Appendix A. The correlation between tunnel coupling and barrier height

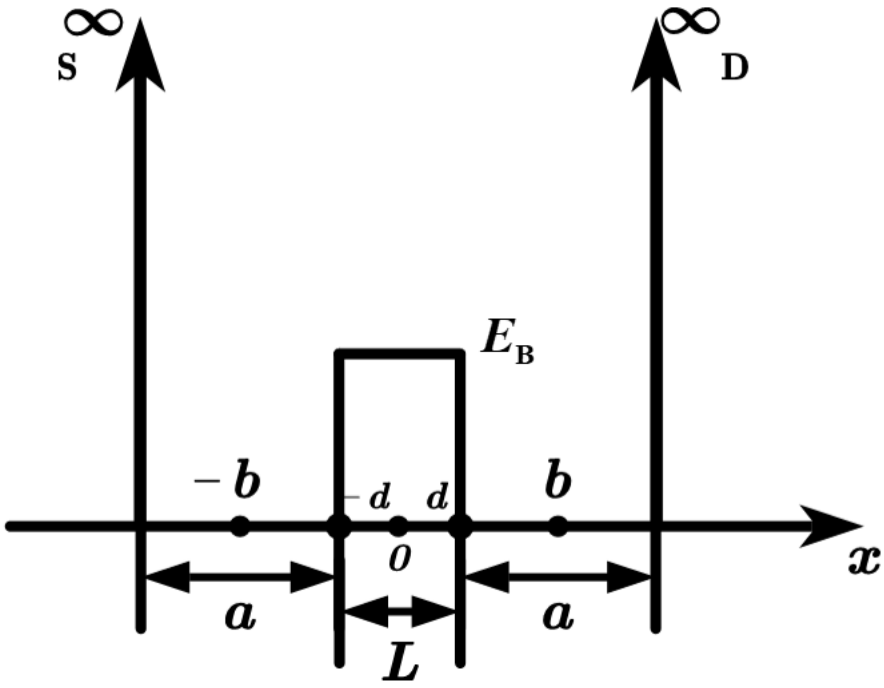

Fig. A. A double well potential model. a is the width of the potential well. L is the barrier width. The $x = b$ and $x = -b$ position is at the center of the well. The $x = d$ and $x = -d$ position is at the edge of the barrier.

In Section 2.3, we have constructed a double well model, as shown in Fig. A. Herein, we assume that the confined electron energy E in each dot is much lower than the barrier height E_B. The correlation between tunnel coupling strength t_c and barrier height E_B can be deduced from single electron stationary Schrodinger equation. We are going to solve the equation in the following three regions.

(1) the central barrier, i.e. $|x| < d$

The stationary Schrodinger equation is in follows:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + E_B\psi = E\psi \tag{A.1}$$

Because the barrier E_B is symmetric with respect to $x = 0$, the wave function inside should be either symmetric or antisymmetric:

$$\begin{cases} \psi(x) = C\cosh(qx) & \text{even} \\ \psi(x) = C\sinh(qx) & \text{odd} \end{cases} \tag{A.2}$$

where $q = \sqrt{2m(E_{\mathrm{B}} - E)/\hbar^2}$ is wave number of the electron in the barrier, C is the amplitude of the electron wave function inside the barrier.

(2) right quantum dot between d and $a+d$

The stationary Schrodinger equation in this region is in follows:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \tag{A.3}$$

The solution of Eq. (A.3) is in follows:

$$\psi(x) = D\sin(kx + \varphi) \tag{A.4}$$

Where D is the amplitude of the electron wave function in right quantum dot. $k = \sqrt{2mE/\hbar^2}$ is wave number of the electron in the quantum dot. φ can be determined by the boundary condition of $\psi(b + a/2) = 0$. The phase φ is in follows:

$$\varphi = n\pi - k\left(b + \frac{a}{2}\right) \quad n = 0, \pm1, \pm2... \tag{A.5}$$

Combining Eq. (A.5), we can rewrite the Eq. (A.4) in follows:

$$\psi(x) = D\sin k\left(b + \frac{a}{2} - x\right) \tag{A.6}$$

(3). left quantum dot between $-(a+d)$ and $-d$

The wave function of the electron in this region is symmetric or antisymmetric with Eq. (A.6) and can be written as in follows:

$$\psi(x) = D\sin k\left(b + \frac{a}{2} + x\right)$$
$$\text{or } \psi(x) = -D\sin k\left(b + \frac{a}{2} + x\right) \tag{A.7}$$

