

# Numerical Analysis of Steady Current and Temperature Distributions and Characteristics of Transverse Mode in VCSEL \*

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**Abstract** The eigen transverse modes of the cylindrical Vertical-Cavity Surface-Emitting Laser (VCSEL) are investigated by numerical simulation. The steady current and temperature distributions in the cavity are calculated self-consistently by the finite element method. At the steady state, the solution of the transverse mode equation in the active region is obtained by the matrix eigenvalue method, taking the material thermal properties and all the significant heat resources into consideration. The output light patterns above the threshold current are also demonstrated in this paper.

**PACC:** 4255P, 734Q

## 1 Introduction

In the past few years, the Vertical-Cavity Surface-Emitting Lasers (VCSEL) was improved considerably in its performance due to the introduction of the intracavity dielectric aperture for the electrical and optical confinement, which was normally formed by the non-conductive  $\text{Al}_2\text{O}_3$  from the selective wet oxidation of  $\text{AlAs}$  <sup>[1,2]</sup>. Because the gain-waveguide structure of the VCSEL is rather weak, J. P. Zhang <sup>[3]</sup>, Y. S. Zhang <sup>[4]</sup> and Morozov *et al.* <sup>[5]</sup> drew a conclusion that the transverse mode was affected by the gain-distribution in the active layer. Rahman *et al.* <sup>[6]</sup> noticed the effects of the carriers distribution on the transverse mode. And the thermal effects in the cavity were discussed by Zhang <sup>[7]</sup>. They took these effects into account separately, but in fact these factors took effects simultaneously. The current (or the driving carriers) distribution is the basic characteristic of an optoelectronic device, but the thermal property plays a significant role as well because of the non-omission of its effect on the gain-distribution in the small cavity. The VCSEL trans-

\* This program is supported by National Natural Science Foundation of China (Grant Nos. 69896260 and 696897003)  
Received 8 March 1999, revised manuscript received 21 May 1999

verse mode can be obtained from the solution of Maxwell equations in the gain-wave guide cavity. In this paper, the VCSEL simulation is presented and the current distribution, the thermal distribution and the optical transverse mode are concerned in the steady state. The finite element method, the finite difference method and the matrix eigenvalue method are used to obtain above distributions and the eigenmode. As a result, the distributions of the current, temperature and light power in the cavity are presented at the different driving currents under the confinement conditions.

## 2 Calculation Model

The schematic diagram of the cylindrical VCSEL cavity is showed in Fig. 1.  $W_0$  is the diameter of light output window,  $R_0$  is the radius of a cavity,  $a_0$  is the radius of a current window. The transverse modes are determined by the following factors

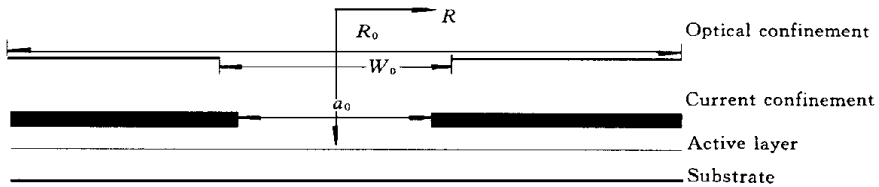


FIG. 1 Schematic diagram of a VCSEL

### 2.1 The current distribution

At the steady state, once the bias-voltage is given, the static distribution of the current is determined by Maxwell equations and the boundary conditions. It can be formulated by using the conventional  $(r, \theta, z)$  on cylindrical coordinates. Due to the axis symmetric structure of the VCSEL, the 3-dimension equation is reduced to a two-dimension one, with the current being the solution of the equation

$$\begin{cases} \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0, \\ \vec{j} = -\sigma(\vec{r}) \nabla(\vec{r}), \end{cases} \quad (1)$$

where  $\sigma(r)$  is the conductivity distribution in the cavity.

