# HBT′s High Frequency Noise Modeling and Analysis*

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**Abstract**: A T-equivalent high frequency heterojunction bipolar transistor (HBT) noise model is reported. This model is derived from Hawkins noise model commonly used in Si-BJT. The main modifications include the influence of the ideality factor, emitter resistance, intrinsic base-collector capacitance, extrinsic base-collector capacitance and other parasitic elements of HBT represented in equivalent circuit topology. In order to calculate accurate noise parameters from the equivalent circuit, the noise correlation matrix method is used to avoid any simplifications generated in circuit transformations and complex noise measurements. The analysis of the influence of the equivalent circuit elements on the minimum noise figure is reported, the results of analysis agree well with the physics explanations. By means of the formulae derived from device physics of HBT, the influence of device parameters on the minimum noise figure is also represented.

**Key words**: heterojunction bipolar transistor; noise modeling; noise correlation matrix
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## 1 Introduction

Due to maximum high oscillation frequency ($f_{max}$), high cut off frequency ($f_T$), good linearity and high gain, heterojunction bipolar transistor (HBT) is emerging as a very useful device for power amplifier at microwave wave band. However, the high frequency noise performance of HBT remains lower than that of high electron mobility transistor (HEMT) because the correlation between the main noise sources in HEMT severely reduces the noise contribution of those noise sources. But the noise level of HBT is acceptable in some applications such as optical receiver front end. Once more, HEMT devices need more precise photolithography than HBT devices for the same noise performance because the current in HBT flows through vertical direction, but in HEMT flows through cross direction. The difficulty of photolithography in HBT devices is transferred to the material growth.

A high frequency noise model of HBT is proposed based on Hawkins noise model in Section 2. Section 2 is also dedicated to the derivation of the four noise parameters from the proposed model. An analysis of the influence of circuit elements and device parameters on the minimum noise figure as well as the comparison between the proposed model and Hawkins model are reported in Section 3.

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## 2 Noise model and noise parameter derivation

An equivalent circuit is generally needed to calculate the four noise parameters. The noise model proposed by Hawkins[1] in such a way for bipolar transistor has been successfully used in Si-BJT. Hawkins's noise model, Fig. 1[1], is essentially a low-to-medium frequency model in that it neglects all device capacitances except to the base-emitter junction capacitance ($C_e$), and all parasitic resistances except the base resistance ($R_b$). The high frequency noise model of HBT is different from Hawkins model. However, we can derive the noise model suitable for HBT devices by using the Hawkins model with some modifications, which include the influence of the ideality factor, emitter resistance, intrinsic base-collector junction capacitance, extrinsic base-collector junction capacitance and other parasitic elements of HBT.

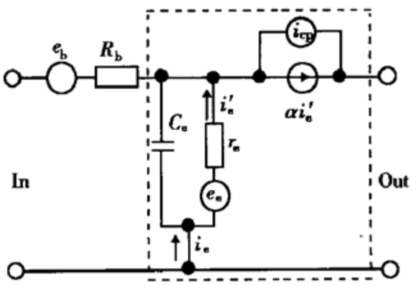


Fig. 1　Hawkins noise model[1]

In the circuit of Fig. 1, $e_b$ represents the thermal noise in the base resistance, $e_e$ and $i_{cp}$ are the equivalent emitter shot noise source and the collector partition noise source. Thus

$$\overline{e_b^2} = 4kTR_b$$
$$\overline{i_{cp}^2} = 2kT(\alpha_0 - |\alpha|^2)/r_e$$
$$\alpha = \frac{\alpha_0}{1 + jf/f'_b} \qquad (1)^{[1]}$$
$$\overline{e_e^2} = 2kTr_e$$
$$\overline{i_{cp}e_e^*} = 0$$

taking emitter dynamic resistance $r_e = kT/qI_e$ where $T$ is the absolute temperature and $\alpha$ is common base current gain. $\alpha$ is a single pole function where $\alpha_0$ is common base current gain at DC and $f'_b$ is cut-off frequency of base transit time plus base-collector depletion layer transit time. Although physical noise sources of the emitter and collector in reality are correlated in a bipolar transistor, Hawkins transformed these sources to a T-equivalent circuit configuration that cancels the correlation of the equivalent circuit noise sources[1].

