Nonlinear Auto-Companding Method for Behavioral Modeling of Switched-Current Circuits*

Wang Wei[1], Zeng Xuan[1], Tao Jun[1], Su Yangfeng[2] and Tang Pushan[1]

(1 ASIC & System State Key Laboratory, Microelectronics Department, Fudan University, Shanghai 200433, China)
(2 Department of Mathematics, Fudan University, Shanghai 200433, China)

Abstract: A wavelet collocation method with nonlinear auto-companding is proposed for behavioral modeling of switched-current circuits. The companding function is automatically constructed according to the initial error distribution obtained through approximating the input-output function of the SI circuit by conventional wavelet collocation method. In practical applications, the proposed method is a general-purpose approach, by which both the small signal effect and the large signal effect are modeled in a unified formulation to ease the process of modeling and simulation. Compared with the published modeling approaches, the proposed nonlinear auto-companding method works more efficiently not only in controlling the error distribution but also in reducing the modeling errors. To demonstrate the promising features of the proposed method, several SI circuits are employed as examples to be modeled and simulated.

Key words: wavelet collocation method; behavioral modeling; switched-current circuits; nonlinear auto-companding
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1 Introduction

During the past one decade, switched-current (SI) technique[1] has been considered as a promising methodology for the monolithic implementation of mixed analog and digital VLSI. With the remarkable evolution of SI techniques, the need for more advanced behavioral modeling technology of the switched-current circuits has become increasingly urgent. First, in top-down designs, the simulation based on behavioral models can provide the necessary information for selecting correct architectures to implement the analog functions before investing time in detail circuit implementation. Second, in bottom-up verifications, because the behavioral simulation is computationally cheap, it enables designers to verify the complex system efficiently.

When modeling nonlinear SI circuits, we first partition the whole system into building blocks. Then the input-output function of each block is expressed as a nonlinear function[2~5]. Compared with the conventional theoretical analysis method[2,3], polynomial expansion approximation[4] is more effi-

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Wang Wei　male, PhD candidate. His research interests are analog circuit design and behavioral modeling methodology.

Zeng Xuan　female, professor. Her research interests include VLSI high-speed interconnect circuit analysis and design, VLSI physical design and analog circuit design automation.

Tao Jun　female. Her research interests include analog circuit behavioral modeling and simulation.

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cient and flexible. Unfortunately, the polynomial expansion approach cannot control the error distribution efficiently and does not consider the large signal effect[5].

Reference [5] proposes a wavelet collocation method to expand the input-output functions of switched-current building blocks by wavelets. Due to the compact support of wavelet basis functions, a nonlinear companding algorithm is introduced to control the modeling error distribution. It is worth noting that the companding function in Ref. [5] is obtained manually based on the varied trend of the given input-output curve. However, automatically generating the nonlinear companding function has not been discussed yet.

In this paper, we propose a novel nonlinear auto-companding algorithm for behavioral modeling of nonlinear SI circuits. The companding function can be automatically generated according to the error distribution.

The rest of the paper is organized as follows. In section 2, we review the wavelet collocation method with nonlinear companding. Then, we illustrate how to construct the companding function automatically step by step in section 3. Numerical experiments are presented in section 4 to demonstrate the computational accuracy and efficiency of the proposed method. Finally, we draw conclusions in section 5.

2 Review of wavelet collocation method with nonlinear companding

In this section, we shall review wavelet collocation method with nonlinear companding. With the nonlinear companding, we can regulate the error distribution so that the modeling error can satisfy certain specifications given by designers.

2.1 Algorithm of nonlinear companding

As referred in Reference[6], changing the singularity of wavelet bases can modify the modeling error distribution. This issue can be realized by a nonlinear companding algorithm reviewed in the following.