The total wave function reads

$$\begin{cases} \psi(x) = D\sin k\left(b + \frac{a}{2} + x\right) & -(a+d) < x < -d \\ \psi(x) = C\cosh(qx) & |x| < d \\ \psi(x) = D\sin k\left(b + \frac{a}{2} - x\right) & d < x < a+d \end{cases} \tag{A.8}$$

$$\begin{cases} \psi(x) = -D\sin k\left(b+\frac{a}{2}+x\right) & -(a+d)<x<-d \\ \psi(x) = C\sinh(qx) & |x|<d \\ \psi(x) = D\sin k\left(b+\frac{a}{2}-x\right) & d<x<a+d \end{cases} \tag{A.9}$$

Here, Eq. (A.8) is in even parity and Eq. (A.9) is in odd parity. The continuity of the wave function and its first derivative allows Eq. (A.8) written in follows:

$$\begin{cases} \psi(d) = C\cosh(qd) = D\sin ka \\ \frac{d\psi(d)}{dx} = Cq\sinh(qd) = -Dk\cos ka \end{cases} \tag{A.10}$$

By combining the two equations in Eq. (A.10), we can obtain

$$\tan(ka) = -\frac{k}{q}\coth qd \tag{A.10}$$

When $E \ll E_B$, that is to say q is large enough, we use $\lim_{x\to\infty}\coth x = 1+2e^{-2x}$ to rewrite Eq. (A.10) in follows:

$$\tan(ka) = -\frac{k}{q}\left(1+2e^{-qL}\right) \tag{A.11}$$

At the low-energy limit, the ratio of k to q is nearly zero, $k/q \to 0$. The right hand side of Eq. (A.11) is, thus, nearly zero. Left hand side of Eq. (A.11) is also nearly zero. We can consider the Taylor expansion of $\tan(ka)$ in follows:

$$\tan(ka) \approx \tan(k_0 a) + \sec^2(k_0 a)(k-k_0)a = (k-k_0)a \tag{A.12}$$

Where k_0 is defined in follows:

$$k_0 a \equiv n\pi \qquad n = 0, \pm 1, \pm 2... \tag{A.13}$$

Because k is close to k_0, the wave number q is close to q_0, which can be defined in follows:

$$q_0 \equiv \sqrt{\frac{2m(V_0 - E_n^{(0)})}{\hbar^2}} \tag{A.14}$$

Where $E_\text{n}^{(0)} \equiv \frac{\hbar^2 k_0^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$ is the energy of electron with wave number k_0.

Combining Eqs. (A.12) and (A.14), Eq. (A.11) can be rewritten in follows:

$$(k-k_0)a=-\frac{k_0}{q_0}\left(1+2e^{-q_0L}\right) \tag{A.15}$$

From Eq. (A.15) we can write k in follows:

$$k=k_0-\frac{k_0}{aq_0}\left(1+2e^{-q_0L}\right) \tag{A.16}$$

And we can also write k^2 from Eq. (A.16) in follows:

$$k^2=k_0^2-2\frac{k_0^2}{aq_0}(1+2e^{-q_0L})+o\left(\frac{1}{q_0^2}\right) \tag{A.17}$$

The energy of even-parity wave state can be written in follows:

$$E_{\mathrm{S}}=\frac{\hbar^2k^2}{2m}=\frac{\hbar^2k_0^2}{2m}\left[1-\frac{2}{aq_0}(1+2e^{-q_0L})\right]=E_n^{(0)}\left[1-\frac{2}{aq_0}-\frac{4}{aq_0}e^{-q_0L}\right] \tag{A.18}$$

In same way, the energy of odd-parity wave state (A.9) can be written in follows:

$$E_{\mathrm{AS}}=E_{\mathrm{n}}^{(0)}\left[1-\frac{2}{aq_0}+\frac{4}{aq_0}e^{-q_0L}\right] \tag{A.19}$$

The tunnel coupling is the half energy difference between E_{AS} and E_{S}, and can be written in follows:

$$t_{\mathrm{c}}=\frac{E_{\mathrm{AS}}-E_{\mathrm{S}}}{2}=\frac{4e^{-q_0L}}{aq_0}E_{\mathrm{n}}^{(0)} \tag{A.20}$$