A multivariate process capability index model system

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Abstract: This paper presents a systematic multivariate process capability index (MPCI) method, which may provide references for assuring and improving process quality levels while achieving an overall evaluation of process quality. The system method includes a spatial MPCI model for multivariate normal distribution data, MPCI model based on factor weight for multivariate no-normal distribution application, and MPCI model based on yield for yield application. At last, examples for calculating MPCI are given, and the experimental results show that this systematic method is effective and practical.

Key words: microelectronics process; multivariate; process capability index; yield; factor weight
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1. Introduction

With the increasing development in technologies, the minimum size in integrated circuit manufacture reaches nanometer level. Microelectronics processes are becoming more complex: a manufacture system includes more processes and each process has more specifications to satisfy the advanced nanometer process requirements. Therefore, more requirements for processes exist in evaluating microelectronics processes level. Moreover, few differences happen between the advanced process, and traditional methods used to evaluate process level are not effective for evaluating and differentiating the nanometer process level. Thus, the differentiating and evaluation of these advanced processes require a competitive and systematic solution.

A process capability index (PCI) is a numerical summary that compares the behavior of a microelectronics process characteristic to its engineering specifications. The measure is also often called capability or performance indices or ratios. We use the capability index as the generic term. A capability index relates specification limits to the performance of the process. A large value of the index indicates that the current process is capable of producing parts that, in all likelihood, will meet or exceed the specification's requirements. Over the last two decades, most developments in PCI focus on the univariate process. For example, Kane (1986), Chan *et al.* (1988), Choi and Owen (1990), Boyles (1991), Singhal (1991), Pearn *et al.* (1992), and Boyles (1994)[1−6]. However, it is not uncommon in microelectronics manufacturing that one often encounters processes which involve many correlated variables of interest. In such a situation, simply calculating the univariate PCI of individual variables and combining them together will inevitably fail to value the level of processes[7]. Therefore, it is more desirable to assess the process capability using the multivariate process capability index (MPCI).

Recently, several attempts to develop MPCI have been carried out by various researchers such as Chen (1994)[8], Pearn, Kotz and Johnson (1992)[9], Wang and Du (2000)[10], and Kotz and Lovelace (1998)[2]. However, most existing MPCIs such as Chen (1994) require that the data from process be normal distribution, and are largely dependent on the variance-covariance structure of the underlying distribution. The others are uneasy to apply them into practices due to the difficulty of computing[11].

In this paper, a systematic method has been demonstrated to evaluate the microelectronics process ability and apply the multivariate process capability index into practices. The system method divides all cases into three situations: one is multivariate normal distribution application, the second case is multivariate no-normal distribution application, and the last one is yield application. In the systematic method, the MPCI model based on yield is for yield application; the MPCI model based on factor weight is for multivariate no-normal distribution application; and the spatial MPCI model is for multivariate normal distribution application.

2. Univariate PCI

A process capability index relates the engineering specification (usually determined by the customer) to the observed behavior of the process. The capability of a process is defined as the ratio of the distance from the process center to the nearest specification limit divided by a measure of the process variability. Some basic capability indices that have been widely used in the manufacturing industry include C_{p}, and C_{pk}, explicitly defined as follows[1]:

$$C_{\text{p}} = \frac{\text{USL} - \text{LSL}}{6\sigma}, \tag{1}$$

$$C_{\text{pk}} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \quad \frac{\mu - \text{LSL}}{3\sigma}\right\}, \tag{2}$$

where USL and LSL are the upper and the lower specification limits respectively, μ is the process mean, and σ is the process standard deviation. Equation (1) is effective when the mean of process data is equal to the median of specification limits;

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however, Equation (2) is used when the mean of process data is departure from the median of specification limits.

3. Multivariate PCI

Nowadays, the microelectronics process level has been improved extraordinarily, and process capability analysis often entails characterizing or assessing processes and products based on more than one engineering specification on quality characteristic, which needs to use multivariate process capability index to assess such process. Univariate process capability indices have been investigated extensively. However, MPCI are comparatively neglected. Some of the MPCIs defined by researchers are complexly computed and applied. In the section, a solution system for multivariate process capability index has been demonstrated. The system includes MPCI model based on yield, MPCI model based on factor weight, and a spatial MPCI model. These models are introduced in the following sections.