In order to derive the HBT's noise model based on Hawkins model, the characteristic difference of HBT from Si-BJT must be considered. First, the ideality factor of HBT must be included accurately for emitter dynamic resistance. Considering the ideality factor $n_e$ of HBT, the expression of $r_e$ is changed to

$$r_e = n_e kT/qI_e \qquad (2)$$

Thus from the expressions of noise sources we derive

$$\overline{e_e^2} = \overline{i_e^2}r_e^2 = 2qI_e r_e^2 = 2n_e kTr_e$$
$$\overline{i_{cp}^2} = 2q\alpha_0 I_e - |\alpha|^2 2qI_e = 2n_e kT\frac{(\alpha_0 - |\alpha|^2)}{r_e} \qquad (3)$$

Secondly, the effect of intrinsic base-collector capacitance $C_{bc}$ and emitter resistance $R_E$ on the noise parameters must be included. Because of the small size of emitter and its small base resistance, the thermal noise of emitter resistance is not negligible compared to that of base resistance. Meanwhile, in HBT devices base layer is doped so heavily that the base-collector capacitance becomes more important than that of Si-BJT. Thirdly, in order to get more accurate noise parameters of HBT devices, we must include noise contribution provided by the parasitic elements and the influence of parasitic elements on overall device noise properties.

Concerning the above three modifications of Hawkins noise model, we can propose high frequency noise T-equivalent circuit (Fig. 2) of HBT, where $i_{cp}$ collector partition noise source, $i_{nc}$ collector shot noise source, $e_{re}$ emitter shot noise source, $e_{RE}$ thermal noise source of emitter resistance $R_E$, $e_{rb}$

thermal noise source of intrinsic base resistance $r_b$, $e_{R_C}$ thermal noise source of collector parasitic resistance $R_C$, $e_{R_B}$ thermal noise source of base parasitic resistance $R_B$, $L_B$ base parasitic inductance, $L_C$ collector parasitic inductance, $C_{be}$ intrinsic base-emitter capacitance, $C_{bc}$ intrinsic base-collector capacitance, $C_{ex\text{-}bc}$ extrinsic base-collector capacitance and $r_c$ intrinsic base-collector resistance. Because we only discuss the high frequency noise of HBT, the low frequency noise of HBT that results from the superimposition of both $1/f$ noise and generation-recombination noise can be ignored in this model. The collector shot noise source $i_{nc}$ can be found to be negligible since base-collector junction is reverse-biased.