### 2.2 The thermal distribution

In the cylindrical coordinates, the basic heat equation given which governs the steady-state heat flow by Poisson equation

$$\nabla[\kappa(r) \cdot \nabla T(r)] = -Q(r), \quad (2)$$

Where  $T(r)$ ,  $Q(r)$  and  $\kappa(r)$  are respectively the spatial variations of the temperature, the internal heat sources and the material thermal conductivity. The following assumptions are applied: (1) at the bottom junction between the VCSEL substrate and the heat sink, which contacts with the VCSEL substrate, the temperature is keeping constant,  $T = T_0$ ;

(2) at the top or the side wall with adiabatic conduction,  $\left. \frac{\partial T}{\partial r} \right| = 0$ . Besides, there are two types of heat sources:

(i) the heat generated in the active region by nonradiation recombination

$$q(r) = \frac{U(r)(1 - \eta_{sp} f_{sp})}{d} [j_{th} + (j(\bar{r}) - j_{th})(1 - \eta)] \quad (3)$$

(ii) the Joule's heat in the other regions

$$q(\bar{r}) = j(\bar{r})^2 \cdot \rho(\bar{r}), \quad (4)$$

Where  $U(r)$  and  $j(r)$  are the voltage drop and the current density in the quantum well (QW) region. While  $j_{th}(r)$ ,  $\eta_{sp}$  and  $f_{sp}$  are the threshold current density, the internal quantum efficiencies of spontaneous and the stimulated emission, respectively. When the driving current is at equal to or higher than the threshold, the voltage drop  $U(r)$  can be regarded as a constant  $U = V_g$ .  $V_g$  is the built-in field. For GaAs,  $V_g = 1.2$  eV.

### 2.3 The carriers distribution in the active region

The carriers enhance the gain in the active region then form a gain wave guide to modify the transverse mode of the laser. Because the QW's width, 8nm, is much less than the effective length of the cavity, the distribution equation can be reduced into one-dimension

$$D_n \nabla^2 N(\bar{r}) - \frac{N(\bar{r})}{\tau_s} - v_g \Gamma_r g(N(\bar{r})) \sum_i P_i = - \frac{I(\bar{r})}{ed}, \quad (5)$$

$D_n$ ,  $v_g$ ,  $\Gamma_r$  and  $\tau_s$  are the diffusion coefficient of the carriers, the group velocity of the light wave in the cavity, the confinement factor and the lifetime of the carrier, respectively.  $P_i$  is the photon energy for the  $i$ -th mode;  $g(N)$  is the gain of the cavity, depending on the carriers density.

$$g(N(\bar{r})) = a(T) \cdot (N(\bar{r}) - N_{th}), \quad (6)$$

$N_{th}$  is the density of the transparent carriers

### 2.4 Optical field

The optical property is mainly determined by the gain profile, the carrier distribution and the temperature distribution in the active region. As assumed above, it can also be reduced into one-dimension. The  $p$ -th mode of light is the solution of the wave equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) + \left( k_0^2 \frac{\tilde{\epsilon}}{\epsilon} - \frac{m^2}{r^2} - \beta_z^2 \right) \Psi = 0 \quad (7)$$

$$\Phi(\theta) = \frac{1}{\sqrt{2m}} e^{im\theta} \quad (8)$$

Where  $k_0$  is the plane wave vector,  $\beta_z$  is the complex propagation constant in the  $z$  direction. Where  $\tilde{\epsilon}$  and  $\epsilon$  are the complex dielectric constant and the vacuum dielectric constant respectively.

$$\frac{\tilde{\epsilon}}{\epsilon} = n_0^2 + 2(\delta n)n_0 + i \frac{[g(\bar{r}) - \alpha_{total}]}{2k}, \quad (9)$$

where  $n_0$ ,  $g$  and  $\alpha_{total}$  are dielectric refraction index, gains and total losses of the cavity respectively.

