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New Method for Determining Characteristic Parameters of Normal Distribution

MU Fu-chen(穆甫臣), TAN Chang-hua(谭长华) and XU Ming-zhen(许铭真)

(*Institute of Microelectronics, Peking University, Beijing* 100871, *China*)

Abstract: By proportional differentiating cumulative distribution function of normal distribution, the spectroscopy characteristics are found. The characteristic parameters can be extracted directly from the height and position of the spectroscopy peaks. On this basis, a new method for determining these parameters of normal distribution is developed. This method can be applied to microelectronics reliability study.

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1 Introduction

Normal distribution is widely used in every fields, especially in microelectronics. It is a popular tool in microelectronics reliability study. The characteristic parameters, μ and σ, are key to this distribution. Some estimate methods, such as Maximum Likelihood Estimate (MLE), Minimum Variance Unbiased Estimate (MVUE) etc., have been developed. But they are all complicated in computation. XU, TAN *et al.* have developed a proportional difference theory[1,2]. By using difference analysis method, some monotonic function can be changed into a new function with spectroscopy characteristics. These peaks (height and position) relate with the characteristic parameters of the system directly. Therefore, the

MU Fu-chen (穆甫臣) was born in 1970 in Hebei province, China. He is now working for his Ph. D. Degree in small dimension device physics, in the Department of Computer Science and Technology, Peking University. His interests include small-size MOS devices and reliability, device modeling and characterization, time-dependent dielectric breakdown, hot-carrier effects, electromigration and reliability statistics.

TAN Chang-hua(谭长华) was born in 1936. He graduated from Moscow University, USSR. Currently, he is a professor in Institute of Microelectronics, Peking University. His current interests include physics and reliability of small dimensional devices.

XU Ming-zhen(许铭真) was born in 1939. She graduated from the Department of Physics, Peking University, China. Currently, she is a professor in Institute of Microelectronics, Peking University. Her current interests include physics and characterization of small dimensional devices and reliability of semiconductor material and devices.

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characteristic parameters of the complicated system can be extracted from these peaks. Using Proportional Difference Operator (PDO), proportional difference characteristics of the cumulative function of normal distribution were analyzed. The proportional difference of the cumulative distribution is a spectroscopy function. Based on these characteristics, a new method——proportional Difference Estimate(PDE) method, for direct estimation of the characteristic parameters of normal distribution was developed.

2 Difference Damped Function Theory[1,2]

If $f(x)$ is strictly monotonic over $[0, \infty)$, continuous and differentiable on $[0, \infty)$, and if $f(0) = A$, and $\lim_{x \to \infty} f(x) = B$, where A and B are two constants, then there exists $x_p \in [0, \infty)$, such that

$$F'(x_p) = \left.\frac{dF}{dx}\right|_{x = x_p} = 0 \tag{1}$$

where

$$F(x) = \Delta_p f(x) = f(kx) - f(x) \quad k > 1 \tag{2}$$

is the proportional difference of $f(x)$. Δ_p is the proportional difference operator. k is a constant larger than 1.0. The proof is straightforward from the arithmetic theorem of continuous functions. If $f(kx)$ and $f(x)$ are strictly monotonic, continuous and differentiable on $[0, \infty)$, respectively, then

$$F(0) = 0, \text{ for } x = 0 \tag{3}$$

and

$$\lim_{x \to \infty} F(x) = 0, \text{ for } x \to \infty \tag{4}$$

by Rolle's theorem[3], there must exist $x_p \in [0, \infty)$, such that $F'(x_p) = 0$. Therefore, the above theorem is a common mathematical basis.

3 Proportional Difference Estimate(PDE)

The probability density function of normal distribution is defined as

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \tag{5}$$

The cumulative distribution function is

$$F(t) = \int_0^t \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \tag{6}$$

where μ and σ are two constants: position and scale factors, respectively. They are also called as the mean value and standard deviation of the normal distribution. It is obviously that the cumulative function (Equation (6)) is monotonic on $[0, \infty)$, and $F(0) = 0$, $\lim_{t \to \infty} F(t) = 1$ are two constants. Therefore, the proportional difference of $F(t)$ over $[0, \infty)$ must exist a peak at $t = t_p$.

According to Leibnitz's rule[4], proportional differentiating Equation (6) yields,

$$\Delta_p F(t) = F(k_p t) - F(t) = \int_t^{k_p t} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \tag{7}$$

where k_p is a constant larger than 1. The peak condition of $\Delta_p F(t)$ is

$$\left.\frac{d\Delta_p F(t)}{dt}\right|_{t=t_p} = 0 \tag{8}$$

Combining (7) and (8), rearranging terms yields

$$(k_p^2 - 1)t^2 - 2\mu(k_p - 1)t - 2\sigma^2 \ln k_p = 0 \tag{9}$$

Solving (9) yields

$$t_p = \frac{(k_p - 1)\mu \pm \sqrt{(k_p - 1)^2\mu^2 + 2(k_p^2 - 1)\sigma^2 \ln k_p}}{k_p^2 - 1} \tag{10}$$