Assume the input-output function $f(x)$ is defined and modeled in Input Domain interval $[x_A, x_B]$. And the wavelet basis functions $\{W_i(l); i=1, 2, \Lambda, M\}$ are defined in Companding Domain $[l_A, l_B]$. The relation between them is determined by a nonlinear companding function $l=g(x)$, which is defined in interval $[l_A, l_B]$. Now, with nonlinear companding, the input-output function $f(x)$ is expanded by wavelets, as shown in Eq. (1)[5].

$$f(x) = f[g^{-1}(l)] = \sum_{i=1}^{M} C_i W_i(l) = \sum_{i=1}^{M} C_i W_i[g(x)] \tag{1}$$

where $x=g^{-1}(l)$ is the inverse function of $l=g(x)$, and $\{C_i; i=1, 2, \Lambda, M\}$ are wavelet coefficients. Function $l=g(x)$ should satisfy the following constraints:

(1) $l_A=g(x_A)=x_A$ and $l_B=g(x_B)=x_B$.

(2) Function $l=g(x)$ is monotonically increasing.

Hence, function $l=g(x)$ establishes a one-to-one mapping between the Input Domain $[x_A, x_B]$ and the Companding Domain $[l_A, l_B]$.

2.2 Mechanism of nonlinear companding

As expressed in equation (1), the singularity of $W_i[g(x)]$ will be changed if $g(x)$ is modified. Therefore, by using proper nonlinear companding function $l=g(x)$, we can force the modeling error distribution satisfy certain specifications given by designers.

The first-order derivative functions of $W_i[g(x)]$ are

$$\frac{\mathrm{d}W_i}{\mathrm{d}x} = \frac{\mathrm{d}W_i}{\mathrm{d}l} \times \frac{\mathrm{d}l}{\mathrm{d}x} = \frac{\mathrm{d}W_i}{\mathrm{d}l} g'(x); i = 1, 2, \Lambda, M \tag{2}$$

Equation (2) demonstrates that the derivative of the companded wavelet $\mathrm{d}W_i/\mathrm{d}x$ is $g'(x)$ times of the derivative of the original wavelet $\mathrm{d}W_i/\mathrm{d}l$ after nonlinear mapping. Since the derivative function of a waveform indicates its singularity, thus it implies that the singularity of the original wavelet basis functions dW_i/dl is changed in Input Domain. The companded wavelet basis functions $W_i[g(x)]$ will become much more singular in those regions where the derivative function $g'(x)$ has a large value. Therefore, when these companded wavelet bases are used to represent the input-output function $f(x)$ in Input Domain, the singular basis functions have the potential to approximate $f(x)$ more accurately since they contain more high frequency components. In summary, we shall increase the value of $g'(x)$ in those regions where high modeling accuracy is needed. In other words, the modeling error is reversely proportional to the value of $g'(x)$.

3 Nonlinear auto-companding method

In this section, we shall present how to construct the companding functions automatically according to error distribution.

3.1 Principle of nonlinear auto-companding

The objective of the nonlinear companding is to control the modeling error distribution to satisfy certain specifications provided by designers. For instance, the relative simulation error is needed to be constant at different circuit output values. The initial modeling error distribution, obtained through approximating the circuit input-output function by wavelet collocation method in Ref. [8], can impart useful information to control the error distribution. As discussed in section 2, in order to obtain uniform error distribution, we shall increase the singularities of wavelet basis functions in those regions where the modeling error is large. That is to say, we shall increase the value of $g'(x)$ in those regions. Contrarily, the modeling error will be enlarged in those regions where the singularities of wavelet basis functions are reduced. That offers us an idea to construct the nonlinear companding function automatically according to the initial error distribution.

In the following, we will show how to automatically construct proper companding function step by step according to the initial error distribution.

3.2 Construction of companding function

In practical application, we require the developed behavioral model to have a constant relative error at different circuit output values, since the relative error is a suitable criterion for evaluating the accuracy of behavioral models. Under such modeling requirement, we can construct the nonlinear companding function $l = g(x)$ by the following four steps.

