$ properly involves the energy transfer between the detector and the measured system into the transition rates, and thus implies a detailed balance condition which determines the stationary occupation probabilities.

The last term $[\cdots]$ in Eq. (3) describes the back-action of the detector on the measured system. In the high-voltage limit, formally $V \gg L$, the spectral function $\tilde{C}^{(\pm)}(L) \simeq \tilde{C}^{(\pm)}(0)$, and Eq. (3)

reduces to a Lindblad-type master equation

$$\dot{\rho} = -\mathrm{i}L\rho + \tilde{C}(0)\left[Q\rho Q - \frac{1}{2}(Q^2\rho + \rho Q^2)\right] \quad (4)$$

where $\tilde{C}(0) = \tilde{C}^{(+)}(0) + \tilde{C}^{(-)}(0)$. It is straightforward to check that this equation is the master equation that appeared in Refs. [1,6～10].

In the following, we study the measurement-induced relaxation of a qubit (i.e. N = 2) and a multi-state system, respectively. The detailed-balance property of our QME will be elaborated, and discussions will be highlighted to some important features resulting from the detailed balance, which is absent in the Lindblad-type Eq. (4).

3 Measurement of a qubit

For a single qubit, there is a bound level in each well, ε_a and ε_b. For clarity, our discussion is restricted in the symmetric qubit case ($\varepsilon_a = \varepsilon_b$). Under quantum measurement, a pure state of the qubit state evolves into a statistical mixture. Figure 2 shows such evolution by plotting the time-dependent occupation probabilities on the individual dot states. The relevant parameters for numerical calculations are adopted as follows: the applied voltage over the QPC V = Ω, the inverse temperature $\beta = 1/\Omega$, the DOS in both electron reservoirs $g_L = g_R = 2/\Omega$, and for the QPC, we assume the tunneling amplitudes $\Omega_0 = \Omega$ and $\chi_a = 0.15\Omega$, $\chi_b = 0$.

In the dot-state representation as shown in Fig. 2(a), despite certain quantitative differences on a short time scale, a common final occupation probability of 1 = 2 in each dot is approached, regardless of whether the detailed balance is satisfied. However, in contrast to Ref. [1], this feature is valid only for symmetric qubits, while in the asymmetric case, it is only valid in the limit of large measurement voltage, as we will show in the multi-level system.

Moreover, even in the symmetric case, the impact of the detailed balance on the qubit relaxation can be revealed by transforming the result in Fig. 2(a) into the qubit eigenstate representation, as shown in Fig. 2(b). Noting that initially the electron is located in the left dot, which is equivalent to a 1 = 2 probability in each eigenstate of the symmetric qubit, the constant dashed line in Fig. 2(b) indicates an equal occupation probability of

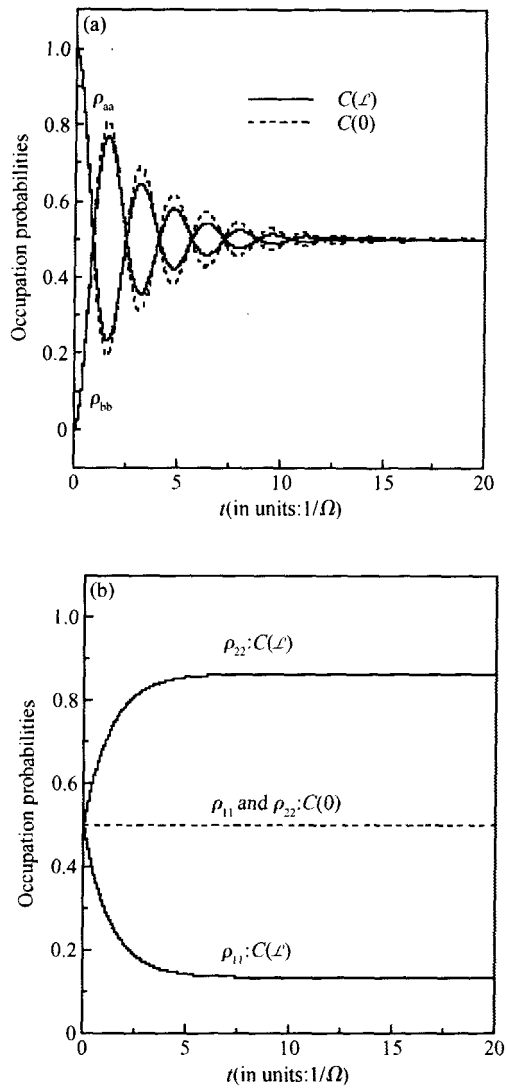

Fig. 2 Measurement induced qubit relaxation in the individual dot-state representation (a) and the eigenstate representation (b) The results in the presence and the absence of the detailed balance are symbolized by "C(L)" and "C(0)", respectively.

the two eigenstates in the absence of detailed balance. However, if the energy exchange between the measured qubit and the QPC detector is correctly accounted for, relaxation between the eigenstates will lead to quite different occupation probabilities as shown by the solid curves in Fig. 2(b).