3.1. MPCI model based on yield

Inspired by the recent works of Chen (2003)[12] and Chao (2005)[13], in which Chao (2005) proposes a universal PCI with considering a very specific view that a proper value of the process capability index represents the true yield of the process, we incorporate the approach with Chen (2003) to propose and study a MPCI based on yield. The new MPCI, which we denote by $\mathrm{MC_y}$, is not limited to data scale, and its result is relative to yield of process level.

3.1.1. Univariable PCI

It is generally agreed that the original motives underlying the introduction of PCI are related to the proportion of nonconforming products[14]. Therefore, when we have a value of PCI = 1 and without considering random influence, the yield is 99.73%; and if the situation is not the case, the yield is less than 99.73%. Most PCIs do not provide a precise meaning of yield. Chao (2005) proposes a PCI which can present the true yield of process. Let $F(x)$ be the distribution function. The univariate PCI is defined as

$$C_{\mathrm{y}} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}(F(\mathrm{USL}) - F(\mathrm{LSL}) + 1)\right], \tag{3}$$

where $\Phi(x)$ denotes the cumulative distribution function of the standard normal distribution. And the equality implies the relationship between the index C_{y} and yield, which can be expressed by process yield $= 2\Phi(3C_{\mathrm{y}}) - 1$[15]. The relationship is true under any situation: whether or not mean or process target coincides with the center of the specification interval and whether or not the process follows a normal distribution.

3.1.2. MPCI ($\mathrm{MC_y}$)

Similar to the univariate PCI, MPCI should imply the true yield of the process. Based on the PCI (C_{y}) which is presented by Chao and combined with the approach to propose the multivariate process capability index which is presented by Chen, we present a multivariate process capability index which does not require process data to satisfy normal distribution. The MPCI called $\mathrm{MC_y}$ is defined as

$$\mathrm{MC_y} = \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{i=1}^{m}(2\Phi(3C_{\mathrm{y}i}) - 1) + 1\right]/2\right\}, \tag{4}$$

where $C_{\mathrm{y}i}$ is the process capability index value of ith characteristic for $i = 1, 2, \ldots, m$, and m is the number of characteristics. The new index, $\mathrm{MC_y}$, may be viewed as a generalization of the single characteristic yield index C_{y}. Let $\mathrm{MC_y} = C$, hence

$$\eta = 2\Phi(3C) - 1, \tag{5}$$

where η is the yield of a process. Equality (5) shows one to one correspondence relationship between the index $\mathrm{MC_y}$ and yield. Then, the MPCI can be used to assess the process level by yield in multivariate situation. For a process with n characteristics, if the requirement for the multivariate process capability index is $\mathrm{MC_y} \geqslant C_0$, a sufficient condition for the requirement to each univariate process capability index can be obtained by the following. Let $C_{\min}$ be the minimum value for each single characteristic, then

$$\frac{1}{3}\Phi^{-1}\left\{\left[\prod_{i=1}^{m}(2\Phi(3C_{\mathrm{y}i}) - 1) + 1\right]/2\right\} \geqslant \frac{1}{3}\Phi^{-1}\left\{\left[\prod_{i=1}^{m}(2\Phi(3C_{\min}) - 1) + 1\right]/2\right\}. \tag{6}$$

If

$$\frac{1}{3}\Phi^{-1}\left\{\left[\prod_{i=1}^{m}(2\Phi(3C_{\min}) - 1) + 1\right]/2\right\} \geqslant C_0, \tag{7}$$

then

$$C_{\min} \geqslant \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[m]{2\Phi(3C_0) - 1} + 1}{2}\right). \tag{8}$$

Therefore, requirements of univariate process capability index are given by

$$C_{\mathrm{y}i} \geqslant \frac{1}{3}\Phi^{-1}\left(\frac{\sqrt[m]{2\Phi(3C_0) - 1} + 1}{2}\right), \quad i = 1, 2, \ldots, m. \tag{9}$$

When inequality (8) is satisfied, then the multivariate process capability index requirement $\mathrm{MC_y} \geqslant C_0$ will be satisfied.

