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Analysis of Characteristic Impedance and Effective Dielectric Constant of Asymmetrical Coplanar Waveguide with Finite Dimensions

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Abstract: Conformal mapping techniques are used to obtain the closed-form expressions of the characteristic impedance and effective dielectric constant of the asymmetrical coplanar waveguide with finite line dimensions and substrate thickness. These closed-form expressions are generally applicable to design various particular types of coplanar waveguide transmission lines.

Key words: coplaner waveguide; characteristics impedance; effective dielectric constant
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1 Introduction

Coplanar waveguide transmission lines have become quite attractive due to its application to microwave or millimeter-wave integrated circuits and devices. Calculations of characteristic impedance and effective dielectric constant of coplanar waveguide by conformal mapping techniques were first presented by Wen etc[1], whose analysis leads to a simple analytical expression for the line parameters. But it is valid only in the case of infinite substrate thickness and dimensions for the two-side ground strips. Several trials have been to calculate line parameters considering the effect of finite substrate thickness and line dimensions[2,3]. Veyres at last got a closed-form expression for the characteristic impedance and effective dielectric constant of symmetrical coplanar waveguide with finite line dimensions and substrate thickness. However, the asymmetrical structure of the coplanar waveguide

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makes the design be of additional flexibility in many applications, such as the integrated circuits and microwave probe. Therefore, it is necessary to investigate the characteristics of asymmetrical coplanar waveguide with finite line dimensions and substrate thickness.

In this paper, the expressions for the line capacitance of the asymmetrical coplanar waveguide transmission lines with finite line dimensions and substrate thickness are derived for the first time using conformal mapping techniques and the closed-form expression of characteristics impedance and its effective dielectric constant are also presented.

2 Transformations of Conformal Mapping for Asymmetrical Coplanar Line with Finite Line Dimensions and Substrate Thickness

A zeroth-order quasi-static approximation is employed, referring to the linear configuration of the asymmetrical coplanar waveguide transmission lines with finite line dimensions and substrate thickness which is shown in Fig. 1. Our analysis is based on the assumption that, when the line dimensions are finite, the line capacity between the center strip and the two ground ones would be equal to the sum of the line capacity C_1 in the absence of the dielectric and the line capacity C_2 assuming that all the electric field is concentrated in the dielectric whose relative permittivity is $(\epsilon_r - 1)$. So we can get the expression of the capacitance in two parts individually.

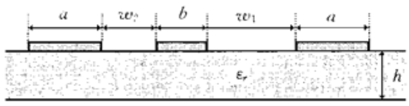

FIG. 1 Asymmetrical Coplanar Waveguide Line with Finite Line Dimensions and Substrate Thickness

The procedures of the conformal mapping to get C_1 and C_2 are expressed in Fig. 2 and Fig. 3 respectively.

2.1 Calculation of Capacity C_1

The line capacity of the asymmetrical coplanar waveguide without substrates can be evaluated analytically by a repeated application of conformal mapping. A series of transformations are shown in Fig. 1. As the first step towards the solution of this kind of boundary-value problem, we map the boundary of X plane (Fig. 2 (a)) into the real axis in T plane (Fig. 2 (b)) using the mapping functions

$$t = j\frac{x}{x_3}\sqrt{(x_3^2 - x_1^2)/(x_1^2 - x^2)} \quad \text{for } |x| > |x_1| \tag{1a}$$

and

$$t = \frac{x}{x_3}\sqrt{(x_3^2 - x_1^2)/(x^2 - x_1^2)} \quad \text{for } |x| \leqslant |x_1| \tag{1b}$$

The configuration in T plane in turn can be mapped into another boundary value problem in Z plane in turn (Fig. 2(c)) using the function:

$$Z = \int_0^t \frac{dt}{\sqrt{(1 - t^2)(1 - k_1 t^2)}} \tag{2}$$

Where

$$k_1 = \frac{t_3}{t_2} = \frac{x_3}{x_2}\sqrt{(x_2^2 - x_1^2)/(x_3^2 - x_1^2)}$$
$$\text{for } (0 < k_1 < 1)$$

Then it also can be easily transformed into another boundary value problem in U plane (Fig. 2(d)) using the mapping functions:

$$U_4 = -j(Z_1 + Z_4) \quad (3a)$$
$$U_1 = -2jZ_1 \quad (3b)$$
$$U_2 = jZ_2 \quad (3c)$$