The parameters referenced in this paper are listed in Table 1.  $N$ ,  $q_e$  and  $\omega$  are the carriers' density, electronic charge, and the frequency of the light  $n_0$ , and  $m_0$  are the effective index of the active layer and the effective mass of the carrier.

Table 1

Parameter	Value
$V_g$ the group velocity	$8 \times 10^9$ cm/s
$q_e$ electronic charge	$1.6 \times 10^{-19}$ C
$N_{th}$ transparent carriers density	$1.5 \times 10^{18}$ cm $^{-3}$
$D_n$ carrier diffusion coefficient	$4.8 \times 10^{-2}$ cm $^2$ /s
$\tau_c$ carriers life time	2 ns
$R_1, R_2$ reflect coefficient for top and bottom DBR	0.99, 1.0
$d$ width of the well	8nm
$\sigma_1$ conductivity of the confinement region	$10^{-1} \Omega^{-1} \cdot \text{cm}^{-1}$
$\sigma$ conductivity of other regions	$5 \times 10^2 \Omega^{-1} \cdot \text{cm}^{-1}$
$\omega$ frequency of the light	$3 \times 10^{14}$ Hz

### 3 Calculation method

Three calculation methods were adopted in this simulation: the finite element method (FEM), the finite difference method (FDM) and the matrix eigen method. The finite element method<sup>[8]</sup> is a powerful engineering tool for its flexibility and versatility, especially in complicated boundary structure. It was employed to solve the electric and heat distribution equations. In this procedure, the concerned structure was first automatically divided into small subregions, triangle elements. With these elements, the complicated structure of the cavity can be modulated. The memory requirement of the FEM program does not beyond 32Mb to a general personal computer for the safe work of the system. In the finite element method, the 2-dimension Poisson's equation could be expressed as following matrix equation.

$$[K] \cdot [\Psi] - [P] = [0], \quad (10)$$

$[K]$  is a structure factor, which could be obtained from combination with the finite elements.  $[\Psi]$  is the distribution of the field and  $[P]$  is the field resource. The elements of matrix  $[K]$  are obtained through the following expressions

$$K_{ii} = \sum_i k_{ii}^e \quad K_{ij} = \sum_{ij} k_{ij}^e, \quad (11)$$

$i$  is the summation of the triangle elements having the common vertex;  $ij$  is the summation of the triangle elements having the common hypotenuse.

The thermal distribution was obtained by the traditional FEM. Without the source in the electric field, the electric equation was reduced to Laplace equation. The carrier distribution equation was treated upon FDM<sup>[9]</sup> and the Optical field equation was reduced to a set of linear equations, which was treated by the matrix eigen process<sup>[10]</sup>. The final results were obtained by recursion self-consistently. The output of the  $n$  step was set as the input of the  $n+1$  step. And the stepped calculation would be stopped until the difference of results between two continuous steps is less than 0.01%.

### 4 Numerical Results

Two structures, with the current confinement window radii,  $2\mu\text{m}$  and  $10\mu\text{m}$  respec-

tively, are simulated as the examples. The used structure parameters are given in Table 2.

**Table 2**

Parameter	Value
$R_0$ radii of the cavity	$15\mu\text{m}$
$L_{\text{eff}}$ effective length of the cavity	$1.5\mu\text{m}$
$w_0$ the radii of the light-output window	$6\mu\text{m}$
$d$ the position of resistance region	$0.6\mu\text{m}$

The results show that the threshold of first structure was reduced to  $0.515\text{mA}$ , while that of the second was  $1.47\text{mA}$ . However, the current densities in the active regions reach  $2500\text{A}/\text{cm}^2$  and  $1300\text{A}/\text{cm}^2$  for the strong and weak confinement structures respectively.

The contour map of the confined cavity is presented in Fig. 2 and Fig. 3.