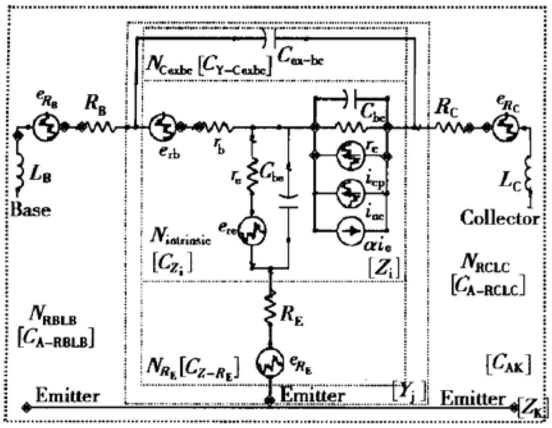

Fig. 2 Modified noise T-equivalent circuit of HBT

This circuit is divided into five networks: the intrinsic network represented by $N_{\text{intrinsic}}$, the network represented by $N_{\text{Cexbc}}$, in which only the extrinsic base-collector capacitance is included, the network represented by $N_{R_E}$ in which only the emitter resistance is included, and the networks represented by $N_{\text{RBLB}}$ and $N_{\text{RCLC}}$ consisting of the parasitic elements $R_B$-$L_B$ and $R_C$-$L_C$, respectively. $[Z_i]$ represents the impedance matrix of the intrinsic network. $[Y_j]$ represents the admittance matrix of the network in which the extrinsic base-collector capacitance network, the emitter resistance network and the intrinsic network are taken into account. $[Z_K]$ represents the impedance matrix of the overall network. Because those five networks can be regarded as the interconnections of two-port linear network, we can use the noise correlation matrix method proposed by Hillbrand and Russer[2] to calculate four noise parameters according to the noise equivalent circuit represented in Fig. 2. The noisy two-ports are represented either by an admittance representation with a noise free part and two noise current sources, or by an impedance representation with two noise voltage sources, or by a chain representation with a noise current source and a noise voltage source both at the input side. These representations and the corresponding correlation matrices are shown in Fig. 3. The noise free circuit can be represented by the admittance, impedance, and chain matrix.

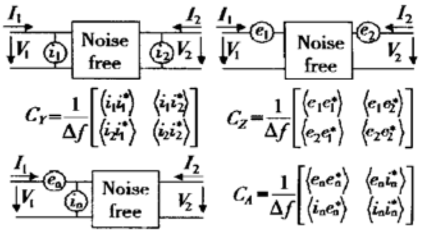

Fig. 3 Three representations and correlation matrices[2]

If the two-port network considered consists of only passive elements, the thermal noise from it results in a correlation matrix of either of the two forms:

$$\begin{aligned} C_Y &= 4kT\,\mathrm{Re}[Y] \\ C_Z &= 4kT\,\mathrm{Re}[Z] \end{aligned} \quad (4)^{[2]}$$

where $k$ is Boltzmann's constant.

From the expressions of the chain correlation matrix of active two-ports, we can derive

$$\begin{aligned} Y_{\text{SOF}} &= \sqrt{C_{A,22}/C_{A,11} - (\mathrm{Im}[C_{A,12}]/C_{A,11})^2} \\ &\quad + \mathrm{j}(\mathrm{Im}[C_{A,12}]/C_{A,11}) \\ F_{\min} &= 1 + (C_{A,12} + C_{A,11}Y^*_{\text{SOF}})/kT \\ R_n &= C_{A,11} \end{aligned} \quad (5)^{[2]}$$

where $R_n$ is the equivalent noise resistance, $F_{\min}$ is the minimum noise figure and $Y_{\text{SOF}}$ is the corresponding optimum source admittance.

These matrices can be transformed to any of the three forms shown in Fig. 3 by

$$C' = TCT^{+} \qquad (6)^{[2]}$$

where $C$ and $C'$ denote the correlation matrix of the original and resulting representation, respectively. The transformation matrix $T$ is given in Fig. 4 and the dagger '+' denotes the Hermitian conjugate.

| $T$ | From impedance | From admittance | From chain |
|---|---|---|---|
| To impedance | $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ | $\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}$ | $\begin{bmatrix}-Y_{11} & 1\\ -Y_{21} & 0\end{bmatrix}$ |
| To admittance | $\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}$ | $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ | $\begin{bmatrix}1 & -Z_{11}\\ 0 & -Z_{21}\end{bmatrix}$ |
| To chain | $\begin{bmatrix}0 & B\\ 1 & D\end{bmatrix}$ | $\begin{bmatrix}1 & -A\\ 0 & -C\end{bmatrix}$ | $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ |

Fig. 4 Transformation matrix $T$[2]

Interconnections between two two-ports in series, in parallel or in cascade result in a correlation matrix given by

$$\begin{aligned} C_Z &= C_{Z1} + C_{Z2} \\ C_Y &= C_{Y1} + C_{Y2} \\ C_A &= C_{A1} + A_1 C_{A2} A_1^{+} \end{aligned} \qquad (7)^{[2]}$$

where the subscripts 1 and 2 refer to the two-ports to be connected.