This configuration in U plane can be further mapped into following boundary value problem in S plane (Fig. 2(e)) using the following function:

$$S = \mathrm{Sn}(U, k_2) \quad (4a)$$

Where $\mathrm{Sn}(u, k)$ is the Jacobi elliptic function. k_2, the modulus of elliptic integral, is given by the following relationship:

$$\frac{K(k_2)}{K'(k_2)} = 2\frac{K'(k_1)}{K(k_1)} \quad (4b)$$

Finally the configuration in S plane can be transformed into a rectangle in W plane (Fig. 2 (f)) using the following function:

$$\frac{dw}{ds} = \frac{A}{\sqrt{(s^2 - s_1^2)(s^2 - s_2^2)}} \quad (5)$$

Please note that the determinantal equations:

$$\frac{w_4}{w_2} = \frac{K(k_3)}{K'(k_3)} \quad (6)$$

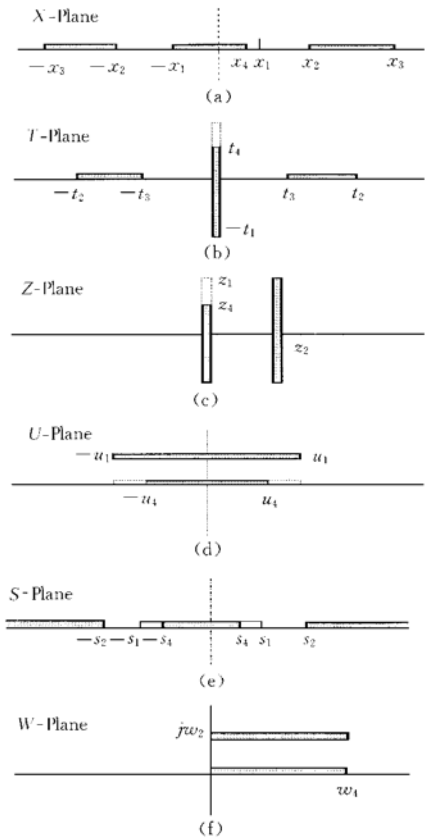

FIG. 2 A Series of Conformal Mapping Transformations for Calculation of Line Capacitance C_1 in Absence of Dielectric

$$\frac{F(k_4, k_2)}{K(k_2)} = \frac{U_4}{U_1} \quad (7)$$

$$k_3 = k_2k_4 \quad (8)$$

Where $K(k)$ is the complete elliptic integral of the first kind, and $K'(k) = K(k')$. $k' = \sqrt{1 - k^2}$ is the complementary modulus of elliptic integral. $F(k, m)$ is the elliptic function of the first kind.

Hence, the capacitance per unit length of the asymmetrical coplanar line is twice as much as that of the mapping of one-quarter section in the air and equal to:

$$C_1 = 2\epsilon_0\frac{K(k_3)}{K'(k_3)} \quad (9)$$

2.2 Calculation of Capacity C_2

The boundary problem X plane (Fig. 3 (a)) can be transformed into another boundary value problem in T plane (Fig. 3 (b)) using the mapping function

$$t = \sinh\left(\frac{\pi x}{2h}\right) \tag{10}$$

Thus the finite thickness of the substrate can be transformed to the infinite thickness by conformal mapping. This boundary value problem in T plane can be mapped into a rectangle in the final W plane using the same transformations utilized in the structure of Fig. 2.

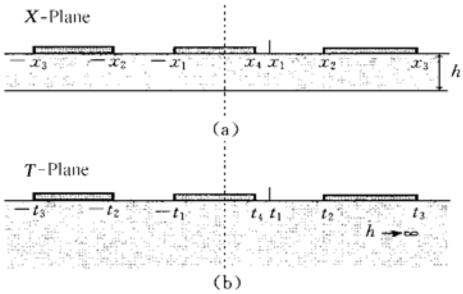

FIG. 3 Conformal Mapping Transformations for Calculation of Line Capacitance C_2

The capacitance per unit length of the asymmetrical coplanar line is twice the value of the mapping of the half section in W' plane considering that all the electric field lines are concentrated in the dielectric whose relative permittivity is $(\epsilon_r - 1)$,

$$C_2 = \epsilon_0(\epsilon_r - 1)\frac{K'(k_{-3})}{K(k_{-3})} \tag{11}$$

Where the meaning of k_{-3} is just like that of k_3 in (C_1), but the expressions are different.