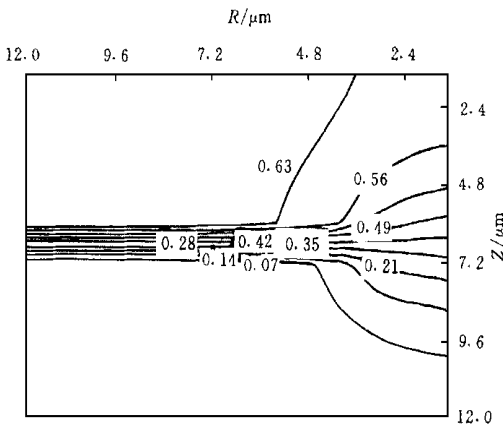


FIG. 2 The contour map of the electric field of the strong confinement structure

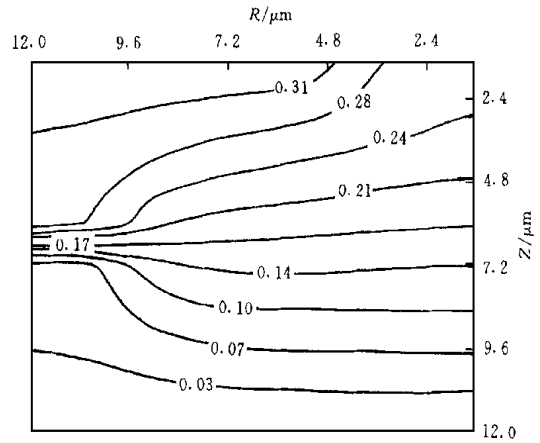


FIG. 3 The contour map of the electric field of the weak confinement structure

Fig. 4 and Fig. 5 (see last page I) respectively show the output pattern of light intensity for two structures, when the current  $I$  is 3 times of the threshold current  $I_{\text{th}}$ . The output pattern as the transverse mode is the numerical result of the optical equation in the active layer. In our model, all the gain-guide elements were treated numerically, and the eigen transverse mode series contain the modes which are like the ground and the first mode, not same as the existing of higher mode that in the dielectric guide. But for the weak confinement one, the output pattern is concentrated in the center.

Fig. 6 and Fig. 7 (see last page I) show the current density distributions in the cavity. The strong confinement benefits to concentrate the current density in the active region to a higher level, at the same time of the reduction of the threshold.

The temperature distributions are also presented in Fig. 8 and Fig. 9 (see last page II). In Fig. 7, the current peak is far away from the heat peak, which means that the heat resource is dominated by the non-radiation process. The temperature peak of the strong confined structure is slightly higher than the weak one, though the average temperature is lower than the latter.

With the increment of the driving current, the hole-burning takes place. At about  $5I_{\text{th}}$  it appears in the output pattern of the weak confinement device while at about  $6I_{\text{th}}$  in the

strong one. The confinement hampers the hole burning. Fig. 10 and Fig. 11 (see last page II) prove the output pattern in the burning-hole region.

Furthermore, the relationship between the threshold current  $I_{th}$  and the radius  $W_0$  of the confinement window is fitted

In Fig. 12,  $I_{th}$  can be given as the function of  $W_0$

$$I_{th} = - \frac{0.00288}{1 + \left(\frac{W_0}{x_0}\right)^p} + 0.00333, \quad (12)$$

where  $x_0 =$

12.87454,  $p = 2.33218$ . This result agrees very well with the experimental results of Oh<sup>[11]</sup>

