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Temperature Dependence of Vacuum Rabi Splitting in a Single Quantum Dot-Semiconductor Microcavity*

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Abstract: The quantum dynamics of an exciton dressed by acoustic phonons in an optically driven quantum dot-semiconductor microcavity at finite temperatures is investigated theoretically by quantum optics methods. It is shown that the temperature dependence of the vacuum Rabi splitting is $2\sqrt{2}g \times \exp[-\sum_q \lambda_q (N_q + 1/2)]$, where $N_q = 1/[\exp(\omega_q/k_B T) - 1]$ is the phonon population, g is the single-photon Rabi frequency, and λ_q corresponds to exciton-phonon coupling.

Key words: semiconductor microcavity; quantum dot; exciton; Vacuum Rabi splitting

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1 Introduction

Quantum computation has been shown to perform certain tasks much faster than classical computers. The most important role to this speedup is the entanglement due to the superposition of states in quantum bits (qubits)[1~3]. But the requisite quantum coherence is very fragile, and can be destroyed by interactions with the environment or other noise sources. Therefore qubits must be sufficiently isolated from the environment so that they can maintain their coherence throughout computation. But for quantum computations based on solid-state qubits, it is impossible to isolate the qubits from lattice vibrations, even at zero temperature. Overcoming the obstacle arising from decoherence caused by the interaction of the electrons with the lattice vibrations (phonons) and controlling this decoherence have become very important. Meanwhile, in recent years a semiconductor quantum dot (QD) embedded in a semiconductor microcavity has become a novel basic system for quantum information processing[4~8]. In such systems, excitons in the quantum dots constitute an alternative two-level system instead of the usual two-level atomic systems. In general, these small quantum dots are characterized by strong exciton-phonon interactions[9~12]. Thus the effects of exciton-phonon interactions and exciton-exciton interactions play an important role in this quantum dot-cavity system and also make a natural difference from that in the usual two-level atom-cavity system. How the quantum decoherence due to the exciton-phonon interactions and exciton-exciton interactions affect the quantum information processing based on the QD cavity-QED is one of the hottest research subjects in current quantum information science. Wilson-Rae and Imamoglu[13] have shown that for super-ohmic spectral functions, the main role of exciton-phonon interactions is the reduction of the quantum dot-cavity coupling strength. On the other hand, excitonic Rabi oscillations in single quantum dots have been observed in recent experiments[14~16]. Zrenner *et al.*[17] have demonstrated that such excitonic coherent oscillations in a quantum dot two-level system can be converted into deterministic photocurrents and found that this can

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function as an optically triggered single-electron turnstile device. The observation of such nonclassical oscillations would be a first step towards quantum information processing in semiconductor nanostructures. More recently, three groups have demonstrated vacuum Rabi splittings for a single quantum dot inserted into an optical microcavity[18~20]. In this paper, we will further study the influence of strong exciton-phonon interactions on excitonic Rabi oscillations in such quantum dot-cavity systems at finite temperatures. We find that the quantum Rabi oscillations of exciton dressed by phonons still persist with a new single-photon Rabi frequency $g \times \exp[-\sum_q \lambda_q \times (N_q + 1/2)]$ in which $N_q = 1/[\exp(\omega_q/k_B T) - 1]$ is the phonon population, g is the single-photon Rabi frequency without exciton-phonon interaction, and λ_q corresponds to exciton-phonon coupling.

2 Theory and analytical results

In what follows, we assume a simple two-level model for a semiconductor quantum dot which consists of the electronic ground state $|1\rangle$ and the lowest-energy electron-hole (exciton) state $|2\rangle$. Such a quantum dot is placed inside a high-Q single mode cavity. We consider that this quantum dot via the exciton interacts with a single mode of a radiation field of frequency ω_s and is coherently driven by a quantized control field of frequency ω_c. As usual this two-level system can be characterized by the pseudospin-1/2 operators $S^{\pm}$ and S^z, and the cavity field (the control field) is characterized by the annihilation and creation operators a_s (a_c) and a_s^+ (a_c^+) with the Bose commutation relation $[a_s, a_s^+] = 1$ ($[a_c, a_c^+] = 1$). In this model, we no longer consider the exciton-phonon interaction as a perturbation, but take into account the new eigenstates resulting from the strong coupling of the exciton with the bulk acoustic phonons. The total Hamiltonian for this coupled photon-exciton-phonon system is written as ($\hbar = 1$)[13]

$$H = \omega_{ex}(S^z + 1/2) + \omega_s a_s^+ a_s + \omega_c a_c^+ a_c + g_s(S^+ a_s + S^- a_s^+) + g_c(S^+ a_c + S^- a_c^+) + \sum_q \omega_q b_q^+ b_q + (S^z + 1/2)\sum_q M_q(b_q^+ + b_q) \quad (1)$$

where ω_{ex} is the exciton frequency, g_s is the coupling constant of the exciton-cavity photon, and g_c is the coupling constant of the exciton and the control field photon. b_q^+ (b_q) is the creation (annihilation) operator of the phonon (with momentum q and frequency ω_q). The last term is the exciton-phonon interaction characterized by the matrix elements M_q. For simplicity, we also assume that the off-diagonal exciton-phonon interactions are negligible[12,13].