3.1.3. Result and application

Based on equalities (4) and (5), the correspondence values of $\mathrm{MC_y}$ and yield for the number characteristics from $n = 1$ to $n = 14$ are obtained, as shown in Table 1. From Table 1, the correspondence relationship between MPCI and yield is similar to univariate situation, thus we can know application of the $\mathrm{MC_y}$ model is practicable, and we can use the values in Table 1 to class the process capability level.

In fact, in order to meet customers' requirement, process level needs to be classed. When process level is qualified to the specification, how to precisely monitor the variation of process and how to class the products is an important subject. For this reason, we set up the process capability zone for process level

Table 1. Correspondence value of MCy and yield.

n	$\mathrm{MC_y}$ (%Yield)			
1	1.0 (0.997300204)	1.33 (0.999933927)	1.67 (0.999999456)	2.00 (0.999999998)
2	1.068 (0.998649190)	1.384 (0.999966963)	1.714 (0.999999728)	2.037 (0.999999999)
3	1.107 (0.999099257)	1.414 (0.999977975)	1.739 (0.999999819)	2.059 (0.999999999)
4	1.133 (0.999324367)	1.436 (0.999983481)	1.757 (0.999999864)	2.074 (1.000000000)
5	1.153 (0.999459457)	1.452 (0.999986785)	1.770 (0.999999891)	2.085 (1.000000000)
6	1.170 (0.999549527)	1.465 (0.999988987)	1.781 (0.999999909)	2.095 (1.000000000)
7	1.183 (0.999613868)	1.477 (0.999990561)	1.791 (0.999999922)	2.103 (1.000000000)
8	1.195 (0.999662126)	1.486 (0.999991741)	1.799 (0.999999932)	2.110 (1.000000000)
9	1.205 (0.999699662)	1.495 (0.999992658)	1.806 (0.999999940)	2.116 (1.000000000)
10	1.214 (0.999729692)	1.502 (0.999993392)	1.812 (0.999999946)	2.121 (1.000000000)
11	1.222 (0.999754262)	1.509 (0.999993993)	1.818 (0.999999951)	2.126 (1.000000000)
12	1.230 (0.999774738)	1.515 (0.999994494)	1.823 (0.999999955)	2.130 (1.000000000)
13	1.236 (0.999792064)	1.520 (0.999994917)	1.828 (0.999999958)	2.135 (1.000000000)
14	1.243 (0.999806915)	1.526 (0.999995280)	1.832 (0.999999961)	2.138 (1.000000000)

Table 2. Limit value of C_{yi}.

Number of characteristic	Limit value of C_{yi}	
	Value of upper limit	Value of lower limit
1	1.000	1.670
2	1.068	1.714
3	1.107	1.739
4	1.133	1.757
5	1.153	1.770
6	1.170	1.781
7	1.183	1.791
8	1.195	1.799
9	1.205	1.806
10	1.214	1.812
11	1.222	1.818
12	1.230	1.823
13	1.236	1.836
14	1.243	1.840

according to specification and then class them to conformity or unconformity. When we know the value of specification, the value of univariate PCI for single characteristic can be obtained by inequality (9). Now we give an example to show this point. The process capability specification for a process level is $1.0 \leqslant \mathrm{MC_y} \leqslant 1.67$ which is common value in most factories. According to inequality (9), the process capability zone for a single characteristic is shown Table 2.

The original motives underlying the introduction of PCI are to relate to the proportion of non-conforming products. In the section we propose a new MPCI. The proposed index, $\mathrm{MC_y}$, can be applied to a variety of specification zones without considering data distribution. Therefore, it has greater flexibility than the existing MPCI's. Furthermore, the proposed index is directly related to the yield of process. Thus, the index $\mathrm{MC_y}$ can be used to assess to what extent the process is producing non-conforming products, of which its computing is simple and practitioners will not limit to theoretical.

3.2. Spatial MPCI model

In the section, based on the multivariate process capability index definition, an effective and workable spatial MPCI model has been developed. The model can solve the problem that MPCI definition cannot achieve MPCI values when process quality characteristics are greater than three. Then a practical application using the model is given.