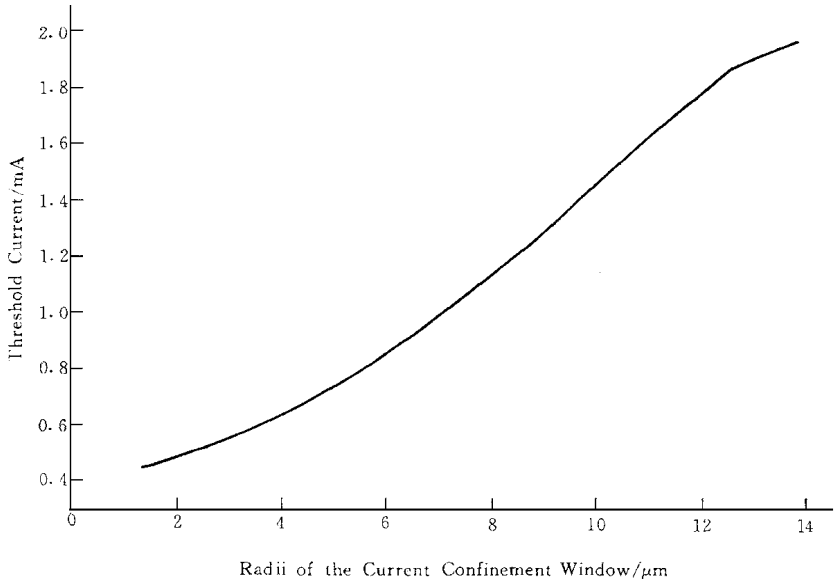


FIG. 12 The threshold current  $I_{th}$  as the function of the radii of the confinement window  $W_0$

### 5 Conclusion

Our simulating results demonstrate several characters of VCSEL at steady state. The key role of the high resistance confinement layer is presented clearly. The strong confined structure not only focuses the current, but also compacts the current pattern effectively. Because it concentrates the current density in the active region (Fig. 7) to a higher level, the threshold current is reduced and the hole burning is hampered do further. The other dynamic effects such as microcavity effect *et al*, will be considered in future simulation.

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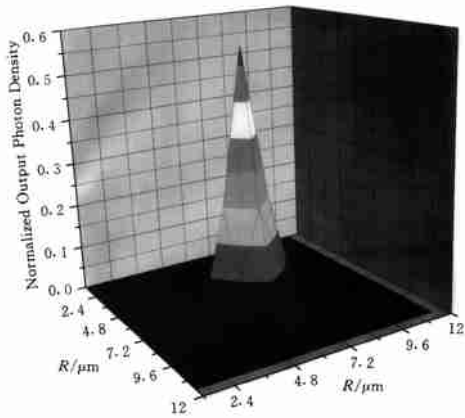


FIG 4 The output photon density distribution of the strong confinement structure at  $I = 3I_{th}$

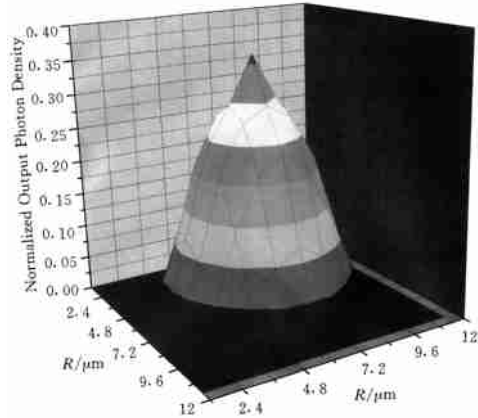


FIG 5 The output photon density distribution of the weak confinement structure at  $I = 3I_{th}$

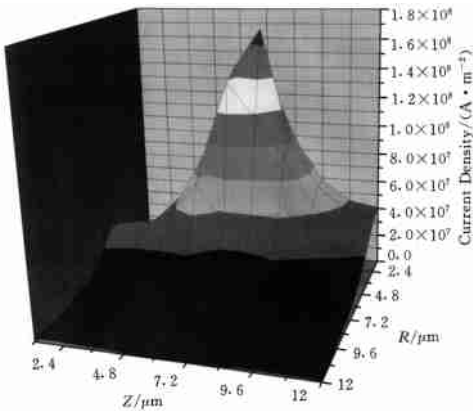


FIG 6 The current density distribution of the strong confinement structure at  $I = 3I_{th}$