We first apply a canonical transformation to the Hamiltonian Eq. (1)[21]

$$H' = \exp(S) H \exp(-S) \quad (2)$$

where the generator S is

$$S = (S^z + 1/2)\sum_q \frac{M_q}{\omega_q}(b_q^+ - b_q) \quad (3)$$

The transformed Hamiltonian is given by

$$H' = (\omega_{ex} - \Delta)(S^z + 1/2) + \omega_s a_s^+ a_s + \omega_c a_c^+ a_c + \sum_q \omega_q b_q^+ b_q + g_s(S^+ X^+ a_s + S^- X a_s^+) + g_c(S^+ X^+ a_c + S^- X a_c^+) = H_0' + H_I' \quad (4)$$

where

$$H_0' = (\omega_{ex} - \Delta)(S^z + 1/2) + \omega_s a_s^+ a_s + \omega_c a_c^+ a_c + \sum_q \omega_q b_q^+ b_q$$
$$H_I' = g_s(S^+ X^+ a_s + S^- X a_s^+) + g_c(S^+ X^+ a_c + S^- X a_c^+) \quad (5)$$

where $\Delta = \sum_q M_q^2/\omega_q$ and

$$X = \exp\left[-\sum_q \frac{M_q}{\omega_q}(b_q^+ - b_q)\right], \quad X^+ = X^{-1} \quad (6)$$

As in the treatment of a small polaron[21,22] and quantum transport through a single-molecule transistor[23], here it is reasonable to replace the operator X with its expectation value at thermal equilibrium, $\langle X \rangle = \exp[-\sum_q \lambda_q (N_q + 1/2)]$, where $N_q = 1/[\exp(\omega_q/k_B T) - 1]$ is defined as the population of phonons at temperature T, and $\lambda_q = (M_q/\omega_q)^2$ corresponds to exciton-phonon coupling. It should be noted that this is an important approximation made in the present paper, which is valid only when the single photon Rabi frequency is small in comparison to the exciton-phonon coupling strength.

In what follows, it is convenient to work in the interaction picture with H_0'. The Hamiltonian is the given by

$$H'' = e^{iH_0' t} H_I' e^{-iH_0' t} = g_s \exp[-\sum_q \lambda_q (N_q + 1/2)] \times [S^+ a_s e^{i\delta_s t} + S^- a_s^+ e^{-i\delta_s t}] +$$

$$g_c \exp[-\sum_q \lambda_q (N_q + 1/2)] \times [S^+ a_c e^{i\delta_c t} + S^- a_c^+ e^{-i\delta_c t}] \quad (7)$$

where $\delta_s = \omega_{ex} - \Delta - \omega_s$, $\delta_c = \omega_{ex} - \Delta - \omega_c$.

In order to understand the basic physical features of the quantum dot with strong exciton-phonon couplings and strong exciton-photon couplings, we only consider the case of exact resonance, i.e. $\omega_{ex} - \Delta = \omega_s$ and $\omega_{ex} - \Delta = \omega_c$ which correspond to the case in which the control field and the cavity field are all on resonance with the zero-phonon line. For analytical simplicity, we also assume $g_s = g_c = g$[24,25]. Therefore the Hamiltonian Eq. (7) becomes

$$H'' = g\exp[-\sum_q \lambda_q (N_q + 1/2)] \times [S^+(a_s + a_c) + S^-(a_s^+ + a_c^+)] \quad (8)$$

It is useful to define the normal-mode operators by[25]

$$A = \frac{1}{\sqrt{2}}(a_c + a_s) \quad (9)$$

$$B = \frac{1}{\sqrt{2}}(a_c - a_s) \quad (10)$$

These are annihilation operators just like a_s and a_c and obey the Bose commutation relations,

$$[A, A^+] = 1, \quad [B, B^+] = 1 \quad (11)$$

Moreover, the normal-mode operators commute with each other,

$$[A, B] = 0, \quad [A, B^+] = 0 \quad (12)$$

In terms of these operators, the Hamiltonian Eq. (8) becomes independent of the antisymmetric normal-mode combination and is given by

$$H_{eff} = \sqrt{2} g\exp[-\sum_q \lambda_q (N_q + 1/2)][S^+ A + S^- A^+] \quad (13)$$

Next we proceed to solve the equation of motion for $|\psi(t)\rangle$, i.e.

$$i\frac{d}{dt}|\psi(t)\rangle = H_{eff}|\psi(t)\rangle \quad (14)$$

In general, the state vector $|\psi(t)\rangle$ is a linear combination of the states $|1, m\rangle|ph\rangle$ and $|2, m\rangle|ph\rangle$. Here $|2, m\rangle$ is the state in which the quantum dot is in the excited state $|2\rangle$ (i.e., with the presence of the exciton) and the symmetric normal mode of the field has excitation number m ($A^+ A|m\rangle = m|m\rangle$). A similar description exists for the state $|1, m\rangle$. $|ph\rangle$ is the phonon state. As we are using the interaction picture, we use the slowly varying probability amplitudes $c_{1,m,ph}(t)$ and $c_{2,m,ph}(t)$. The state vector is therefore[26]

$$|\psi(t)\rangle = \sum_m [c_{1,m,ph}(t)|1, m\rangle|ph\rangle + c_{2,m,ph}(t)|2, m\rangle|ph\rangle] \quad (15)$$