To apply the approach of noise correlation matrix, we transform the intrinsic network $N_{\mathrm{intrinsic}}$ of noise model in Fig. 2 to an equivalent one consisting of two noise voltage sources with a noiseless network (see Fig. 3). These two noise sources can be expressed in terms of the original noise sources by a linear transformation:

$$\begin{aligned} \overline{e_1^2} &= \overline{e_{\mathrm{rb}}^2} + \overline{e_{\mathrm{re}}^2}/(1 + \mathrm{j}w C_{\mathrm{be}} r_{\mathrm{e}})^2 \\ \overline{e_2^2} &= \frac{\overline{e_{\mathrm{re}}^2}}{(1 + \mathrm{j}w C_{\mathrm{be}} r_{\mathrm{e}})^2} + \frac{\overline{i_{\mathrm{cp}}^2} r_{\mathrm{e}}^2}{(1 + \mathrm{j}w C_{\mathrm{be}} r_{\mathrm{e}})^2} \\ \overline{e_1 e_2^*} &= \overline{e_1^* e_2} = \overline{e_{\mathrm{re}}^2}/(1 + \mathrm{j}w C_{\mathrm{be}} r_{\mathrm{e}})^2 \end{aligned} \qquad (8)$$

We can get the intrinsic noise correlation matrix represented by $[C_{Zi}]$ using the expression in Fig. 3.

Because the network $N_{R_E}$ of the emitter resistance $R_E$ is connected to the intrinsic network in series, we can get the noise correlation matrix which includes the network of $N_{R_E}$ and $N_{\mathrm{intrinsic}}$ by means of the formula (7) in impedance representation. In a similar way, because the network $N_{\mathrm{Cexbc}}$ of the extrinsic base-collector capacitance $C_{\mathrm{ex\text{-}bc}}$ is connected to the intrinsic network in parallel, we can calculate the noise correlation matrix $[C_{Y_j}]$ by means of the formula (7) in admittance representation. The noise correlation matrix can be transformed to its any representation through the expression (6). Since the network of $N_{\mathrm{Cexbc}}$ and $N_{R_E}$ are passive, their noise correlation matrices are easily known taking account of the expression (4).

Because the networks of $N_{\mathrm{RBLB}}$ and $N_{\mathrm{RCLC}}$ are connected with the network $N_{Y_j}$ in cascade, respectively, the overall noise correlation matrix $[C_{AK}]$ can be derived according to the formula (7) in cascade representation. The correlation matrices of networks $N_{\mathrm{RBLB}}$ and $N_{\mathrm{RCLC}}$ consisting of parasitic base and collector elements are expressed by using the formula (4). The noise correlation matrix $[C_{Y_j}]$ is transformed to its chain representation $[C_{A_j}]$ by means of the expression (6). The overall noise correlation matrix $[C_{AK}]$ is shown below:

$$\begin{aligned} C_{AK} = {} & C_{A\text{-RBLB}} + A_{\mathrm{RBLB}}\big(T_{Y\to A}\big(C_{Y\text{-Cexbc}} \\ & + T_{Z\to Y}(C_{Zi} + C_{Z\text{-RE}})T_{Z\to Y}^{+}\big)T_{Y\to A}^{+} \\ & + A_j C_{A\text{-RCLC}} A_j^{+}\big) A_{\mathrm{RBLB}}^{+} \end{aligned} \qquad (9)$$

The final step of the noise calculations is to obtain the four noise parameters. Therefore we use the expression (5) to calculate the four noise parameters.

This method does not involve any simplifications. It takes all parasitic elements of the equivalent noise circuit in Fig. 2 into account and avoids the complex noise measurements.