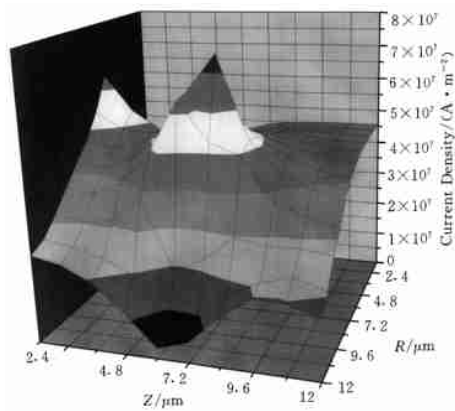


FIG 7 The current density distribution of the weak confinement structure at  $I = 3I_{th}$

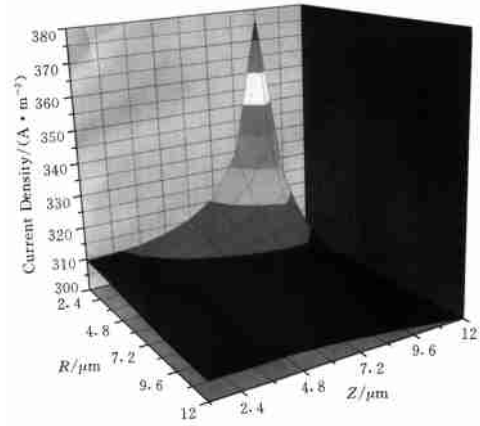


FIG 8 The temperature distribution of the strong confinement structure of  $I = 3I_{th}$

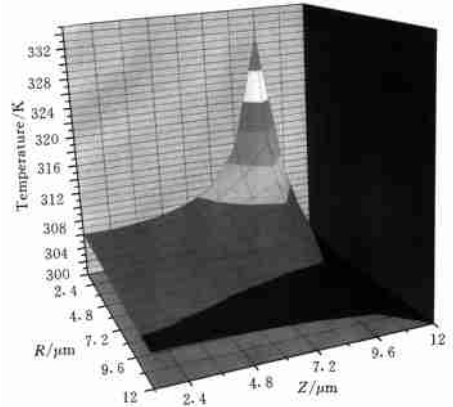


FIG 9 The temperature distribution of the weak confinement structure of  $I = 3I_{th}$

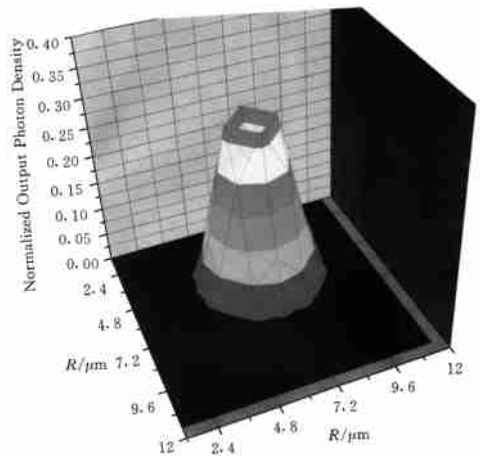


FIG 10 The output photon density distribution of the strong confinement in the burning-hole region

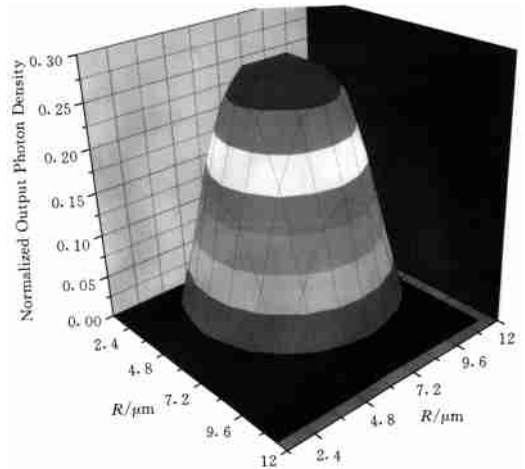


FIG 11 The output photon density distribution of the weak confinement structure in the burning-hole region