3.2.1. MPCI model

As a general case, define $\boldsymbol{X}$ as a $m \times n$ sample matrix, where m is the number of process quality characteristics measured on a part and n is the number of parts measured. That is, each column in the matrix represents the p measurements recorded from a sampled part. These n observations represent samples drawn from a multivariate distribution with correlation among the m variables. Engineering specifications for the processes are assumed to exist for each of the m dimensions. Analogously to univariate process capability indices, also multivariate capability indices, relate the allowed process region such as some measure of the specification region, to the actual process region such as some measure of the process region. Therefore, a multivariate process capability index is called MVCp which is defined as[7]

$$\mathrm{MVCp} = \frac{\mathrm{vol}(R_1)}{\mathrm{vol}(R_2)}, \tag{10}$$

where R_1 represents an engineering specification region and R_2 is for a scaled 99.73% process region. In particular, if the process data satisfy multivariate normal, then R_2 is for an elliptical region. For Eq. (10), its computation in high dimension variables is complex and difficult. When the variable number in multivariate normal distribution is greater than three, achieving MPCI results using Eq. (10) is very difficult. Therefore, we need to transform the form and to build a simple model.

The specification for the i-th quality variable x_i is usually given by the triple of lower specification limit LSL_i, mean value μ_i and upper specification limit USL_i. For the m process quality characteristics, a multivariate normal distribution with mean vector μ and positive definite covariance matrix $\Sigma = (\sigma_{ij}), i, j = 1, \cdots, m$ is commonly assumed. This multivariate PCI, here denoted by MVCp, is given by

$$\mathrm{MVCp} = \frac{\prod\limits_{i=1}^{m} (\mathrm{USL}_i - \mathrm{LSL}_i)}{\mathrm{vol}\left[(x-\mu)' \sum^{-1} (x-\mu) \leqslant \chi^2_{m,0.9973}\right]}, \tag{11}$$

where $\chi^2_{p,0.9973}$ denotes the 99.73%-quantile of the χ^2-distribution with p degrees of freedom. Note that the volume of the ellipsoid does not depend on the value of μ. A drawback of this index as a generalization of the univariate C_p is that its value is not 1 if indeed 99.73% of the distribution is inside the specification. To overcome this drawback, Taam *et al.* (1993)[7] proposed to use a modified specification region R^*, which is the greatest ellipsoid with generating matrix Σ entirely contained inside the specification region. Then the corresponding index is defined as

$$\mathrm{MVCp}^* = \frac{\mathrm{vol}\left[(x-\mu)' \sum^{-1} (x-\mu) \leqslant K^2\right]}{\mathrm{vol}\left[x-\mu)'\Sigma^{-1}(x-\mu) \leqslant \chi^2_{m,0.9973}\right]}, \tag{12}$$

where K is determined by the modified specification region and K^2 is chosen so that the resulting ellipsoid is the greatest volume ellipsoid inside the specification.

3.2.2. Model transformation

When the distribution function of process data are multivariate normal distribution and process region is an elliptical region, we can achieve the relationship as follows:

$$\begin{aligned} &\mathrm{vol}\left[(x-\mu)'\Sigma^{-1}(x-\mu) \leqslant K^2\right] \\ &\qquad = |\Sigma|^{1/2} (\pi K^2) \left[\Gamma(m/2+1)\right]^{-1}. \end{aligned} \tag{13}$$

Then, based on equalities (12) and (13), we can see that

$$\mathrm{MVCp}^* = \left(\frac{K}{\chi_{m,0.9973}}\right)^m. \tag{14}$$

To compute the MVCp value, we need to find the value of K. Then we will achieve a greatest volume ellipsoid, which is a tangent ellipsoid to the original specification region, with the sth specification limit and $1 \leqslant s \leqslant m$. For the mth variable this tangent is easy to find replacing this variable by its specification limit. We employ a permutation matrix which exchanges the s-th for the m-th dimension and let $x(s) = (x_1, \cdots, x_{s-1}, x_m, x_{s+1}, \cdots, x_{m-1})'\sqrt{b^2-4ac}$, $\mu = 0$ (for simple discussion), and $\Sigma^{-1} = A = (a_{ij}) \quad (i, j = 1, \cdots, m)$ which is a symmetric matrix. Then we can further the matrix into