(1) Obtain the initial error distribution. When the input-output function $f(x)$ is expanded by wavelet basis functions, the initial relative error distribution is obtained. The relative error is defined as in Eq.(3).

$$\mathrm{Err} = \frac{|f(x) - \hat{f}(x)|}{|f(x) + \alpha|} \tag{3}$$

where $\hat{f}(x)$ is the initial approximation of $f(x)$. α is a small positive value to guarantee that when $f(x)$ is approaching zero, the relative error will not go to infinity. α is set to 1% of the maximum value of $f(x)$ in following examples. Apparently, the relative error can be exploited to decide in which region the singularity of basis function should be increased to reduce the original error.

(2) Extract the envelope of the error distribution. Assuming there exist two adjacent small regions $[x_1, x_2]$ with minimal error e_1 and $[x_2, x_3]$ with maximal error e_2 as shown in Fig. 1(a). If $e_2 \gg e_1$, in the companding procedure, we will reduce the singularities of the basis functions in the region $[x_1, x_2]$ and increase the singularity of the basis functions in the region $[x_2, x_3]$. As a result, error e_1 is increased and e_2 is decreased after companding. However, if the companding function is not properly designed, the approximation errors after companding may result in $e_1 \gg e_2$, as shown in Fig. 1 (b). Such a phenomenon is called ultra-companding

which makes it hard to control the uniform modeling error distribution. Therefore, we will not directly choose the original error distribution as $g'(x)$. In this step, in order to minish the exacerbation of uniform error distribution, we extract the envelope of the initial error distribution to reduce the difference of errors in two adjacent small regions. As shown in Fig. 1, the difference between error e'_1 (obtained via linear interpolation) and e_2 is reduced after envelope extraction.

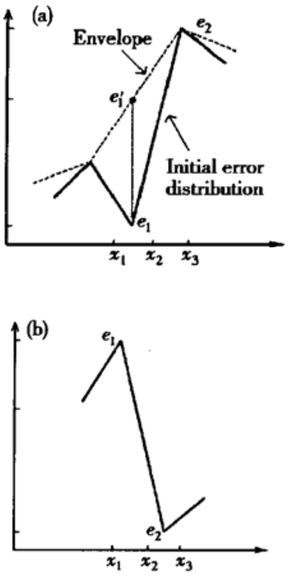

Fig. 1 (a) Envelope extraction; (b) Approximation errors after companding

(3) Build the prototype of nonlinear companding function. As shown in Eq. (2), $g'(x)$ means the singularity change of wavelet basis functions. Usually, since finite numbers of basis functions are employed to model a circuit's behavior, we hope the change of singularity of wavelet basis functions is restricted in a certain range $[p_A, p_B]$ to control the companding degree, where $0< p_A < 1< p_B$. In practical application, we set the range in interval [0.5, 2.0]. In this step, the envelope obtained in step 2 is mapped into the interval $[0.5, p]$, where p is a parameter restricting the singularity of wavelet basis functions and its value is to be determined by an optimization process in step 4. Then, assume $e(x)$ is the function of mapped envelope, the prototype of the companding function $l= g(x)$ is built as given in Eq. (4).

$$g(x) = x_A + \frac{1}{G}\int_{x_A}^{x} e(t)\,\mathrm{d}t \tag{4}$$

Since $e(x) > 0$, $g(x)$ is monotonically increasing and can satisfy constraint (2) of the companding function in section 2. Constant G in Eq. (4) is defined as

$$G = \frac{1}{x_B - x_A}\int_{x_A}^{x_B} e(t)\,\mathrm{d}t \tag{5}$$

which makes $g(x)$ satisfy constraint (1) in section 2.

(4) Refine the prototype of $l= g(x)$ repeatedly, to meet the given requirements of error distribution. In this step, we can use the merit function in Ref. [5] to optimize parameter p by the Golden Section Search method[9]. As long as the minimum value of the merit function is reached, the optimal p is found and consequently the proper companding function $l= g(x)$ is determined.

4 Numerical experiments

In this section, a SI memory cell, a SI delay cell and a 4th order SI filter are modeled by different modeling methods and are simulated to examine the validity of the proposed modeling method. In this paper, we prefer to use the wavelet basis functions in Refs. [7, 8], because they are proved to have a high convergence rate.