Since $t \in [0, \infty)$, the negative t_p should be deleted. Let k_p be a constant little larger than 1. Make $k_p = 1 + \xi$, where $\xi \ll 1$. So $\ln k_p = \ln(1 + \xi)$ can be approximated as

$$\ln(1 + \xi) \approx \xi. \tag{11}$$

Substituting k_p by $1+\xi$, $\ln(1+\xi)$ by ξ the ratio of the two parts in the square root can be written as

$$\frac{2(k_p^2 - 1)\sigma^2 \ln k_p}{(k_p - 1)^2\mu^2} \approx 2(2 + \xi)\left(\frac{\sigma}{\mu}\right)^2 \approx 4\left(\frac{\sigma}{\mu}\right)^2 \tag{12}$$

In reliability study, the standard deviation σ is much smaller than the mean value μ, so (12) is much smaller than 1. Therefore, $2(k_p^2 - 1)\sigma^2 \ln k_p$ can be approximated as 0 with respect to $(k_p - 1)^2\mu^2$. Therefore, the peak position is

$$t_p \cong \frac{2\mu}{k_p + 1} \cong \mu \tag{13}$$

Combining (7) and (13) yields

$$\Delta_p F(t_p) = \int_{t_p}^{k_p t_p} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{k_p - 1}{\sqrt{2\pi}\sigma} t_p \tag{14}$$

Thus, the standard deviation is got from (14),

$$\sigma = \frac{k_p - 1}{\sqrt{2\pi}\Delta_p F(t_p)} t_p \tag{15}$$

Like the relationship between σ and $f(t)$, σ is proportional the reciprocal of the peak height, $\Delta_p F(t_p)$. $F(t)$ (Equation (6)) is a monotonic function over $[0, \infty)$, and has no peak. But the proportional differnece of $F(t)$ has a peak. This peak (position and height) relates with the characteristic parameters μ and σ directly. Therefore, μ and σ can be derived from the peak. In practice, the first thing we get is the cumultive failure rate ($F(t)$), then we get the probability density $f(t)$ by differentiating $F(t)$. Therefore, to estimate the characteristic parameters of normal distribution, PDE method is direct and convenient with respect to that through probability density function $f(t)$.

In microelectronics reliability study, t (in Equation (5) and (6)) commonly refers to the time being (e.g. the stress time). Accordingly, the parameters μ is the mean time (e.g. the lifetime of the devices). Therefore, PDE method can be applied to reliability study.

Here is an application of this method. Figure 1 shows the distribution of activation energy of aluminum electromigration[5]. In this figure, the proportional difference characteristics of $F(Q)$ are also shown. From this figure the peak position and height were obtained: $Q_P = 0.4221\text{eV}$, $\Delta_P F(Q_P) = 0.0726$. Putting them into (13) and (15) yields, $\bar{Q} = \mu = 0.422\text{eV}$, $\sigma = 0.023$. The mean activation energy $\bar{Q}$ and standard deviation σ are determined to be 0.423eV and 0.023, respectively, through the widely used graphical method[5]. Therefore, these results agree well with those through conventional method. It is important to notice that the variable in this example is activation energy Q, not stress time t. This is just an example to illustrate the PDE method. Certainly, this new method can be applied in every field where normal distribution is used.

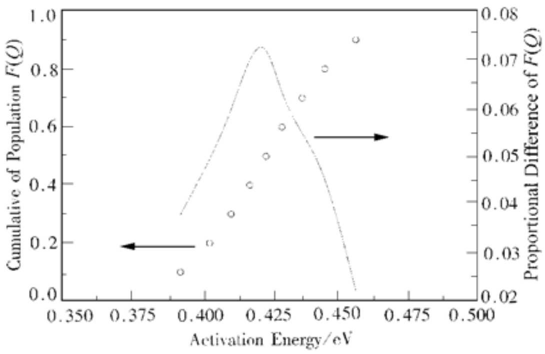

FIG. 1 Proportional Difference Characteristics of Normal Distribution of Activation Energy
Circles: Cumulative of Population $F(Q)$;
Line: Proportional Differenc Characteristics of $F(Q)$

Due to the approximation of Eq. (11), k_P must be small enough. The constant k_P is set to be 1.01 in this case. When k_P is smaller than 1.01, the estimated results are almost the same as those when k_P equals 1.01. Nevertheless, when k_P is larger than 1.01, the larger the k_P is, the larger the deviation is. But when k_P is too small, it is very difficult to estimate the characteristic parameters because a large noise will be induced in proportional difference of $F(t)$. Therefore, k_P being equal to 1.01 is the best condition for this PDE method.

4 Conclusion

The proportional difference of the cumulative distribution function of normal distribution has spectroscopy characteristics, and the characteristic parameters are related to the spectroscopy peak. A new method——PDE method, for direct estimation of the characteristic parameters of normal distribution, was developed on this basis. If experimental data agree with normal distribution, this method can determine the characteristic parameters directly and accurately. So PDE method is a useful tool in microelectronics reliability study, especially in metal electromigration field.

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