$$\begin{cases} a_s^* = (a_{s,1}, \cdots, a_{s,s-1}, a_{s,m}, a_{s,s+1}, \cdots, a_{s,m-1})', \\ A_s^* = \begin{bmatrix} a_{11} & \cdots & a_{1,s-1} & a_{1,m} & a_{1,s+1} & \cdots & a_{1,m-1} \\ \vdots & \vdots & & \vdots & & \vdots & \\ a_{s-1,1} & \cdots & a_{s-1,s-1} & a_{s-1,m} & a_{s-1,s+1} & \cdots & a_{s-1,m-1} \\ a_{m,1} & \cdots & a_{m,s-1} & a_{m,m} & a_{m,s+1} & \cdots & a_{m,m-1} \\ a_{s+1,1} & \cdots & a_{s+1,s-1} & a_{s+1,m} & a_{s+1,s+1} & \cdots & a_{s+1,m-1} \\ \vdots & \vdots & & \vdots & & \vdots & \\ a_{m-1,1} & \cdots & a_{m-1,s-1} & a_{m-1,m} & a_{m-1,s+1} & \cdots & a_{m-1,m-1} \end{bmatrix}, \end{cases} \tag{15}$$

where A_s^* is the left upper part of the matrix A of dimension $(m-1, m-1)$ and the s-th row and s-th column of A have been replaced by the m-th row and m-th column. It can determine the tangent of the ellipsoid with s-th specification limit. The quadratic form of interest is given by

$$\begin{aligned} &Q(x_1, \cdots, \mathrm{LSL}_s, \cdots, x_m) \\ &\quad = (x_1, \cdots, \mathrm{LSL}_s, \cdots, x_m)' A (x_1, \cdots, \mathrm{LSL}_s, \cdots, x_m) \\ &\quad = x(s)' A_s^* x(s) + 2\mathrm{LSL}_s (a_s^*)' x(s) + a_{s,s} \mathrm{LSL}_s^2, \end{aligned} \tag{16}$$

$$\frac{\partial Q(x_1, \cdots, \mathrm{LSL}_s, \cdots, x_m)}{\partial x(s)} = a A_s^* x(s) + a \mathrm{LSL}_s a_s^*. \tag{17}$$

Setting the first derivative equal to null, which lead to $x_{(s)}^{\min} = -\mathrm{LSL}_s (A_s^*)^{-1} a_s^*$, the value of the quadratic form at the point is

$$\begin{aligned} &Q(x_1^{\min}, \cdots, \mathrm{LSL}_m, \cdots, x_p^{\min}) \\ &\qquad = \mathrm{LSL}_s^2 (a_s^*)' (A_s^*)^{-1} A_s^* a_s^* \\ &\qquad\quad - 2\mathrm{LSL}_s^2 (a_s^*)' (A_s^*)^{-1} a_s^* + a_{s,s} \mathrm{LSL}_s^2 \\ &\qquad = \mathrm{LSL}_s^2 (a_{s,s} - (a_s^*)' (A_s^*)^{-1} a_s^*). \end{aligned} \tag{18}$$

Let $B = \begin{pmatrix} A_s^* & a_s^* \\ (a_s^*)' & a_{s,s} \end{pmatrix} = C_s \Sigma^{-1} C_s = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$, then we can get C_m.

$$C_m = \begin{pmatrix} & & 0 & & & \\ I_{s-1} & & \vdots & & 0_{s-1} & \\ 0 & \cdots & 0 & \cdots & 0 & 1 \\ & & \vdots & & & 0 \\ 0_{m-s} & & 0 & & I_{m-s-1} & \vdots \\ & & 1 & & & 0 \end{pmatrix}, \tag{19}$$

where I_k and $I = 0_k$ denote the identity matrix and null matrix of dimension K. Thus

Table 3. Two variable process data for Brinell hardness and tensile strength.