## 3 Results and discussion

The HBT device data sheet used in this work was provided by Philips Semiconductors[5]. The Intrinsic circuit parameters are obtained from $S$-parameter transformation after deembedding the parasitic elements from $S$-parameters measurements under cut off mode bias and normal bias condition.

Figure 5 compares noise measurements and calculations of the minimum noise figure $F_{\min}$ as a function of the frequency under $V_{CE}= 2\mathrm{V}$, $I_C= 1\mathrm{mA}$

bias condition using the Hawkins model and the modified model, respectively. We can conclude that the calculations using the modified model show better agreement with measurements than those of Hawkins model. The agreement using the modified model is about 0.4dB over the whole frequency range. This can be attributed to the fact that the calculations depend on the accuracy of the parameters extraction. Figure 5 also presents the equivalent noise resistance $R_n$ at the right axis. A good agreement is observed between measurements and calculations using the modified model since the equivalent noise resistance is difficult to measure accurately. Hawkins model has not the expression of the equivalent noise resistance.

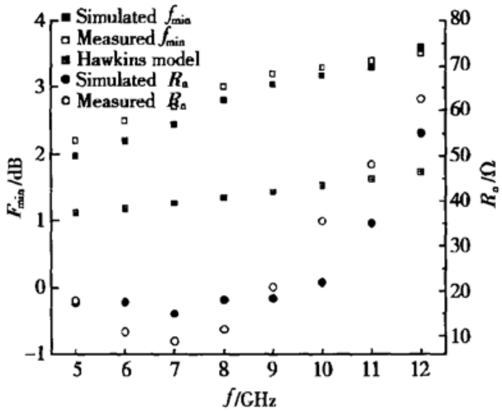


Fig. 5 Calculated and measured results

Furthermore, We can analyze the influence of the equivalent circuit elements on the minimum noise figure by using this modified noise model. Figure 6 shows $F_{min}$ as a function of circuit elements $r_b$, $R_B$, $L_B$, $R_C$, $L_C$, $r_c$ and $r_e$. Figure 7 displays $F_{min}$ as a function of capacitances $C_{be}$, $C_{bc}$, and $C_{ex\text{-}bc}$.

Our simulation shows that the minimum noise figure $F_{min}$ is insensitive to the elements $R_C$, $r_c$, $r_e$, $L_C$, and $C_{be}$, whereas the elements $r_b$, $R_B$, $R_E$, $L_B$, $C_{bc}$, and $C_{ex\text{-}bc}$ have some influence on it. The more sensitive parameters are the base resistances $r_b$, $R_B$ and emitter resistance $R_E$. Those results agree well with the physics explanations and show that including those elements in the equivalent noise circuit is necessary at high frequency range.

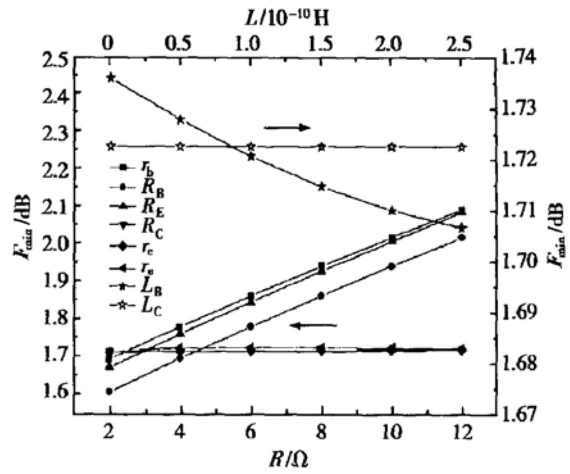


Fig. 6 $F_{min}$ as a function of $r_b$, $R_B$, $L_B$, $R_C$, $L_C$, $r_c$, and $r_e$

