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A Generalized Reynolds´ Equation For Squeeze-Film Air Damping in MEMS*

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Abstract: A differential equation that is generally effective for squeeze-film air damping of perforated plate and non-perforated plate as well as in MEMS devices is developed. For perforated plate, the thickness and the dimensions of the plate are not limited. With boundary conditions, pressure distribution and the damping force on the plate can be found by solving the differential equation. Analytical expressions for damping pressure and damping force of a long strip holeplate are presented with a finite thickness and a finite width. To the extreme conditions of very thin plate and very thin hole, the results are reduced to the corresponding results of the conventional Reynolds´ equation. Thus, the effectiveness of the generalized differential equation is justified. Therefore, the generalized Reynolds´ equation will be a useful tool of design for damping structures in MEMS.

Key words: squeeze-film air damping; MEMS; Reynolds´ equation

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1 Introduction

Squeeze-film air damping is a critical factor for most MEMS transducers and actuators, such as micromachined microphones, micro accelerometers, vibratory gyroscopes, micro switches etc. The squeeze-film air damping of MEMS devices is traditionally analyzed with the well-known Reynolds´ equation[1~3].

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=\frac{12\mu}{h^3}\times\frac{\partial h}{\partial t} \quad (1)$$

where P is the damping pressure caused by squeeze-film air damping effect. μ is the coefficient of viscosity of air, and h is the distance between the plate and the nearby wall against which the plate is moving.

For example, for a long strip plate with a width of $2a$, the solution to the equation is[1~3]

$$P(x)=-\frac{6\mu}{h^3}(a^2-x^2)\frac{\mathrm{d}h}{\mathrm{d}t} \quad (2)$$

and the damping force on a strip plate with length L is

$$F_{\mathrm{lr}}=L\int_{-a}^{a}P(x)\mathrm{d}x=-\frac{8\mu a^3 L}{h^3}\dot{h} \quad (3)$$

The squeeze-film air damping is usually a limited factor for the performance of most MEMS devices. To reduce the air damping effect. the scheme

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most commonly used is to perforate the plate so that air is allowed mainly to flow through the holes instead of the borders. As a matter of fact, perforation is inherently needed in surface micromachining technique for the etching of sacrificial layer. Other schemes (such as vacuum package and the use of structures with slide-film air damping) are also possible, but major structural changes or sophisticated packaging technique will be needed.

For a perforated thin plate (as is often seen in MEMS devices by surface micromachining technique[4]), the air flowing through the border can be neglected (i.e., the infinite plate approximation). The force of squeeze-film air damping is[3,5]

$$F_{\text{thin}} = - A \frac{3\mu\pi r_c^2}{2\pi h^3} \dot{h} K(\beta) \tag{4}$$

where $K(\beta) \equiv (4\beta^2 - \beta^4 - 4\ln\beta - 3)$, A is the area of the plate, r_c is the equivalent radius of a cell area corresponding to a hole and β is the relative hole radius, i.e., the ratio between the radius of hole r_0 and the equivalent radius of cell r_c, $\beta = r_0/r_c$.

The main difficulty of above-mentioned treatment is that the damping effect of air flowing in holes, the air flowing among neighboring cells, and the air flowing through the borders are not considered. As a matter of fact, thick silicon plates perforated with thin holes are getting popular for micromechanical transducers and actuators, recently. For a thick holeplate the damping effect of air flowing in holes becomes significant and the boundary conditions may be more important than that for a thin holeplate. Therefore, new effective methods for the squeeze-film air damping of perforated thick plates are needed very much for the design consideration.

2 Generalized Reynolds' equation

To establish a differential equation that is generally effective for the squeeze-film air damping of holeplate and non-perforated plate as well, let us consider a thick plate with uniform perforated holes as shown in Fig. 1. When the plate is moving towards the wall, pressure is induced under the plate. According to Poiseuille Equation, for a hole with radius r_0 and length H, the volume of air passing through the hole in a unit time is related to the pressure difference P_H between the two ends of the holes.

$$V = \frac{dV}{dt} = \frac{\pi r_0^4}{8\mu} \times \frac{p_H}{H} \tag{5}$$

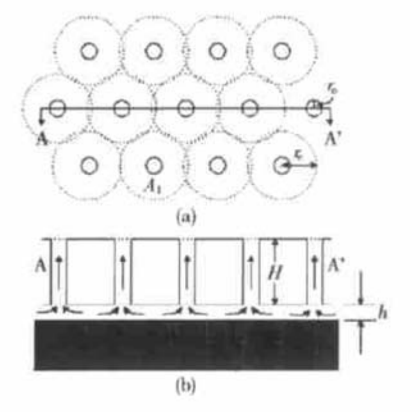

Fig. 1 Schematic structure of a hole-plate (a) Top view; (b) View of cross-section

According to Reference[3] the damping force developed under the plate in a cell by the lateral air-flow to the hole is

$$F_1 = \frac{3\mu A_1}{2\pi h^3} K(\beta) V \tag{6}$$

where $A_1 = \pi r_c^2$. Therefore, the average damping pressure in the cell area is

$$P = P_H + \frac{F_{11}}{A_1} = P_H \left[1 + \frac{3r_0^4 K(\beta)}{16H h^3}\right] = \eta(\beta) P_H \tag{7}$$

where $\eta(\beta) \equiv \left[1 + \frac{3r_0^4 K(\beta)}{16H h^3}\right]$.