Hence, the effective dielectric constant of the asymmetrical coplanar line with finite line dimensions and substrate thickness can be defined as:

$$\epsilon_{\mathrm{eff}} = \frac{(C_1 + C_2)}{C_1} = 1 + \frac{C_2}{C_1} \tag{12}$$

From transmission line theory, it is known that

$$Z_0 = \frac{1}{(C_1 + C_2)u_{\mathrm{ph}}} \tag{13}$$

where u_{ph} is the phase propagation velocity.

Therefore we obtain the effective dielectric constant of the asymmetrical coplanar line.

$$\epsilon_{\mathrm{eff}} = 1 + \frac{\epsilon_r - 1}{2}\frac{K(k_{-3})}{K'(k_{-3})}\frac{K'(k_3)}{K(k_3)} \tag{14}$$

The line characteristic impedance Z_0 can be obtained from the form:

$$Z_0 = \frac{60\pi}{\sqrt{\epsilon_{\mathrm{eff}}}}\frac{K(k_3)}{K'(k_3)} \tag{15}$$

Hence, Z_0 and ϵ_{eff} can be calculated easily by the aid of computer for any finite line configuration and substrate thickness.

3 Calculation Results

The asymmetrical coplanar waveguide with finite line dimensions and substrate thickness has been studied with a range of geometric parameters. In Fig. 4, the finite center strip width, ground plane width and the substrate thickness were kept constant. Figure 4 shows the typical results and gives the characteristic impedance and the effective dielectric constant as a function of the width ratio w_1/w_2 with the relative dielectric constants as parameters respectively for the asymmetrical coplanar waveguide in Fig. 1.

The characteristic impedance increases when the width ratio w_1/w_2 increases, but the effective dielectric constant will decrease though the effect of the width ratio w_1/w_2 to the effective dielectric constant is very small. The circle data dots are calculated by the formula of symmetrical coplanar line from the reference [3] which quite agree with the curves we calculated when the width ratio w_1/w_2 is one. Figure 4 shows that, when the relative permittivity increases, the characteristic impedance will decrease but the effective dielectric constant increases.

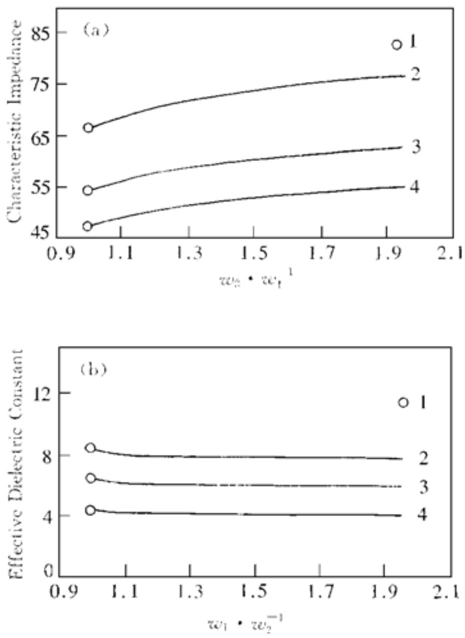

FIG. 4 (a) Characteristic Impedance and (b) Effective Dielectric Constant as a Function of Width Ratio w_1/w_2 with Relative Dielectric Constant and Finite Ground Plane Width as Parameters Respectively (Where 1 for $w_1/w_2 = 1$; 2 for $\epsilon_r = 9.6$; 3 for $\epsilon_r = 15$; 4 for $\epsilon_r = 20$)

The calculated results also show that the characteristic impedance and the effective dielectric constant become smaller as the width of the center strip increases, or the ground strip width increases. When the substrate thickness increases, the characteristic impedance will decrease while the effective dielectric constant will increase.

4 Conclusions

In conclusion, the closed-form expressions of impedance and effective dielectric have been obtained for the asymmetrical coplanar waveguide with finite line dimensions and substrate thickness by using conformal mapping techniques. These closed-form expressions are quite common and are applicable to design various particular types of coplanar waveguide transmission lines.

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