H	S	H	S
143	34.3	186	57.0
200	57.0	172	49.4
160	47.5	182	57.2
181	53.4	177	50.6
148	47.8	204	55.1
178	51.5	178	50.9
162	45.9	196	57.9
215	59.1	160	45.5
161	48.4	183	53.9
141	47.3	179	51.2
175	57.3	194	57.5
187	58.5	181	55.6
187	58.2		

$$Q(x_x^{\min},\cdots,\mathrm{LSL}_s,\cdots,x_m^{\min})=\frac{\mathrm{LSL}_s^2}{\sigma_s^2}. \tag{20}$$

Then we have

$$K=\min_{i=1,\cdots,m}\left(\frac{\mathrm{USL}_i-\mu_i}{\sigma_i},\frac{\mu_i-\mathrm{LSL}_i}{\sigma_i}\right). \tag{21}$$

This implies that the value of MVCp depends only on the process quality characteristics with the greatest variance in relation to the corresponding specification width. Therefore, we can use the MVCp* in Eq. (22) to compute the multivariate process capability index.

$$\mathrm{MVCp}^*=\min_{i=1,\cdots,m}\left(\left[\frac{\mathrm{USL}_i-\mu_i}{\chi_{m,0.9973}\sigma_i}\right]^m,\quad\left[\frac{\mu_i-\mathrm{LSL}_i}{\chi_{m,0.9973}\sigma_i}\right]^m\right). \tag{22}$$

3.2.3. Experimental result

Table 3 has been used in the case study. Chan *et al.* (1991) use the bivariate process data to examine their definition of a multivariate PCI over an ellipsoid zone. In Table 3, $H=X_1$ represents the Brinell hardness of chips, and $S=X_2$ represents the tensile strength of chips. Setting the upper specification limit of H is $\mathrm{US}L_H=233$, and lower specification limit of H is $\mathrm{LSL_H}=122$. For tensile strength, its upper and lower specification limits respectively are $\mathrm{USL}_S=70$ and $\mathrm{LSL}_S=35$. Based on the data in Table 3, we can get sample mean $\overline{X}_H=177.2$ and $\overline{X}_S=52.32$, and their standard deviation are $\sigma_H=18.38$ and $\sigma_S=5.8$. Then according to Eqs. (10) and (22), the multivariate process capability index can be achieved as $\mathrm{MVCp}=1.4783$ and $\mathrm{MVCp}^*=1.4004$, where the value 1.4783 is computed using the MPCI definition form and the value 1.4004 is obtained from Eq. (22). The two results are close to each other. Therefore, the spatial MPCI model is effective when the model is used for multivariate normal distribution data.

In this section, a spatial MPCI model use to compute multivariate process capability index has been presented. The Spatial MPCI model requires the data from process satisfy multivariate normal distribution. A case study shows the model is effective. Moreover, the Spatial MPCI model is based on the basic definition of multivariate process capability index. Therefore, the model is meaningful and reasonable in its application.

3.3. MPCI model based on factor weight

In section 3.2, we build MPCI model for multivariate normal distribution data. However, in many cases, the multivariate data did not satisfy multivariate normal distribution. Thus, in the section, a MPCI model based on factor weigh has been build to compute the MPCI value for no-normal distribution data.

3.3.1. Factor analysis

As for multivariate process quality characteristics, the aim of factor analysis[10] is to describe covariance relation between them using several factors. The basic way is to group the factors according correlation, in which the factor having more correlation is classified to a group, and the correlation among different group is very weak, then such group is regarded as a factor. Assuming X is one $m\times n$ step matrix, m is the number of process quality characteristics and n is the sample number.

$$X=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n}\\ x_{21} & x_{22} & \cdots & x_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ x_{m1} & x_{m2} & \cdots & x_{mn}\end{bmatrix}. \tag{23}$$

The factor model is defined as[16, 17]

$$X=L\times F+\varepsilon, \tag{24}$$

where F is a $p\times 1$ step matrix. p, less than m, is the number of factors which are chosen based on principles. F_i is the i-th factor. L is one $m\times n$ step matrix and L_{ij} expresses the load of the i-th variable on the j-th factor. ε, one $m\times 1$ step matrix, is the special factor of x. And we have

$$\begin{cases}\mathrm{cov}(F,\ \varepsilon)=0,\\ V(F)=1,\quad V(\varepsilon)=\begin{bmatrix}\sigma_2^2 & & 0\\ & \ddots & \\ 0 & & \sigma_n^2\end{bmatrix}.\end{cases} \tag{25}$$