4.1 Modeling methodology

For modeling a SI circuit, we first partition the SI circuit into building blocks. Then each building block is modeled by representing its non-ideal input-output relation as a nonlinear function. After that, we construct the behavioral model of the SI circuit by a signal-flow-graph (SFG), which is derived from the circuit topology. Such a SFG-based model is then simulated by MATLAB SIMULINK to verify the accuracy of the developed model.

The input-output relation of a building block circuit is acquired by SPICE transient simulation. Taken the SI memory cell in Fig. 2 as an example, we apply an input signal covering both small and large signal range in Fig. 3 to this circuit. The memory cell will sink the input signal I_{in} at phase Φ_1 and output the signal at phase Φ_2, which consists of non-ideal errors. In phase Φ_1, the output current is zero. The output current I_{out} is obtained in Fig. 4 (an enlarged potion). By sampling the input and

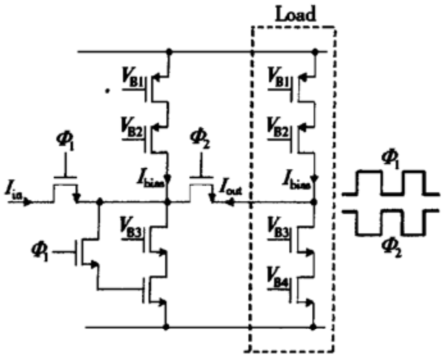

Fig. 2　SI memory cell

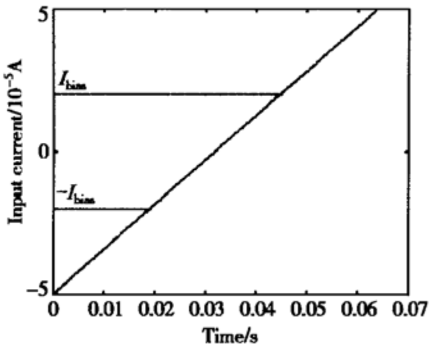

Fig. 3　Input signal for transient simulation of SPICE

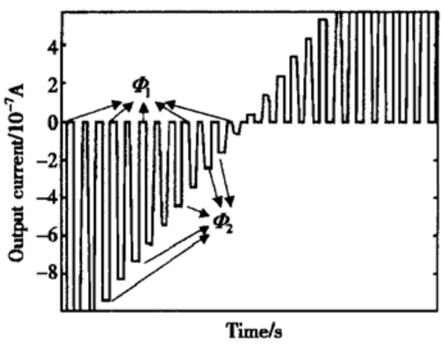

Fig. 4　An enlarged potion of output current

output currents at all phase Φ_2 time points (dots in Fig. 4), we obtain the input-output relation of the SI memory cell in Fig. 4. By applying the wavelet method proposed in the last section, the behavioral model of the memory cell can be constructed.

4.2　Simulation results

4.2.1　Switched-current memory cell

A SI memory cell is shown in Fig. 2 with $I_{bias} = 20\mu A$ and its input-output relation is shown in Fig. 5. The output current I_{out} is restricted by the bias current I_{bias} under large signal input. In order to model the effect of large input, the linear input signal is ranged in [$-50\mu A$, $50\mu A$] to obtain the input-output relation of the SI memory cell.

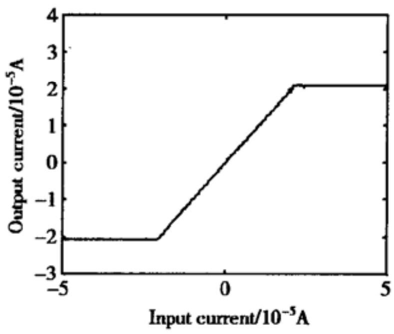

Fig. 5　Input-output function of SI memory cell

At first, three methods, conventional wavelet expansion without companding, nonlinear companding method in Ref. [5] and the proposed auto-companding method, are employed to approximate the input-output relation in Fig. 5 to build the behavioral model of the SI memory cell. The whole relative error (as defined in Eq. (3)) distributions are shown in Fig. 6 and the modeling results are listed in table 1.

