Appendix. Coulomb potential of an electron in a quantum well

For a quantum well occupying the region |z|<*d*/2, the potential can be represented as follows for $|z_1| < d/2$:

$$\varphi_q(z_1,z_2)=\begin{cases}\frac{2\pi e}{q\kappa_2}\left[\exp(-q|z_2-z_1|)+\frac{\exp(qz)\delta\left[\exp(q(d-z_1))+\delta\exp(-q(z_1+d))+(\delta+1)\exp(qz_1)\right]}{\exp(qd)-\delta^2\exp(-qd)}\right], & z_2<-d/2\\ \frac{2\pi e}{q\kappa_2}\left[\exp(-q|z_2-z_1|)+\frac{2\delta\left[ch(q(z_2+z_1))+\delta\exp(-qd)ch(q(z_2-z_1))\right]}{\exp(qd)-\delta^2\exp(-qd)}\right], & |z_2|<d/2\\ \frac{2\pi e}{q\kappa_2}\left[\exp(-q|z_2-z_1|)+\frac{\delta\left(\exp(q(z_1+d))+\delta\exp(q(z_1-d))+(\delta+1)\exp(-qz_1)\right)}{\exp(qd)-\delta^2\exp(-qd)}\right], & z_2>d/2\end{cases}$$

, (A1)

where $\delta=\frac{\kappa_2-\kappa_1}{\kappa_2+\kappa_1}$, and $\kappa_{1,2}$ are high-frequency permittivities of the quantum well and barrier, respectively.

For $z_1 < -d/2$:

$$\varphi_q(z_1,z_2)=\begin{cases}\frac{2\pi e}{q\kappa_1}\exp(q|z_1-z_2|)-\frac{2\pi e}{q\kappa_1}\frac{\delta\exp(qz_1+qd+qz_2)sh(qd)}{\exp(qd)-\delta^2\exp(-qd)}, & z_2<-d/2\\ \frac{2\pi e}{q\kappa_1}\exp(q|z_1-z_2|)+\frac{\pi e\delta}{q\kappa_1}\frac{\exp(qz_1)\left[\delta\exp(q(-d-z_2))-\exp(q(d-z_2))+(1-\delta)\exp(qz_2)\right]}{\exp(qd)-\delta^2\exp(-qd)}, & |z_2|<d/2\\ \frac{2\pi e}{q\kappa_1}\exp(q|z_1-z_2|)-\frac{2\pi e\delta^2}{q\kappa_1}\frac{\exp(q(z_1-z_2))sh(qd)}{\exp(qd)-\delta^2\exp(-qd)}, & z_2>d/2\end{cases}$$

. (A2)

For $z_1 > d/2$

$$\varphi_q(z_1,z_2)=\begin{cases}\frac{2\pi e}{q\kappa_1}\exp(-q|z_2-z_1|)-\frac{2\pi e}{q\kappa_1}\frac{\delta^2\exp(q(z_2-z_1))sh(qd)}{\exp(qd)-\delta^2\exp(-qd)}, & z<-d/2\\ \frac{2\pi e}{q\kappa_1}\exp(-q|z_2-z_1|)+\frac{2\pi e\exp(-qz_1)}{q\kappa_1}\frac{\delta\left[\delta\exp(q(z_2-d))+(1-\delta)\exp(-qz)-\exp(qd+qz_2)\right]}{\exp(qd)-\delta^2\exp(-qd)}, & |z|<d/2\\ \frac{2\pi e}{q\kappa_1}\exp(-q|z_2-z_1|)-\frac{2\pi e\delta}{q\kappa_1}\frac{\exp(qd-qz_1-qz)sh(qd)}{\exp(qd)-\delta^2\exp(-qd)}, & z>d/2\end{cases}$$

. (A3)

Using Eq. (6) and Eqs. (A1) — (A3) we obtain for $|z_1| < d/2$

$$g(z_1,z_2)=\begin{cases}\dfrac{2\pi e}{\kappa_1}\left[\dfrac{d}{2}\left(1-\dfrac{\kappa_2}{\kappa_1}\right)+z_2-\dfrac{\kappa_1}{\kappa_2}z_1\right], & z_2<-d/2\\ \dfrac{2\pi e}{\kappa_2}\left[\dfrac{d}{2}\left(1-\left(\dfrac{\kappa_2}{\kappa_1}\right)^2\right)-|z_2+z_1|\right], & |z_2|<d/2\\ \dfrac{2\pi e}{\kappa_1}\left[\dfrac{d}{2}\left(1-\dfrac{\kappa_2}{\kappa_1}\right)-z_2+\dfrac{\kappa_1}{\kappa_2}z_1\right], & z_2>d/2\end{cases}.$$

(A4)

For $z_1<-d/2$

$$g(z_1,z_2)=\begin{cases}\dfrac{2\pi e}{\kappa_1}\left(\dfrac{d}{2}\left(\dfrac{\kappa_1}{\kappa_2}-\dfrac{\kappa_2}{\kappa_1}\right)-\left|z_2-z_1\right|\right), & z_2<-d/2\\ \dfrac{2\pi e}{\kappa_1}\left(\dfrac{d}{2}\left(1-\dfrac{\kappa_2}{\kappa_1}\right)+z_1-z_2\dfrac{\kappa_1}{\kappa_2}\right), & |z_2|<d/2\\ \dfrac{2\pi e}{\kappa_1}\left(z_1-z_2-\dfrac{d}{2}\dfrac{\kappa_1}{\kappa_2}\left(1-\dfrac{\kappa_2}{\kappa_1}\right)^2\right), & z_2>d/2\end{cases}.$$

(A5)

For $z_1>d/2$

$$g(z_1,z_2)=\begin{cases}\dfrac{2\pi e}{\kappa_1}\left[d\left(1-\dfrac{\kappa_1}{2\kappa_2}-\dfrac{\kappa_2}{2\kappa_1}\right)+z_2-z_1\right], & z_2<-d/2\\ \dfrac{2\pi e}{\kappa_1}\left[\dfrac{d}{2}\left(1-\dfrac{\kappa_2}{\kappa_1}\right)+\dfrac{\kappa_1}{\kappa_2}z_2-z_1\right], & |z_2|<d/2\\ \dfrac{2\pi e}{\kappa_1}\left[\dfrac{d}{2}\left(\dfrac{\kappa_1}{\kappa_2}-\dfrac{\kappa_2}{\kappa_1}\right)-|z_2-z_1|\right], & z_2>d/2\end{cases}.$$

(A6)