For the symmetric qubit, since $\varepsilon_a = \varepsilon_b$, the final equal occupation probability of 1 = 2 in each dot is anticipated. However, as shown by Gurvitz[1], in the asymmetric case (i.e. for non-identical coupled dots), a final equal occupation prob-

ability of 1 = 2 in each dot will also be approached. A similarly confusing feature also existed in the breakdown of the Anderson localization, where equal occupation probabilities on each site of the disordered chain were concluded[14]. We would like to emphasize here that the results in Refs. [1,14] are valid only in the limit of large measurement voltage.

4 Measurement of a multi-state system

Numerically, we further consider N = 20 wells with randomly distributed energy levels (ε_j). To distinguish the electron's position in each quantum well, we assume quantum-well-state-dependent tunneling coefficients through the QPC, i. e. $\Omega_0 + \chi_j$, and $\chi_j = \Omega \sqrt{4 + (j - 1)^2}$ with j = 1, 2, ···, 20. The numerical results for measurement-induced relaxation are shown in Fig. 3, where we assume $\Omega_j = \Omega$, and other parameters are the same as in the qubit measurement.

Rather than working in a large voltage limit as in Ref. [14], here we particularly focus on the low voltage regime, say, $V < \Delta$, with Δ the disorder strength. Initially (at time t = 0), the electron is assumed to be in the ground state, with a distribution probability dominantly localizing in the eighteenth well, as shown by the solid curves in Fig. 3. As a result of the measurement, the state relaxation gradually takes place, i. e. delocalization leads to the redistribution of electron probability in each well. Note that our result shown in Fig. 3 (a) differs considerably from that in Fig. 3 (b) based on the Lindblad-type master equation, Eq. (4). The latter shows that after sufficient relaxation each well is occupied with an identical probability, which was proven analytically in Ref. [14]. However, our treatment leads to unequal occupation probabilities in each well. This discrepancy originates from whether the inelastic energy exchange between the detector and the measured system is properly included in the transition rates[11,12], which leads to a non-trivial detailed balance condition. Remarkably, we notice that ignoring this inelastic effect in the transition rates is equivalent to assuming the large voltage limit. This is in particular illustrated by Fig. 3 (c) in comparison with Fig. 3 (b).

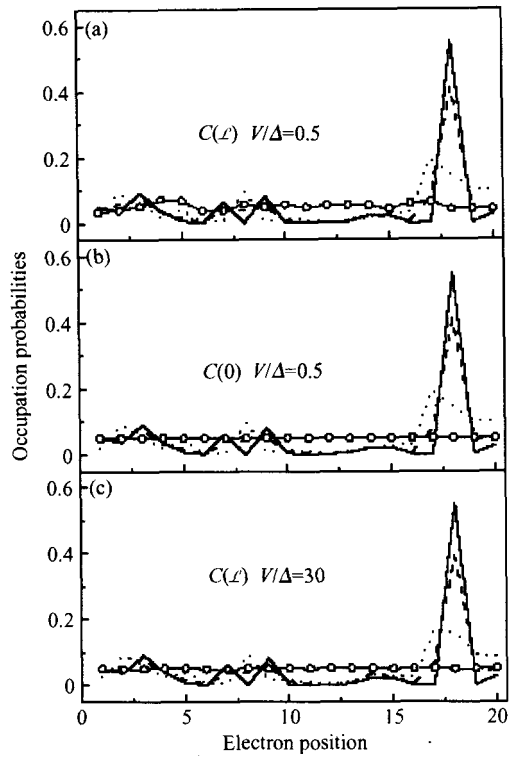

Fig. 3　Measurement-induced delocalization of the electron, which is initially dominantly localized in the eighteenth quantum well (in the ground state), as denoted by the solid curves. Shown in the figure by the dashed, dotted, and symbol curves are the distribution probabilities in each well, at times 0. 4, 0. 8, and $500\Omega^{-1}$, respectively. In the low voltage regime, the detailed-balance preserved result in (a) differs considerably from that in (b) obtained from the Lindblad master equation, Eq. (4). In the high voltage limit, the result in (c) from Eq. (3) recovers the prediction by Eq. (4).

5 Conclusion

We have studied the relaxation nature of a qubit and a multistate system under the quantum measurement of a mesoscopic detector. Differing from the Lindblad-type master equation[i.e. Eq. (4)], which is in fact the starting point of some recent literature[1,6~10,14], our treatment properly accounts for the energy exchange between the measured system and the detector, which leads to the valid detailed-balance relation and correct relaxation behavior. The present work may shed new light on the future study of solid-state quantum measurement and quantum feedback control.

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介观测量仪器对单电子态的量子测量*

胡学宁† 骆钧炎 李新奇

(中国科学院半导体研究所 超晶格和微结构国家重点实验室, 北京 100083)

摘要: 对于一个实际的测量装置,即被介观测量仪器所测量的两态(量子比特)或多态量子系统做了理论研究. 为了正确描述测量引起的反作用,发展了一个保持细致平衡条件的量子主方程的方法,建立的这套方案适用于任意电压和任意温度.

关键词: 量子测量; 细致平衡; 弛豫

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† 通信作者. Email: xnHu@red.semi.ac.cn

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