3.3.2. Computing L and σ_i [18]

When computing L and σ_i, an extensively method name principal component analysis has been chosen. Then the sample covariance matrix S of X matrix is as follows:

$$S=\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1m}\\ s_{21} & s_{22} & \cdots & s_{2m}\\ \vdots & \vdots & \vdots & \vdots\\ s_{m1} & s_{m2} & \cdots & s_{mm}\end{bmatrix}, \tag{26}$$

where S is a symmetrical nonsingular matrix, s_{ii} is variance of X_i, and s_{ij} is covariance of X_i and X_j. Thus

$$s_{ij}=\frac{1}{n-1}\sum_{k=1}^{n}(x_{ik}-\overline{x}_i)(x_{jk}-\overline{x}_j), \tag{27}$$

$$\overline{x}_i = \frac{1}{n}\sum_{j}^{n} x_{ij}. \tag{28}$$

After making an orthonormalization $D = E_x^T S E_x$, a diagonal matrix D will be obtained, the diagonal element λ_1, λ_2, $\ldots$, λ_m ($\lambda_1 > \lambda_2 > \ldots > \lambda_m$) are characteristic roots of S matrix and E_1, E_2, $\ldots$, E_m are characteristic vector of S matrix. E_i is the load of every variable on i-th factor. Contribution rate of each factor is as follows:

$$r_i = \lambda_i \bigg/ \sum_{i=1}^{m} \lambda_i, \quad i = 1, 2, \ldots, m. \tag{29}$$

We can get the load matrix L as

$$L = (E_1,\ E_2,\ \cdots,\ E_m) \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sqrt{\lambda_m} \end{bmatrix}. \tag{30}$$

3.3.3. Factor MPCI

In order to get the process capability index value for every factor, the factor specification and the target value for quality standard should be determined first.

$$\mathrm{LSL_{FA}} = L^{-1}(\mathrm{LSL} - \varepsilon), \tag{31}$$

$$\mathrm{USL_{FA}} = L^{-1}(\mathrm{USL} - \varepsilon), \tag{32}$$

$$T_{\mathrm{FA}} = L^{-1}(T - \varepsilon), \tag{33}$$

where LSL, USL and T are lower limit of allowance, upper limit of allowance and process quality expected target value of every process quality characteristic respectively. $\mathrm{LSL_{FA}}$, $\mathrm{USL_{FA}}$ and T_{FA} are lower limit of allowance, upper limit of allowance and process quality expected target value of every factor. Then we obtain process capability indices and its mean for the i-th factor:

$$C_{p,\mathrm{FAi}} = \frac{\mathrm{USL}_{\mathrm{FA}i} - \mathrm{LSL}_{\mathrm{FA}i}}{6\sqrt{\lambda_i}}, \tag{34}$$

$$C_{pk,\mathrm{FA}i} = \min\left(\frac{\mathrm{USL}_{\mathrm{FA}i} - \hat{\mu}_{Fi}}{3\sqrt{\lambda_i}},\quad \frac{\hat{\mu}_{Fi} - \mathrm{LSL}_{\mathrm{FA}i}}{3\sqrt{\lambda_i}}\right), \tag{35}$$

$$\hat{\mu}_{Fi} = \frac{1}{n}\sum_{j=1}^{n} Fi_j, \quad i = 1, 2, \ldots, m. \tag{36}$$

3.3.4. MPCI based on factor weight

Now we consider that each factor has different contribution ratio to random vector X. The fluctuation of process variable causes the fluctuation of common factor, and we can use contribution ratio to measure the fluctuant influence of common factor to overall quality. Therefore, the multivariate process capability index based on factor weight is defined to be:

$$\mathrm{MCp} = \sum_{i=1}^{p} r_i C_{p,\mathrm{FA}i}, \tag{37}$$

$$\mathrm{MCpk} = \sum_{i=1}^{p} r_i C_{pk,\mathrm{FA}i}, \tag{38}$$

where r_i is contribution ratio of common factor. We can achieve the contribution ratio using Eq. (29).