Table 1　Modeling results of SI memory cell

	Maximum relative error	Mean square error	Number of basis function used
No companding	8.59%	0.0139	15
Companding method in Ref. [5]	6.33%	0.0065	15
Proposed auto-companding	3.64%	0.0041	15

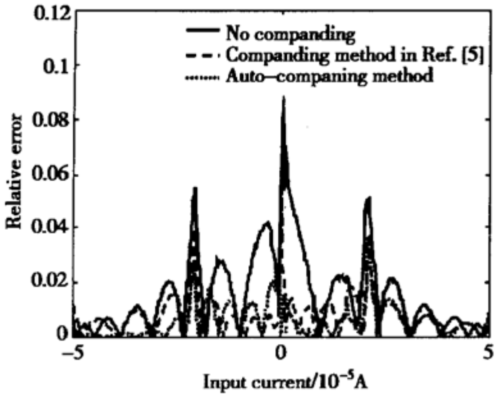

Fig. 6　Relative error distribution of modeling

Then, we test the above three different behavioral models of the SI memory cell with sinusoidal input of different amplitude $\pm i\mu A$ ($i = 5, 10, \Lambda, 50$). Figure 7 depicts the relative simulation error Err_R, defined in Eq. (6), of the three different approaches with 15 basis functions.

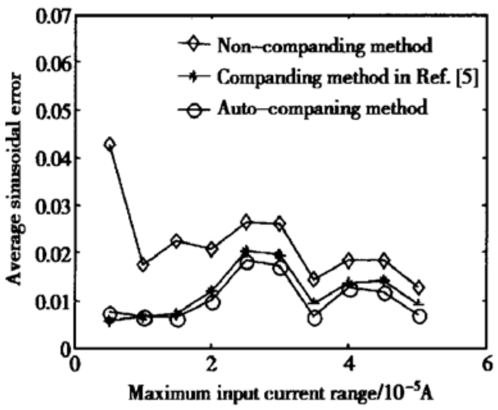

Fig. 7　Relative simulation error

$$Err_R = \sqrt{\frac{\int (y_{SPICE} - y_{Model})^2 dt}{\int (y_{SPICE})^2 dt}} \tag{6}$$

where y_{SPICE} is the simulation result by SPICE and y_{Model} is the result by the developed behavioral model.

Note that the proposed nonlinear auto-companding method is the most efficient one to control the distribution of the modeling error. Furthermore, the modeling error of the proposed method is less than that of the other two methods.

4.2.2 Switched-current delay cell

The input-output relation of SI delay cell (in Fig. 8) with $I_{bias} = 20\mu A$ is shown in Fig. 9, which is obtained by transient analysis of SPICE with a linear input signal ranging in $[-50\mu A, 50\mu A]$.

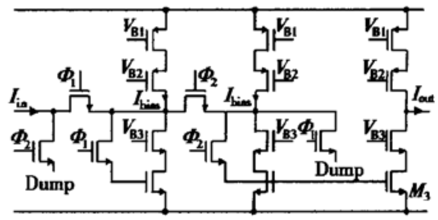

Fig. 8　Schematic of the SI delay cell

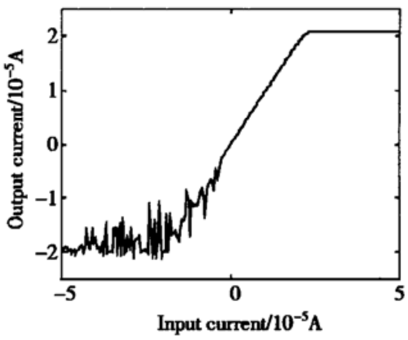

Fig. 9　Input-output relation of SI delay cell

We approximate the input-output relation in Fig. 9 by three different methods, which are the wavelet collocation method without companding, nonlinear companding in Ref. [5], and the proposed auto-companding method, to build the behavioral model of SI delay cell. The modeling results are given in Table 2.