The interaction Hamiltonian Eq. (13) can only cause transitions between the states $|1, m+1\rangle|ph\rangle$ and $|2, m\rangle|ph\rangle$. We therefore consider the evolution of the amplitudes $c_{1,m+1,ph}(t)$ and $c_{2,m,ph}(t)$. The equations of motion for the probability amplitudes $c_{2,m,ph}(t)$ and $c_{1,m+1,ph}(t)$ are obtained by first substituting for $|\psi(t)\rangle$ and H_{eff} from Eqs. (15) and (13) in Eq. (14) and then projecting the resulting equations onto $\langle ph|\langle m, 2|$ and $\langle ph|\langle m+1, 1|$, respectively. We then obtain

$$\frac{d}{dt}c_{2,m,ph}(t) = -i\sqrt{2} g \times \exp[-\sum_q \lambda_q (N_q + 1/2)]c_{1,m+1,ph}(t) \quad (16)$$

$$\frac{d}{dt}c_{1,m+1,ph}(t) = -i\sqrt{2} g \times \exp[-\sum_q \lambda_q (N_q + 1/2)]c_{2,m,ph}(t) \quad (17)$$

The above coupled set of equations can be solved exactly subject to certain initial conditions. If initially the quantum dot is in the excited state $|2\rangle$ (i.e., with the presence of the exciton) then $c_{2,m,ph}(0) = 1$ and $c_{1,m+1,ph}(0) = 0$. We then obtain

$$c_{2,m,ph}(t) = \cos(\sqrt{2} g \times \exp[-\sum_q \lambda_q (N_q + 1/2)]\sqrt{m+1}\, t) \quad (18)$$

$$c_{1,m+1,ph}(t) = -i\sin(\sqrt{2} g \times \exp[-\sum_q \lambda_q (N_q + 1/2)]\sqrt{m+1}\, t) \quad (19)$$

Thus the probability $|c_{2,m,ph}(t)|^2$ of finding the quantum dot in the excited state $|2\rangle$ (i.e., with the presence of the exciton) and the cavity field in the $|m\rangle$ state and the phonon in the $|ph\rangle$ state, and the probability $|c_{1,m+1,ph}(t)|^2$ of finding the quantum dot in the ground state $|1\rangle$ and the cavity field in the $|m+1\rangle$ state and the phonon in the $|ph\rangle$ state oscillate according to

$$|c_{2,m,ph}(t)|^2 = \cos^2[\sqrt{2} g\exp[-\sum_q \lambda_q (N_q + 1/2)]\sqrt{m+1}\, t] \quad (20)$$

$$|c_{1,m+1,ph}(t)|^2 = \sin^2[\sqrt{2} g\exp[-\sum_q \lambda_q (N_q + 1/2)]\sqrt{m+1}\, t] \quad (21)$$

When the initial cavity field is in the vacuum state

($m=0$), the coherent Rabi oscillations are dressed by phonons, and the excitonic vacuum Rabi splitting is $2\sqrt{2}g\times\exp[-\sum_q\lambda_q(N_q+1/2)]$. This analytical result indicates that the effect of strong exciton-phonon coupling on the excitonic Rabi oscillations is just an added factor of $\exp[-\sum_q\lambda_q(N_q+1/2)]$ to the exciton-photon coupling constant g, but the coherent oscillations can still persist like those in the two-level atom-cavity systems. For zero temperature, $N_q=0$, the vacuum Rabi splitting is reduced to our previous result[27]. The result further implies that in the approximation made in the present paper, the decoherence due to the exciton-phonon interactions will not destroy the quantum information processing based on quantum dot cavity-QED, but will suppress the quantum dot-cavity coupling strength.

3 Conclusion

In summary, we have investigated the effect of strong exciton-phonon interactions and temperature dependence on excitonic Rabi oscillations in a coherently driven quantum dot-cavity system at finite temperatures. It is found that the temperature dependence of the vacuum Rabi splitting is proportional to $g\times\exp[-\sum_q\lambda_q(N_q+1/2)]$. Our results indicate that the decoherence due to acoustic phonons will not destroy the quantum information processing based on semiconductor nanostructures in the approximation made in the present paper, but will significantly suppress the quantum dot-cavity coupling strength.

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半导体微腔中单量子点的真空拉比分裂的温度效应 *

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摘要：采用量子光学和极化子正则变换的方法研究了有限温度下半导体微腔中单量子点的激子动力学行为，并解析得到了激子真空拉比分裂随温度变化的函数关系.

关键词：半导体微腔；量子点；激子；真空拉比分裂

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