For typical hole-plate in MEMS devices, the cell size is much smaller than the overall size of the hole-plate. Therefore, the average pressure in a cell can be considered as the damping pressure at the center of the cell and the damping pressure under the plate is a smooth function of position. For the derivation of differential equation for the pressure distribution under the plate, the air flowing through the hole of a cell is approximated as penetrating the cell area uniformly. The penetration rate of air volume through the plate is

$$Q_z = \frac{V}{A_1} = \frac{\beta^2 r_0^2}{8\mu H} \times \frac{P}{\eta(\beta)} \quad (8)$$

With the extra air penetration through the plate, the equation for mass conservation (corresponding to Eq. 3. 17 in Reference [3]) can be found as

$$\frac{\partial(\rho q_x)}{\partial x} + \frac{\partial(\rho q_y)}{\partial y} + \frac{\beta^2 r_0^2}{8\mu H} \times \frac{1}{\eta(\beta)} \rho P + \frac{\partial(\rho h)}{\partial t} = 0 \quad (9)$$

where $q_x = -\frac{h^3}{12\mu} \times \frac{\partial P}{\partial x}$ and $q_y = -\frac{h^3}{12\mu} \times \frac{\partial P}{\partial y}$. Following the derivation procedure given in Reference [3], we can modify the Reynolds´ equation to a generalized form.

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \frac{3\beta^2 r_0^2}{2h^3 H} \times \frac{1}{\eta(\beta)} P = \frac{12\mu}{h^3} \times \frac{dh}{dt} \quad (10)$$

This generalized Reynolds´ equation differs from the conventional Reynolds´ equation by an additional (the third) term on the left side of the equation. The term represents the damping effect of air-flow in the holes. Generally, by solving the equation with appropriate boundary conditions, the distribution of damping pressure under holeplates can be found.

3 Solution to long rectangular holeplate

To find a useful example, let us consider a strip of holeplate with a length L, which is much longer than the width $2a$. The equation is now one dimension .

$$\frac{\partial^2 P}{\partial x^2} - \frac{3\beta^2 r_0^2}{2h_3 H} \times \frac{1}{\eta(\beta)} P = \frac{12\mu}{h^3} \times \frac{dh}{dt} \quad (11)$$

By defining $l = \sqrt{\frac{2h^3 H \eta(\beta)}{3\beta^2 r_0^2}}$ and $R = -\frac{12\mu}{h^3} \times \frac{dh}{dt}$, Equation (11) can be written as

$$\frac{d^2 P}{dx^2} - \frac{P}{l^2} + R = 0 \quad (12)$$

The boundary conditions are

$$P|_{x=\pm a} = 0$$

The solution to the equation is found to be

$$P(x) = Rl^2 \left[1 - \cosh\left(\frac{x}{l}\right) \Big/ \cosh\left(\frac{a}{l}\right) \right] \quad (13)$$

The dependence of pressure distribution on the width of the plate is shown in Fig. 2, where the pressure is normalized to Rl^2.

Fig. 2. Dependence of normalized pressure distribution on a/l

From Eq. (13), the damping force on the plate is

$$F_d = 2aLRl^2 \left[1 - \frac{l}{a} \tanh\left[\frac{a}{l}\right] \right] \quad (14)$$

If the holes are very thin and the plate is very thick so that l is much larger than a, Eq. (13) can be return to Eq. (2)

For another extreme, if the plate is very thin (i. e., $H \to 0$) so that $l \to 0$, Eq. (13) gives a constant pressure distribution under the plate except for the boundary areas.

$$P = Rl^2 = -\frac{3\mu r_c^2}{2h^3} K(\beta) \dot{h}$$

This result coincides with the squeeze-film damping force of a infinite holeplate given in Eq. (4). The results for the two extremes justify the novel differential equation. Therefore, the generalized Reynolds´ equation will be a useful tool for design of many MEMS devices, such as microphone, micro accelerometers, vibratory gyroscopes, resonant sensors, etc.

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微电子机械器件压膜空气阻尼的一般化雷诺方程*

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摘要: 推导建立了一个用于微电子机械(MEMS)器件压膜空气阻尼的一般化微分方程. 该方程很好地解决了分析有限尺寸孔板的边界和孔板的厚度对压膜阻尼的影响的困难. 结合边界条件求解该方程可以得到各种压膜阻尼结构中的阻尼压强分布和阻尼力. 以长条板为例得到了有限宽度和有限厚度孔板压膜阻尼的压强分布和阻尼力的解析解, 在非孔板的条件下该解退化为一般传统雷诺方程的解, 而在无限大薄孔板的极端条件下, 该解也与传统雷诺方程在此条件下的解相一致, 进一步证明了该方程的正确性. 因此, 该一般化的雷诺方程为MEMS器件的压膜阻尼设计提供了有效的手段.

关键词: 压膜空气阻尼; 微电子机械器件; 雷诺方程

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