Table 2　Modeling results of SI delay cell

	Maximum relative error	Mean square error	Number of basis functions used
No companding	0.8930	0.0556	60
Companding method in Ref. [5]	0.5672	0.0431	60
Proposed auto-companding	0.3031	0.0170	60

Compared with the other two methods, our proposed approach in this paper is more efficient to control the error distribution and obtains better modeling accuracy using the same numbers of wavelet basis functions.

4.2.3 Switched-current filter

A 4th order low-pass Butterworth SI filter is behaviorally modeled based on memory cells by a signal-flow-graph, which is derived from the circuit topology in Ref.[1]. Using the memory cell model developed above, we simulate the SFG-based filter model by MATLAB SIMULINK. The simulation results are discussed in the followed three aspects.

(1) Time domain response. The SFG-based filter model is simulated with a sinusoidal input of frequency 1kHz and amplitude of $\pm 10\mu A$. Figures 10 and 11 give the time-domain simulation results obtained from SPICE and the four different models. The simulation errors of these methods are given in table 3. The proposed approach is the most accurate one.

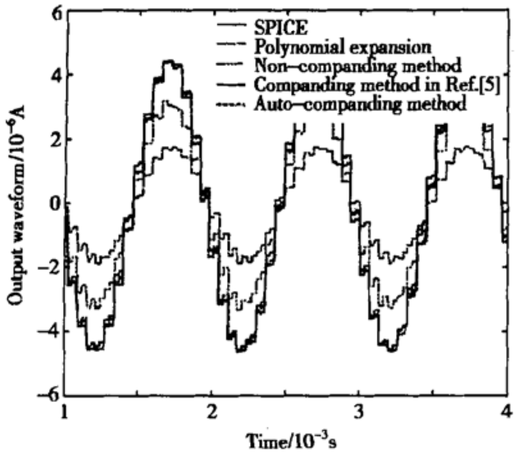

Fig.10 Time domain response of SI filter

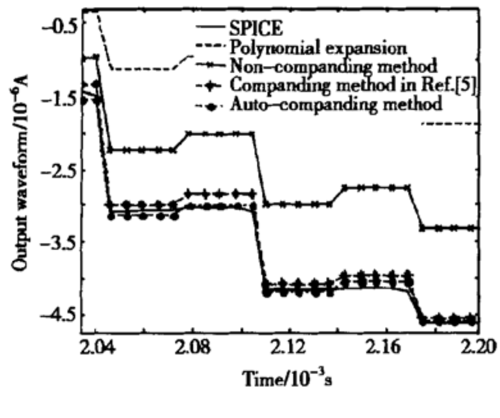

Fig.11 An enlarged potion of time domain response

Table 3 Simulation results of SI filter

	Average relative error*	Simulation time
SPICE	——	384.1s
Polynomial expansion	5.376×10^{-1}	2.0s
Non-companding	2.147×10^{-1}	5.07s
Companding method in Ref.[5]	7.347×10^{-2}	2.83s
Propose auto-companding method	5.432×10^{-2}	3.01s

* Average relative error is defined as the same as in Eq.(6).

(2) Frequency domain response. We simulate the filter model with sinusoidal inputs of amplitude $\pm 10\mu A$ at different frequencies to achieve the frequency domain response of the SI filter. Figure 12 depicts the frequency response obtained from SPICE and four kinds of different models. Note that again the proposed method works better than the other three in terms of accuracy.