Table 4. MPCI value based on factor weight.

	Specification i	Specification ii	Specification iii
MCp	0.6981	1.1737	1.1635
MCpk	0.6710	1.1681	0.8735

3.3.5. Experimental results

(1) Experimental analysis

We use two-variable process data as example which is observed from microelectronic process. There are three different requirements of process target value and process specification, which are:

i: Process specification limit for each process parameter is $\pm 3\sigma$, and process specification center is equal to process distribution center.

Considering X_1, the upper and lower specification limit are USL = 233 and LSL = 122 respectively.

Considering X_2, the upper and lower specification limit are USL = 70 and LSL = 35 respectively.

ii: Process specification limit for each process parameter is $\pm 5\sigma$, and process specification center is equal to process distribution center.

Considering X_1, the upper and lower specification limit are USL = 270.5 and LSL = 87.5 respectively.

Considering X_2, the upper and lower specification limit are USL = 83.6 and LSL = 22.6 respectively.

iii: Process specification limit for each process parameter is $\pm 3\sigma$, and there is 1.5σ deviation between process specification center and process distribution center.

Considering X_1, the upper and lower specification limit are USL = 295 and LSL = 110 respectively.

Considering X_2, the upper and lower specification limit are USL = 82 and LSL = 32 respectively.

The requirement for process target value is $T(X_1)$ = 177 and $T(X_2)$ = 52. Table 4 displays the computing outcome for MCPI based on factor weight.

(2) MPCI result analysis

Firstly, we make a hypothesis test on the experiment data with Chi-square test, A-D test and Kolmogorov test. The result shows that the data cannot be regarded as data from normal distribution. Then we obtain C_{p} and C_{pk} values from X_1 and X_2 respectively without considering correlation between X_1 and X_2, as shown in Table 5.

Table 5 shows the univariate PCI value. Comparing the values in Table 4 with Table 5, for specification i, the MCp $=$ 0.6981 and MCpk $=$ 0.671 which are in the range of C_{p} and C_{pk} of X_1 and X_2. The same results exist for specification ii and iii. Therefore, the results illuminate that the MCp and MCpk in Table 4 be able to represent the process parameter variability approximately.

(3) Experiment summary

In many cases, the multivariate data did not satisfy multivariate normal distribution. Thus, a MPCI model based on factor weigh has been build to compute the MPCI value for no-normal distribution data. Based on the comparing between Tables 4 and 5, the MPCI based on factor weight has the capability to represent such information of process performance. It can be conclude that the approach calculate Multivariate process capability index based on factor weight is feasible. There-

Table 5. C_p and C_{pk} from X_1 and X_2 respectively.

	Specification i		Specification ii		Specification iii	
	C_p	C_{pk}	C_p	C_{pk}	C_p	C_{pk}
X_1	0.7398	0.7120	1.2538	1.2487	1.1257	0.9112
X_2	0.5692	0.5513	1.1557	1.1446	1.6438	0.6945

fore, the experimental results demonstrate the MPCI based on factor weight is workable and effective. The model can achieve the multivariate process capability index in no-normal distribution case.

4. Conclusions

Process capability ultimately decides microelectronics process quality. Based on analyzing process capability index (PCI), microelectronics process capability may be effectively assured. With the rapid development in microelectronics process, Quality evaluation of processes concerns more than one quality characteristics; In this situation, simply calculating the univariate PCI of individual variables and combining them together will inevitably fail to value the level of processes. Therefore, it is more desirable to assess the process capability using multivariate process capability index (MPCI). The paper has presented a system multivariate PCI method, which may provide references for assuring and improving process quality while achieving overall evaluation of process quality. The system method divides all cases into three situations: one is multivariate normal distribution application, the second case is multivariate no-normal distribution application, and the last one is yield application. In the systematic method, MPCI model based on yield is for yield application; MPCI model based on factor weight is for multivariate no-normal distribution application; and spatial MPCI model is for multivariate normal distribution application. Finally, experimental analyses and practical example with the system method demonstrate the method is reasonable and effective.

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