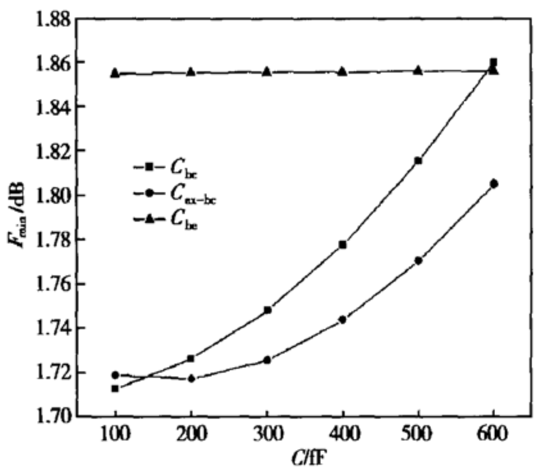


Fig. 7 $F_{min}$ as a function of $C_{be}$, $C_{bc}$ and $C_{ex\text{-}bc}$

According to the formulae given in Liu's book[6], the device parameters can be connected with the equivalent circuit elements in Fig. 2. Figure 8 displays $F_{min}$ as a function of device parameters $L_e$, $N_b$, $S_{bc}$, $S_{be}$, $W_b$, $W_c$, $W_e$, $X_b$, $X_c$, $X_e$, and $X_{sc}$. Variable means the range of those device parameters. It can represent either of those device parameters.

From Fig. 8, we can know that the parameters $S_{be}$, $X_b$, $L_e$, $N_b$, and $W_e$ have some impact on the minimum noise figure and other device parameters do not affect it severely because of the cancellation effect between each other. We can take some measures such as BE junction self-align process to reduce the minimum noise figure according to the results shown in Fig. 8.

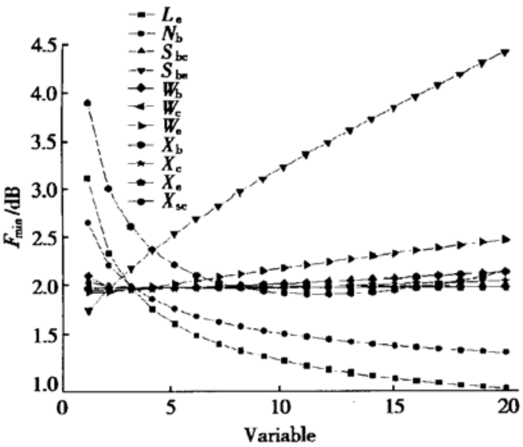


Fig. 8　$F_{min}$ as a function of device parameters

## 4　Conclusion

In this paper, we proposed T-equivalent noise circuit model based on Hawkins noise model for heterojunction bipolar transistor in the high frequency range, including the influence of the ideality factor, emitter resistance, intrinsic base-collector capacitance, extrinsic base-collector capacitance and other parasitic elements on the noise parameters. This noise model satisfactorily describes the noise behavior of HBT and shows better agreement with measured results than Hawkins noise model. The influence of circuit elements and device parameters on the minimum noise figure has also been addressed.

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# 异质结双极晶体管高频噪声建模及分析*

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**摘要**: 提出了一个 T 等效异质结双极晶体管高频噪声电路模型. 该模型是对通常用在硅双极晶体管中的 Hawkins 噪声模型进行改进得到的, 主要的改进包括发射极理想因子、发射极电阻、内部 BC 结电容、外部 BC 结电容和其它寄生元素对器件噪声性能的影响. 为了从等效噪声电路模型中计算出精确的噪声参数, 采用了噪声相关矩阵法来计算噪声参数, 从而避免了在等效电路变换中可能产生的简化和复杂的噪声测量. 进一步利用该模型分析了等效电路元素对器件最小噪声系数的影响, 分析计算结果和物理解释一致. 同时通过基于异质结双极晶体管器件物理的公式, 给出了器件参数对器件最小噪声系数的影响.

**关键词**: 异质结双极晶体管; 高频噪声模型; 噪声相关矩阵

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