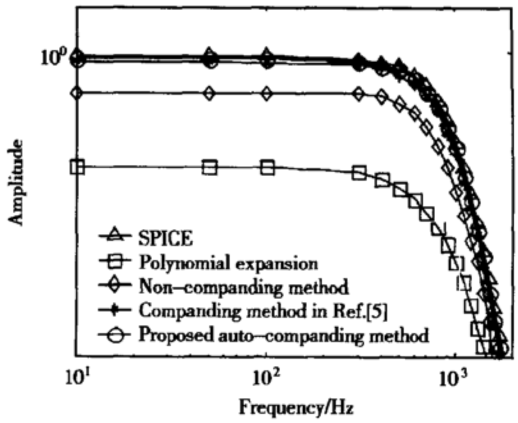

Fig.12 Frequency domain response of the 4th order filter

(3) Simulation speed. All the time-domain simulations are performed on a Pentium Ⅲ-550 computer. Transient simulation is performed for the 4th order SI filter in interval [0ms, 5ms]. The simulation time consumed by these models is listed in table 3. The overall speed up with behavioral models is about two orders in time domain compared with SPICE.

5 Conclusions

In this paper, we propose a wavelet collocation method with nonlinear auto-companding for behavioral modeling of SI circuits. With the proposed method, the companding function can be constructed automatically according to the initial error distribution obtained by conventional wavelet expansion. Since both the small signal effect and the large signal effect are modeled in a unified formulation by the proposed method, the process of modeling and simulation is eased greatly. As a general-purpose approach, it is more flexible to model the building blocks of SI circuits than the method in Ref. [5]. Compared with other published approaches, the proposed auto-companding approach can efficiently control the error distribution and reduce the modeling error. The effectiveness of the proposed method has been demonstrated in the numerical examples.

References

[1] Toumazou C, Hughes J, Battersby N. Switched-currents: an analogue technique for digital technology. Peter Peregrinus Ltd, 1993

[2] Yufera A, Rueda A. Studying the effects of mismatch and clock-feedthrough in switched-current filters using behavioral simulation. IEEE Trans Circuits and Systems Ⅱ: Analog and Digital Signal Processing, 1997, 44(12): 1058

[3] Zeng X, Wang W, Shi J L, et al. Analog behavioral modeling of switched-current building block circuits. Chinese Journal of Electronics, 2001, 10(2): 223

[4] Schneider E, Fiez T. Simulation of switched-current systems. IEEE International Symposium on Circuits and Systems, 1993: 1420

[5] Li Xin, Zeng Xuan, Zhou Dian, et al. Behavioral modeling of analog circuits by wavelet collocation methods. Proc IEEE/ACM ICCAD, 2001: 65

[6] Chui C K. An introduction to wavelets. New York: Academic Press, 1992

[7] Zhou Dian, Cai Wei, Zhang Wu. An adaptive wavelet method for nonlinear circuit simulation. IEEE Trans Circuits and Systems-I, 1999, 46: 931

[8] Zhou Dian, Cai Wei. A fast wavelet collocation method for high-speed circuit simulation. IEEE Trans Circuits and Systems-I, 1999, 46: 920

[9] Press W, Flannery B, Teukolsky S, et al. Numerical recipes in C the art of scientific computing. New York: Cambridge University Press, 1998

非线性自动压扩的开关电流电路行为级建模方法*

王 伟[1] 曾 璇[1] 陶 俊[1] 苏仰峰[2] 唐璞山[1]

(1 复旦大学微电子系 专用集成电路与系统国家重点实验室, 上海 200433)

(2 复旦大学数学系, 上海 200433)

摘要: 提出了一种新的基于非线性压扩函数自动构造的开关电流电路行为级建模方法, 从而简化电路的建模和仿真. 与原有的建模方法相比, 该方法不仅可以对模型的误差分布进行有效地调控, 而且能够降低模型的误差. 为了验证本文所提出的行为级建模方法, 对几种开关电流电路进行了建模和模拟试验.

关键词: 小波配置方法; 行为级模型; 开关电流电路; 非线性自动压扩

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王 伟 男, 博士研究生, 研究方向为模拟电路设计和行为级模型方法研究.

曾 璇 女, 教授, 研究方向包括 VLSI 高速互连电路分析与设计、VLSI 物理设计和模拟电路设计自动化.

陶 俊 女, 研究生, 研究方向为模拟电路的行为级模型方法和仿真.

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